KNR Tutorial Sheet 2 Continued

7. Check that Zariski closed sets form closed sets of a topology (called the Zariski topology). Verify the details that any finite group can be realized as a linear algebraic group (over any algebraically closed field).

Tutorial sheet 3

1. Suppose that all linear repns of a gp G over a field K are semisimple. (equivalently "G is linearly reductive over K").

a) Given V ~~f.d.~~ linear G-rep $/K$, $\exists!$ G-projection $V \xrightarrow{\pi_V} V^G$.

b) π is functorial, i.e., if $\alpha : V \to W$ is a G-linear map, then $\pi_W \circ \alpha = \alpha \circ \pi_V$

2. $K[x_1, \dots, x_n] = R$ std. graded poly ring over a field K. $\mathfrak{m} := (x_1, \dots, x_n)$ If for a homogeneous ideal I, we have $\mathfrak{m}^d \subseteq I$, then I is generated by its homogeneous elements of degree $\leq d$.

1 c) If $v \in V^G$ with $v \neq 0$, show that $\exists$ G-invariant linear form f (i.e., an element of $(V^*)^G$) such that $f(v) \neq 0$.

3. G fin. gp. V f.d. rep of G over a field K. $v \neq 0$, $v \in V^G$. Show that $\exists$ homogeneous elt f of $K[V]^G$ such that $f(v) \neq 0$.

4. K alg. closed field. W f.d. K-vector space. Let $G = GL(W)$ act on $V = \mathrm{End}(W)$ by conjugation. What is $K[V]^G$? What is $\mathrm{MaxSpec}\, K[V]^G$? What is the map $\mathrm{MaxSpec}(K[V]^G) \longleftarrow \mathrm{MaxSpec}(K[V])$ induced by the inclusion $K[V]^G \subseteq K[V]$?

"finite groups are geometrically reductive". [In fact, it is true that given $v \neq 0$ in V^G, $\exists$ hom. elt $f \in K[V]^G$ s/t $f(v) \neq 0$.]