GNU MP

The GNU Multiple Precision Arithmetic Library
Edition 4.1.3
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This manual describes how to install and use the GNU multiple precision arithmetic library, version 4.1.3.

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GNU MP Copying Conditions

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1 Introduction to GNU MP

GNU MP is a portable library written in C for arbitrary precision arithmetic on integers, rational numbers, and floating-point numbers. It aims to provide the fastest possible arithmetic for all applications that need higher precision than is directly supported by the basic C types.

Many applications use just a few hundred bits of precision; but some applications may need thousands or even millions of bits. GMP is designed to give good performance for both, by choosing algorithms based on the sizes of the operands, and by carefully keeping the overhead at a minimum.

The speed of GMP is achieved by using fullwords as the basic arithmetic type, by using sophisticated algorithms, by including carefully optimized assembly code for the most common inner loops for many different CPUs, and by a general emphasis on speed (as opposed to simplicity or elegance).

There is carefully optimized assembly code for these CPUs: ARM, DEC Alpha 21064, 21164, and 21264, AMD 29000, AMD K6, K6-2 and Athlon, Hitachi SuperH and SH-2, HPPA 1.0, 1.1 and 2.0, Intel Pentium, Pentium Pro/II/III, Pentium 4, generic x86, Intel IA-64, i960, Motorola MC68000, MC68020, MC88100, and MC88110, Motorola/IBM PowerPC 32 and 64, National NS32000, IBM POWER, MIPS R3000, R4000, SPARCv7, SuperSPARC, generic SPARCv8, UltraSPARC, DEC VAX, and Zilog Z8000. Some optimizations also for Cray vector systems, Clipper, IBM ROMP (RT), and Pyramid AP/XP.

There are two public mailing lists of interest. One for general questions and discussions about usage of the GMP library and one for discussions about development of GMP. There's more information about the mailing lists at http://swox.com/mailman/listinfo/. These lists are **not** for bug reports.

The proper place for bug reports is bug-gmp@gnu.org. See Chapter 4 [Reporting Bugs], page 27 for info about reporting bugs.

For up-to-date information on GMP, please see the GMP web pages at

http://swox.com/gmp/

The latest version of the library is available at

ftp://ftp.gnu.org/gnu/gmp

Many sites around the world mirror '`ftp.gnu.org`', please use a mirror near you, see http://www.gnu.org/order/ftp.html for a full list.

There are three public mailing lists of interest. One for release announcements, one for general questions and discussions about usage of the GMP library and one for discussions about development of GMP. These lists are **not** for bug reports. For more information, see

http://swox.com/mailman/listinfo/.

The proper place for bug reports is bug-gmp@gnu.org. See Chapter 4 [Reporting Bugs], page 27 for information about reporting bugs.

1.1 How to use this Manual

Everyone should read Chapter 3 [GMP Basics], page 16. If you need to install the library yourself, then read Chapter 2 [Installing GMP], page 4. If you have a system with multiple

ABIs, then read Section 2.2 [ABI and ISA], page 9, for the compiler options that must be used on applications.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

2 Installing GMP

GMP has an autoconf/automake/libtool based configuration system. On a Unix-like system a basic build can be done with

```
./configure
make
```

Some self-tests can be run with

```
make check
```

And you can install (under '/usr/local' by default) with

```
make install
```

If you experience problems, please report them to bug-gmp@gnu.org. See Chapter 4 [Reporting Bugs], page 27, for information on what to include in useful bug reports.

2.1 Build Options

All the usual autoconf configure options are available, run './configure --help' for a summary. The file 'INSTALL.autoconf' has some generic installation information too.

Non-Unix Systems

'configure' requires various Unix-like tools. On an MS-DOS system DJGPP can be used, and on MS Windows Cygwin or MINGW can be used,

```
http://www.cygwin.com/
http://www.delorie.com/djgpp
http://www.mingw.org
```

Microsoft also publishes an Interix "Services for Unix" which can be used to build GMP on Windows (with a normal './configure'), but it's not free software.

