(KNR) Tutorial Sheet 5

1. Let q be a power of a prime p. Observe that, for $0 \le j \le n-1$, $q^n - q^j$ is not in the additive semigroup gen by $q^n - q^{n-1}, q^n - q^{n-2}, \ldots, q^n - q^{j+1}$.

2. The product of the non-zero elements of a finite field always equals —

3. The action of $GL_k(V)$, V a f.d. K-v.s over any field k, is transitive on the set of all m dimensional subspaces of V, $m \le \dim V$.

4. Let W be an n-dim. v.s. over a finite field $\mathbb{F}_q$. Given a linear form φ on an m-dim subspace W' of W, the number of linear forms on W that restrict to φ on W' is _____.

5. ~~G fin gp $\curvearrowright R$ ring by ring automorphisms. Recall $Tr_G(f) = \sum_{g \in G} {}^g f$ G $g \in R$~~
~~Let I be a G-stable ideal of R. Show that $Tr_G(I^{|G|}) \subseteq R^G I$.~~

5. For a finite gp G, observe that $V \mapsto V^G$ is a left exact functor (in fact, it is just $\mathrm{Hom}_G(K, -)$), Show that if Char field $K \nmid |G|$ (from K-linear G representations to K-vector spaces, K a field)
then the functor is exact (and conversely).

6. Compute the Dickson invariants explicitly for small values of n and q:
$GL_2(\mathbb{F}_2)$, $GL_2(\mathbb{F}_3)$, $GL_3(\mathbb{F}_2)$