

### Set 3

1. Prove that the function in the proof of Brouwer fixed pt. theorem is smooth. [Join  $f(x)$  to  $z$  and extend it to meet the sphere  $S^{n-1}$ ]
2. Prove that the fixed points in Brouwer's fixed point theorem need not be an interior point.
3. Find maps from the solid torus to itself not having any fixed pts. (Where does the proof of the Brouwer fixed point theorem fail?)
4. Prove that the Brouwer's theorem fails for the open ball  $|x|^2 < a$ .

### Set 4

- 1) Prove that there is a complex number  $z$  such that  $z^7 + \cos(|z|^2)(1 + 93z^4) = 0$ .
- 2) Prove that intersection is vacuous in contractible manifolds:  
If  $Y$  is contractible with  $\dim Y > 0$ ,  $X, Z$  submanifolds (compact) such that  $\dim X + \dim Z = \dim Y$ , the intersection number of  $X$  and  $Z$  is 0.
- 3) Prove that no compact manifold other than the one-point space is contractible.
- 4) Suppose  $f: X \rightarrow S^k$  is smooth with  $0 < \dim X < k$ . For any submanifold  $Z$  of complementary dimension,  $I_2(X, Z) = 0$ .

5. Prove that  $S^2$  and the torus  $T^2$  are not diffeomorphic.

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EXERCISE:  $Y$  compact submanifold of  $\mathbb{R}^N$ ,  $w \in \mathbb{R}^N$ .

As  $Y$  is compact  $\exists y \in Y$  closest to  $w$ .

Prove that  $w - y \in \nu_y(Y) = (T_y Y)^\perp$ .