

# Syllabus

- I (2 lectures; KNR) Lie algebras and homomorphisms, ideals; representations;  $\mathfrak{sl}_2\mathbb{C}$  and its representations.
- II (3 lectures but only 2 tutorials; UK) linear Lie algebras; derivations; automorphisms. Nilpotent and solvable Lie algebras; Engel's theorem; Lie's theorem. Cartan's criterion for solvability; semisimplicity; Killing form; Cartan's criterion for semisimplicity. (Proofs to be omitted; statements to be emphasized.)
- III (no lecture but 1 tutorial; SV) Basic multilinear algebra.
- IV (1 lecture; KNR) Complete reducibility of representations; Casimir element; Weyl's theorem; preservation of Jordan-Chevalley decomposition. (Proofs to be omitted; statements to be emphasized.)
- V (4 lectures; SV) Root systems: axiomatics, simple roots, Weyl group, bases, irreducibility; Classification: Coxeter graphs, Dynkin diagrams; Construction of root systems.
- VI (2 lectures; UK) Root space decomposition; maximal toral subalgebras; orthogonality, integrality, and rationality properties.
- VII (1 lecture; AKS) Universal enveloping algebras and the PBW theorem; free Lie algebras.
- VIII (2 lectures; AKS) Weights and maximal vectors; standard cyclic modules; finite dimensional modules: weight strings, weight diagrams.
- IX (3 lectures; PB) Central extension of loop algebras; affine algebras; root system; Weyl group; normalized invariant form.
- X (3 lectures; SK) Character and multiplicity formulas: Freudenthal's formula, formulas of Weyl, Kostant, and Steinberg. Harish-Chandra homomorphism.
- XI (1 lecture: KNR) Isomorphism theorem: root system determines the Lie algebra; Cartan subalgebras and their conjugacy; Borel subalgebras and their conjugacy; Automorphism groups. (Proofs to be omitted; statements to be emphasized.)
- XII-A (2 lectures; PB) Highest weight modules over affine Kac-Moody algebras.
- XII-B (2 lectures; KNR) Highest weight modules over affine Kac-Moody algebras (continued); Kac-Weyl character formula.
- XIII (5 lectures but only 4 tutorials; SV) Weight system of integrable highest weight modules; Macdonald's eta function identities.
- XIV (4 lectures but only 3 tutorials; RPS) Chevalley groups: Chevalley basis, construction of the groups; Kostant's theorem; Admissible lattices.
- XV (1 lecture; UK) Generators and relations for complex semisimple Lie algebras; Serre's theorem.

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UK: Upendra Kulkarni

SV: S. Viswanath

AKS: Anupam Kumar Singh

PB: Punita Batra

SK: Senthamarai Kannan

RPS: Ravindra Shukla