SCHUBERT POLYNOMIALS DAY 4

ATMW SCHUBERT VARIETIES 2017

Problem 1 (braid relation of divided difference operators)**.** Prove that $\partial_1\partial 2\partial_1 = \partial_2\partial 1\partial_2$ holds for an arbitrary monomial. The twisted Leibniz rule $(\partial_i(PQ) = (\partial_i P)Q + s_i(P)\partial_i(Q))$ may be helpful.

Problem 2. Suppose w and v are permutations with the same length. Show that $\partial_v(S_w)$ is 1 if $w = v$ and 0 otherwise.

Problem 3. Show that the Schubert polynomials are linearly independent. The previous exercise will be useful.

Problem 4. Let $w = 2431$. Compute S_w using divided difference operators.

Problem 5. Compute S_w by melting Schubert's sweater:
(a) $w = 4132$ (this is easy, if you modify the example done on the board!)
(b) $w = s_3 = 1243$ (beware of non-reduced diagrams!)
(c) 1432 (but only if you are enjoying this!)
(d) 2413 (since Grassmannian permutations are good for health!)

Problem 6. Prove Stanley's formula for S_w, which expresses it as the sum of monomials $x_{b_1} \dots x_{b_l}$ over all words b compatible with reduced words a for w. In this proof, you can assume that Fomin's technique of melting Schubert's sweater is valid.

Problem 7. Let w be a k-Grassmannian permutation. Find a bijection between SSYT on $\lambda(w)$ in the alphabet $\{1, \dots, k\}$ on the one hand, and melted sweaters associated to w on the other hand.

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