REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 4

(1) Find the 5×4 matrix A to which the VRSK algorithm would associate the SSYTs:

$$P = \begin{smallmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 \end{smallmatrix} \text{ and } Q = \begin{smallmatrix} 1 & 1 & 3 & 4 \\ 2 & 5 \end{smallmatrix}.$$

(2) Find the matrices A for which both P and Q have the same shapes as their types.

(3) If λ and μ are partitions such that $\mu \leq \lambda$ (in the reverse dominance order), then show that μ comes after λ in lexicographic (dictionary) order.

(4) List all the partitions of n in reverse lexicographic order:

$$\lambda^{(1)}, \ldots, \lambda^{(p)},$$

where p is the number of partitions of n. Define $p \times p$ matrices

$$M = (M_{\lambda^{(i)},\lambda^{(j)}}), \text{ and } K = (K_{\lambda^{(i)}\lambda^{(j)}}).$$

Compute M and K for $n = 2, 3, 4$. Verify that $M = K'K$.

(5) Recall that the conjugate of a partition λ is defined by

$$\lambda'_i = \#\{j \mid \lambda_j \geq i\}.$$

Show that conjugation reverses dominance:

$$\lambda \leq \mu \text{ if and only if } \mu' \leq \lambda'.$$

(6) Using the notation of problem (4), given n, define a $p \times p$ matrix:

$$N = (N_{\lambda^{(i)},\lambda^{(j)}}), \text{ and } J = (\delta_{\lambda^{(i)},\lambda^{(j)\prime}}).$$

Here δ is the Kronecker delta symbol. For $n = 2, 3, 4$, verify that $N = K'JK$.

Date: 15th June 2017.