The 'macos' directory contains an unsupported port to MacOS 9 on Power Macintosh, see 'macos/README'. Note that MacOS X "Darwin" should use the normal Unix-style './configure'.

It might be possible to build without the help of 'configure', certainly all the code is there, but unfortunately you'll be on your own.

Build Directory

To compile in a separate build directory, cd to that directory, and prefix the configure command with the path to the GMP source directory. For example

```
cd /my/build/dir
/my/sources/gmp-4.1.3/configure
```

Not all 'make' programs have the necessary features (VPATH) to support this. In particular, SunOS and Slowaris make have bugs that make them unable to build in a separate directory. Use GNU make instead.

'--prefix' and '--exec-prefix'

The '--prefix' option can be used in the normal way to direct GMP to install under a particular tree. The default is '/usr/local'.

'--exec-prefix' can be used to direct architecture-dependent files like 'libgmp.a' to a different location. This can be used to share architecture-independent parts like the documentation, but separate the dependent parts. Note however that 'gmp.h' and 'mp.h' are architecture-dependent since they encode certain aspects of 'libgmp', so it will be necessary to ensure both '$prefix/include' and '$exec_prefix/include' are available to the compiler.

'--disable-shared', '--disable-static'
: By default both shared and static libraries are built (where possible), but one or other can be disabled. Shared libraries result in smaller executables and permit code sharing between separate running processes, but on some CPUs are slightly slower, having a small cost on each function call.

Native Compilation, '--build=CPU-VENDOR-OS'
: For normal native compilation, the system can be specified with '--build'. By default './configure' uses the output from running './config.guess'. On some systems './config.guess' can determine the exact CPU type, on others it will be necessary to give it explicitly. For example,

```
./configure --build=ultrasparc-sun-solaris2.7
```

In all cases the 'OS' part is important, since it controls how libtool generates shared libraries. Running './config.guess' is the simplest way to see what it should be, if you don't know already.

Cross Compilation, '--host=CPU-VENDOR-OS'
: When cross-compiling, the system used for compiling is given by '--build' and the system where the library will run is given by '--host'. For example when using a FreeBSD Athlon system to build GNU/Linux m68k binaries,

```
./configure --build=athlon-pc-freebsd3.5 --host=m68k-mac-linux-gnu
```

Compiler tools are sought first with the host system type as a prefix. For example m68k-mac-linux-gnu-ranlib is tried, then plain ranlib. This makes it possible for a set of cross-compiling tools to co-exist with native tools. The prefix is the argument to '--host', and this can be an alias, such as 'm68k-linux'. But note that tools don't have to be setup this way, it's enough to just have a PATH with a suitable cross-compiling cc etc.

Compiling for a different CPU in the same family as the build system is a form of cross-compilation, though very possibly this would merely be special options on a native compiler. In any case './configure' avoids depending on being able to run code on the build system, which is important when creating binaries for a newer CPU since they very possibly won't run on the build system.

In all cases the compiler must be able to produce an executable (of whatever format) from a standard C main. Although only object files will go to make up 'libgmp', './configure' uses linking tests for various purposes, such as determining what functions are available on the host system.

Currently a warning is given unless an explicit '--build' is used when cross-compiling, because it may not be possible to correctly guess the build system type if the PATH has only a cross-compiling cc.

Note that the '--target' option is not appropriate for GMP. It's for use when building compiler tools, with '--host' being where they will run, and '--target' what they'll produce code for. Ordinary programs or libraries like GMP are only interested in the '--host' part, being where they'll run. (Some past versions of GMP used '--target' incorrectly.)

CPU types
: In general, if you want a library that runs as fast as possible, you should configure GMP for the exact CPU type your system uses. However, this may mean the binaries won't run on older members of the family, and might run slower on other members, older or newer. The best idea is always to build GMP for the exact machine type you intend to run it on.

