

TUTORIAL SHEET 15
INDUCED REPRESENTATIONS

Let G be a finite group and H be a subgroup. All representations are finite dimensional over an algebraically closed field K of characteristic 0.

- (1) What do you get when you induce the trivial representation of H to G ? In particular, what is it when H is trivial?
- (2) Show that every irreducible submodule of G arises as a submodule of a module induced from an irreducible H -module. In particular, if H is abelian, then every irreducible representation of G has degree at most $[G : H]$.
- (3) If $G = H \times H'$, and V an H -module, then $\text{Ind}_H^G V$ is naturally isomorphic to $V \otimes KH'$, where KH' denotes the regular representation of H' .
- (4) Let V be a G -module. Show that $V \otimes KG$ is isomorphic as a G -module to $V_{\text{triv}} \otimes KG$, where V_{triv} has the same underlying vector space as V with G acting trivially.