TUTORIAL SHEET 23
GREEN CORRESPONDENCE
(THE TRIVIAL INTERSECTION CASE)

Let k be an algebraically closed field of positive characteristic, G a finite group, P a Sylow p-subgroup of G. Assume that, for every $g \in G$, either ${}^gP \cap P = \{1\}$ or ${}^gP = P$. Let L denote the normalizer of P in G. Let kG and kL be the group rings of G and L with coefficients in k.

(1) Let M, N, Q be kG-modules. Let Q be projective. Suppose that $\varphi : Q \to M$ is a surjective kG-map. Show that any map from N to M that factors through a projective factors through φ.

(2) Let U_1, U_2 be non-projective indecomposables over kG, and V_1, V_2 the corresponding non-projective indecomposables over kL.
   (a) Show that $\varphi \in \mathrm{Hom}_{kG}(U_1, U_2)$ factors through a projective if φ does when considered as an element of $\mathrm{Hom}_{kL}((U_1)_L, (U_2)_L)$.
   (b) Let i be the homomorphism of V_1 onto a direct summand of U_1 and let π be a homomorphism of U_2 onto V_2 with kernel a direct summand. If $\varphi \in \mathrm{Hom}_{kG}(U_1, U_2)$ then $\pi\varphi i \in \mathrm{Hom}_{kL}(V_1, V_2)$. Prove that the map sending φ to $\pi\varphi i$ induces an isomorphism of $\overline{\mathrm{Hom}}_{kG}(U_1, U_2)$ onto $\overline{\mathrm{Hom}}_{kL}(V_1, V_2)$.