TUTORIAL SHEET 10
SIMPLE AND SEMISIMPLE RINGS

(1) A ring is semisimple if and only if every left module over it is semisimple.
(2) Any quotient of a semisimple ring is semisimple.
(3) For any left ideal $\mathfrak{a}$ of a semisimple ring A, there exists $\alpha \in A$ such that $\alpha^2 = \alpha$ and $\mathfrak{a} = \mathfrak{a}\alpha = A\alpha$.
(4) Any simple module over a semisimple ring is isomorphic to a left ideal.
(5) The opposite of a semisimple ring is semisimple.
(6) A ring is semisimple if and only if it is the ring of endomorphisms of a finitely generated semisimple module (over some ring).
(7) For any ring A, $M_n(A)^{\text{opp}}$ is isomorphic to $M_n(A^{\text{opp}})$ through the map $(a_{ij})^{\text{opp}} \leftrightarrow (a_{ji}^{\text{opp}})$.
(8) Is $k \oplus k$ a simple ring (where k is a field)? Why aren't $k \oplus 0$ and $0 \oplus k$ isomorphic to each other as $k \oplus k$ modules?