REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 8

(1) Show that the number of inversions in the transposition (i, j) is $2(j-1-1)+1$, so that $\epsilon((i,j)) = -1$.
(2) Let V and W be representations of S_n. Show that
$$\mathrm{Hom}_{A_n}(V, W) = \mathrm{Hom}_{S_n}(V, W) \oplus \mathrm{Hom}_{S_n}(V, W \otimes \epsilon).$$
(3) Let X be a set with an action of S_n. Given $x \in X$, show that the S_n-orbit of x is a union of two orbits for the action of A_n if and only if $\mathrm{Stab}_{S_n} x \subset A_n$.
(4) How many conjugacy classes does A_5 have? What are their cardinalities?
(5) What must be the dimensions of the irreducible representations of A_5?
(6) Let $p(n)$ denote the number of partitions of n. Let $p_{\mathrm{even}}(n)$ denote the number of partitions of n with an even number of even parts. Let $p_{\mathrm{dop}}(n)$ denote the number of partitions of n with distinct odd parts. Let $p_{\mathrm{sc}}(n)$ denote the number of self-conjugate partitions of n. Prove that:
$$p(n) + 3p_{\mathrm{sc}}(n) = 2p_{\mathrm{even}}(n) + 2p_{\mathrm{dop}}(n).$$
(7) For an odd integer $n > 2$, show that a n-cycle is conjugate to its inverse in A_n if and only if $\lfloor n/2 \rfloor$ is even.
(8) An element of A_n with cycle type λ, where λ has distinct odd parts is conjugate to its inverse if and only if
$$\sum_{i=1}^{l} \lfloor \lambda_i/2 \rfloor$$
is even.

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