REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 9

(1) Show that A_n is generated by $s_1 s_i$, with $2 \leq i \leq n-1$. Here, as usual s_i denotes the simple transposition $(i, i+1)$.

(2) Given a representation $\rho : G \to GL(V)$ of a group, and an automorphism $\sigma : G \to G$, let ρ^σ denote the representation $\rho \circ \sigma : G \to GL(V)$.
   (a) What is the relationship between the character of ρ and the character of ρ^σ?
   (b) Show that, if σ is an inner automorphism, then ρ^σ and ρ are isomorphic as a representations of G.
   (c) If σ and τ are two automorphisms of G, and $\tau^{-1}\sigma$ is an inner automorphism of G, then ρ^σ and ρ^τ are isomorphic as representations of G.

(3) For $w \in A_n$, show that the conjugacy class of w in S_n is a union of two classes in A_n if and only if w has distinct odd parts.

(4) In the character table of A_4 compute $\chi^{\pm}_{(2,2)}$ at the three-cycles $(1,2,3)$ and $(2,1,3)$.

(5) In the character table of A_5, compute $\chi^{\pm}_{(3,1,1)}$ at the 5-cycles $(1,2,3,4,5)$ and $(2,1,3,4,5)$.

(6) Enumerate all the conjugacy classes and irreducible representations of A_8.

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