

TUTORIAL SHEET 4

- (1) Consider the general linear group $\mathrm{GL}(n, \mathbb{F}_q)$ of invertible $n \times n$ matrices over the field \mathbb{F}_q of finite cardinality $q = p^n$, where p is prime. Show that the subgroup consisting of unipotent upper triangular matrices is a Sylow p -subgroup.
- (2) Let V be a finite dimensional representation over a finite field of characteristic p of a p -group. Then there exists a basis of V with respect to which the matrices of all elements of the group are unipotent upper triangular. (Note: More generally, a subgroup consisting of unipotent elements of a general linear group is conjugate to a subgroup of the group of unipotent upper triangular matrices. A proof of this will be outlined in a later tutorial.)