REPRESENTATION THEORY ELECTIVE COURSE

MID-TERM EXAM

INSTITUTE OF MATHEMATICAL SCIENCES, AUGUST–NOVEMBER 2009

24 SEPTEMBER 2009, 1530 TO 1745 HRS, MATSCIENCE ROOM 123

Please hand in your paper no later than at 1745. Answer in the space provided. Sheets for rough work are provided separately and should not be handed in.

(1) Prove or disprove: an Artinian subring of a division ring is a division ring.

(2) Let p be a prime and G be a p-group not admitting $\mathbb{Z}/p\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$ as a quotient. What can you say about G?

(3) Let V be a finite dimensional vector space over a division ring D. Let A be a subring of D-endomorphisms of V. Assume that A is 2-*transitive*, i.e., given any two linearly independent elements v, w of V and any two elements v', w' of V, there exists a in A such that $av = v'$ and $aw = w'$.[1] Compute the commutant and bicommutant of A, even A itself.

[1]This should be taken to mean that A is also 1-transitive (to cover for the situation when there may not exist two linearly independent elements, lest the hypothesis on A become vacuous).

(4) Let A be an algebra of finite dimension over an algebraically closed field k. Let $V_1, \ldots, V_m$ be simple A-modules no two of which are isomorphic. Consider the commutant C of the ring of homotheties of the module $V_1 \oplus \cdots \oplus V_m$. List all the simple C-modules and their dimensions (over k).

(5) Prove or disprove: a module is semisimple if its opposite is so.