

Exercises on Chevalley Groups

Exercise 1. Exercises 25.1, 25.4, 25.5 from Humphreys book.

Exercise 2. Find a Chevalley basis for the Lie algebras of type A_n , C_n and D_n .

Exercise 3. Let $\{x_\alpha \mid \alpha \in \Phi\} \cup \{h_i\}$ be a Chevalley basis of a semisimple Lie algebra L . Show that $\exp(\text{ad}(x_\alpha)) \in \text{Aut}(L(\mathbb{Z}))$.

Exercise 4. Exercise 25.6 from Humphreys book.

Exercise 5. Prove Lemma 26.1 of Humphreys.

Exercise 6. Determine weight lattices of finite dimensional simple faithful modules of Lie algebras of type A_1 and A_2 .

Exercise 7. Let L be a classical matrix Lie algebra (i.e. of type A_l , B_l , C_l and D_l). Let $A \in L$ be a nilpotent matrix. Show that for any $B \in L$ we have $\exp(\text{ad}A)(B) = \exp(A)B\exp(-A)$.