

REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 5

- (1) Show that $N_{\lambda\mu} \geq 2$ for $\mu' < \lambda$ (this is a strengthening of the Gale-Ryser theorem).
- (2) Show that the number of permutations in S_n with cycle type (n) is $(n-1)!$.
- (3) Show that every element of S_n is conjugate to its inverse.
- (4) Show that two elements of S_n that generate the same (cyclic) group are conjugate.
- (5) If w is a permutation in S_n with cycle type λ , show that the cardinality of its centralizer is

$$z_\lambda = \prod_i m_i! i^{m_i},$$

where for each $i = 1, 2, 3, \dots$, m_i is the number of times i occurs in the partition λ . Note that this means that the number of permutations with cycle type λ is $n!/z_\lambda$.

- (6) Recall that the *order* of a permutation w is the smallest positive integer n such that w^n is the identity. What is the order of a permutation of cycle type $(\lambda_1, \dots, \lambda_l)$?
- (7) Show that the number of permutations in S_n with k inversions is equal to the number of permutations in S_n with $\binom{n}{2} - k$ inversions.
- (8) How many permutations in S_n have exactly two inversions?