REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 7

(1) Perform the combinatorial resolution of the partition representations of S_4 to contrcut all the irreducible representations of S_4. Use this to derive the character table of S_4 from first principles.

(2) Let G be a finite group, $\chi : G \to \mathbf{C}^*$ be a character of G. Let X and Y be G-sets. Given $x \in X$ and $y \in Y$, show that there exists a function $k : X \times Y \to \mathbf{C}$ with $k(x, y) \neq 0$ if and only if, for every $g \in G$ such that $g \cdot x = x$ and $g \cdot y = y$, $\chi(g) = 1$.

(3) Let λ be a partition of n, and $(\rho, \mathbf{C}[X_\lambda])$ be the corresponding partition representation. Show that $\det(\rho(w))) = 1$ for all $w \in S_n$ if and only if

$$\sum_{1 \leq i < j \leq l} \binom{n-2}{\lambda_1, \ldots, \lambda_{i-1}, \lambda_i - 1, \lambda_{i+1}, \ldots, \lambda_{j-1}, \lambda_j - 1, \lambda_{j+1}, \ldots, \lambda_l}$$

is even (this is a multinomial coefficient of $n-2$. In the lower row, the ith and jth parts of λ are reduced by 1).

(4) Show that $[S_n, S_n] = A_n$.

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