ASPECTS OF MATHEMATICS
IMSC, JUNE 30 & JULY 01, 2014
ABSTRACTS

Arvind Ayyer: Down the rabbit hole of Alternating Sign Matrices. Starting with a formula for the determinant of a matrix due to Lewis Carroll, we will motivate the definition of alternating sign matrices. Enumerating them was one of the major open problems in combinatorics during the last two decades of the past century. We will go through some of the ideas leading to the proof of their enumeration. Along the way, we will meet some of the main players in the story such as descending plane partitions, totally symmetric self-complementary plane partitions and fully packed loop configurations.

Sinnou David: A journey with Diophantus. We shall go back to the work of Diophantus and explain how the theory of elliptic curves can be put to use to tackle questions which remained out of his reach and elusive to his successors for centuries.

Suresh Govindarajan: Partitions of integers and their higher-dimensional generalisations. Partitions of integers and their higher-dimensional generalisations (such as plane/solid partitions) appear in a variety of counting problems in mathematics, physics and computer science. In this talk, after defining and illustrating partitions, we will discuss the various methods used to enumerate them. We will then discuss their asymptotic behaviour and an exact formula of Hardy-Ramanujan-Rademacher (HRR) for partitions. We then show that there is a HRR-type formula for plane partitions that is asymptotic. Finally, we show that treating partitions in all dimensions on the same footing leads to a surprising result that one needs $[(n-1)/2]$ numbers (rather than $n-1$ numbers) to determine the partitions of a positive integer n in all dimensions.

References:

(1) The Partitions Project at IIT Madras: `http://boltzmann.wikidot.com/the-partitions-project`
(2) S. Govindarajan, Notes on higher-dimensional partitions, `arXiv:1203.4419` [math.CO]
J. of Combinatorial Theory, Series A 120 (3) (2013) 600-622
(3) S. Govindarajan and N. S. Prabhakar, A superasymptotic formula for the number of plane partitions `arXiv:1311.7227` [math.NT]

Prabha Mandayam: Mutually Unbiased Bases: complementary observables in finite-dimensional Hilbert spaces. Mutually unbiased bases (MUBs) lie at the heart of theoretical investigations into complementarity in quantum theory. While much is known about the existence and properties of such bases in certain dimensions, there are interesting mathematical questions that remain to be answered. In particular, constructions of maximal sets of $(d+1)$ MUBs in Hilbert spaces of prime-powered dimensions are known, but not a single example of such a set is known in other dimensions ($d = 6, 10$, etc.). In this talk, we review some of the known constructions of MUBs and discuss the big open problem of whether there exist 7 such bases in a Hilbert space of dimension $d = 6$. We will also briefly illustrate the usefulness of such unbiased bases in the context of quantum information theory.

Ritabrata Munshi: L-functions in the theory of numbers. L-functions were introduced by Dirichlet to study the distribution of prime numbers in arithmetic progression. Later Riemann extended the definition of these functions and chalked down a program to settle the famous problem of Gauss—the Prime Number Theorem. The main hypothesis of Riemann still remains open. The relevance of L-functions and the Riemann hypothesis in modern Mathematics can hardly be undermined. The focus of the talk will be to explain the ideas behind L-functions and their present status in Number Theory.

Ajay Ramadoss: Quivers and their representations: an introduction. Quivers and their representations play an important role in several areas of mathematics and mathematical physics. The purpose of this talk is to give the audience an introduction to quivers, some algebras associated with them and their representations. Among other things, we hope to state (and time permitting, broadly sketch the proof of) Gabriel's theorem classifying Quivers of finite type.

Ref: P. Etingof et al, Introduction to representation theory, `http://math.mit.edu/~etingof/replect.pdf`

S. Sundar: Finite dimensional C^*-algebras. We will start with the spectral theorem for normal operators on finite dimensional Hilbert spaces. Building on it, I will show that finite dimensional commutative C^*-algebras (i.e. *-algebra of matrices) are just functions on a finite space. If time permits, we will see that finite dimensional C^*-algebras are direct sum of matrix algebras.

R. Venkatesh: Unique factorization of symmetric functions. In this talk, I will give an introduction to symmetric functions and prove some of their basic properties. We first realize the symmetric group as a reflection group via its natural action on Euclidean space. We then use this realization to define symmetric functions and their special examples. After this (if time permits), I will discuss the unique factorization of symmetric functions in terms of Schur functions. Some basic knowledge on linear transformations will be assumed.