REPRESENTATION THEORY OF FINITE GROUPS

PROBLEMS SET 10

(1) For a permutation $w \in S_n$ define:

$$\sigma(w) = \prod_{1 \le i < j \le n} \frac{i-j}{w(i)-w(j)}.$$

Show that $\sigma(w)$ coincides with the sign character value $\epsilon(w)$.

(2) Show that the alternating group A_n is generated by the 3-cycles $(1,2,3),(1,2,4),\ldots,(1,2,n)$.

(3) If $n \ge 5$, show that there is no nontrivial conjugacy class in A_n with fewer than n elements.

(4) Given the character values of S_9:

$$\chi_{(5,1^4)}(w_{(9)}) = 1,$$
$$\chi_{(5,1^4)}(w_{(5,3,1)}) = 0,$$
$$\chi_{(3^3)}(w_{(9)}) = 0, \text{ and}$$
$$\chi_{(3^3)}(w_{(5,3,1)}) = -1,$$

compute the character values of A_9:

$$\chi^{\pm}_{(5,1^4)}(w^{\pm}_{(9)}) \text{ and } \chi^{\pm}_{(3^3)}(w^{\pm}_{(5,3,1)}).$$

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