PROPOSED ALGEBRA SYLLABUS

1. Algebra I

Matrices, determinants and linear maps

- Linear maps and matrices, dual = transpose
- determinants
- Equality of row, column and determinantal rank over a commutative ring

Representations of a single endomorphism

- Minimal and characteristic polynomials, eigenvalues and eigenvectors
- Rational and Jordan canonical forms
- S-N decomposition

Bilinear forms and spectral theorems

- Preliminaries and quadratic maps
- Symmetric forms, orthogonal basis, Sylvester's theorem
- Hermitian forms, polarization, Cauchy-Schwarz inequality
- Spectral theorems, polar decomposition

Basic category theory

- Categories and functors
- Universal properties
- Sums, products and limits

Rings and modules over a PID

- Finitely generated abelian groups
- PID $\Rightarrow$ UFD, R UFD $\Rightarrow$ $R[X]$ UFD, Gauss' lemma
- Irreducibility criteria
- Modules over a PID

Tensor products

- Of vector spaces, modules over a ring, basic properties
- connection with Hom, of algebras
- tensor, symmetric and exterior algebras and connection with the determinant

Group theory

- Group actions
- Sylow's theorems
- Free groups and presentations
- Direct and semidirect products
- symmetric and alternating groups

2. Algebra II

Group theory

- simple, solvable and nilpotent groups
- Jordan-Holder theorem

Semisimplicity

- Schur's lemma and semisimple modules
- Jacobson density theorem, DCT
- Structure of semisimple rings
- Structure of simple rings

Representations of finite groups

- Maschke's theorem
- Characters
- Class functions
- Orthogonality relations

Galois theory

- Finite extensions, algebraic extensions, algebraic closure
- Splitting fields and normal extensions
- separable extensions
- Finite fields
- Inseparable extensions
- Galois extensions
- Examples and applications
- Cyclotomic fields
- Independence of characters, norm and trace
- Cyclic extensions
- Solvable and radical extensions

Commutative algebra and Dedekind domains

- Prime, maximal ideals, Zariski topology, CRT
- Localization and its properties
- Integral extensions
- Dedekind domains - characterizations
- Unique factorisation - failure and restoration

Desired outcomes after completion of course

The student should be able to

- define all concepts alluded to in the syllabus,
- give appropriate examples,
- state and prove all the theorems mentioned, and,
- apply these concepts and theorems in problem solving.