TUTORIAL SHEET 3

(1) Show from first principles (using Schur's lemma) that each irreducible representation of an abelian group over an algebraically closed field is of dimension one. Show by means of examples that the hypothesis of algebraic closure cannot be omitted. How large can the dimension of an irreducible representation of a cyclic group over the reals be? over the rationals?

(2) If V is a finite dimensional linear representation of a group, then the space of bilinear forms on V too is naturally a representation: ${}^{g}(v, w) := (gv, gw)$. Show that an irreducible representation over an algebraically closed field admits, up to scaling, at most one invariant bilinear form. (Hint: Think of bilinear forms on V as maps from V to V^* and apply Schur's lemma.)

(3) (The underlying field in this exercise is assumed to be of characteristic zero.) Let X be a finite G-set, ρ the corresponding permutation representation, and χ the character of ρ. The value of χ on an element g of G is the number of elements of X fixed by g.

(4) (See item 1 in the section "More problems from Tutorial 1" on Tutorial sheet 2.) Let X be a G-set and ρ the corresponding permutation representation. Show that the dimension of the sum of trivial sub-representations equals the number of orbits.