The following CPUs have specific support. See 'configure.in' for details of what code and compiler options they select.

- Alpha: 'alpha', 'alphaev5', 'alphaev56', 'alphapca56', 'alphapca57', 'alphaev6', 'alphaev67', 'alphaev68'
- Cray: 'c90', 'j90', 't90', 'sv1'
- HPPA: 'hppa1.0', 'hppa1.1', 'hppa2.0', 'hppa2.0n', 'hppa2.0w'
- MIPS: 'mips', 'mips3', 'mips64'
- Motorola: 'm68k', 'm68000', 'm68010', 'm68020', 'm68030', 'm68040', 'm68060', 'm68302', 'm68360', 'm88k', 'm88110'
- POWER: 'power', 'power1', 'power2', 'power2sc'
- PowerPC: 'powerpc', 'powerpc64', 'powerpc401', 'powerpc403', 'powerpc405', 'powerpc505', 'powerpc601', 'powerpc602', 'powerpc603', 'powerpc603e', 'powerpc604', 'powerpc604e', 'powerpc620', 'powerpc630', 'powerpc740', 'powerpc7400', 'powerpc7450', 'powerpc750', 'powerpc801', 'powerpc821', 'powerpc823', 'powerpc860',
- SPARC: 'sparc', 'sparcv8', 'microsparc', 'supersparc', 'sparcv9', 'ultrasparc', 'ultrasparc2', 'ultrasparc2i', 'ultrasparc3', 'sparc64'
- 80x86 family: 'i386', 'i486', 'i586', 'pentium', 'pentiummmx', 'pentiumpro', 'pentium2', 'pentium3', 'pentium4', 'k6', 'k62', 'k63', 'athlon'
- Other: 'a29k', 'arm', 'clipper', 'i960', 'ns32k', 'pyramid', 'sh', 'sh2', 'vax', 'z8k'

CPUs not listed will use generic C code.

Generic C Build

If some of the assembly code causes problems, or if otherwise desired, the generic C code can be selected with CPU 'none'. For example,

```
./configure --host=none-unknown-freebsd3.5
```

Note that this will run quite slowly, but it should be portable and should at least make it possible to get something running if all else fails.

'ABI'

On some systems GMP supports multiple ABIs (application binary interfaces), meaning data type sizes and calling conventions. By default GMP chooses the best ABI available, but a particular ABI can be selected. For example

```
./configure --host=mips64-sgi-irix6 ABI=n32
```

See Section 2.2 [ABI and ISA], page 9, for the available choices on relevant CPUs, and what applications need to do.

'CC', 'CFLAGS'

By default the C compiler used is chosen from among some likely candidates, with gcc normally preferred if it's present. The usual 'CC=whatever' can be passed to './configure' to choose something different.

For some systems, default compiler flags are set based on the CPU and compiler. The usual 'CFLAGS="-whatever"' can be passed to './configure' to use something different or to set good flags for systems GMP doesn't otherwise know.

The 'CC' and 'CFLAGS' used are printed during './configure', and can be found in each generated 'Makefile'. This is the easiest way to check the defaults when considering changing or adding something.

Note that when 'CC' and 'CFLAGS' are specified on a system supporting multiple ABIs it's important to give an explicit 'ABI=whatever', since GMP can't determine the ABI just from the flags and won't be able to select the correct assembler code.

If just 'CC' is selected then normal default 'CFLAGS' for that compiler will be used (if GMP recognises it). For example 'CC=gcc' can be used to force the use of GCC, with default flags (and default ABI).

'CPPFLAGS'

Any flags like '-D' defines or '-I' includes required by the preprocessor should be set in 'CPPFLAGS' rather than 'CFLAGS'. Compiling is done with both 'CPPFLAGS' and 'CFLAGS', but preprocessing uses just 'CPPFLAGS'. This distinction is because most preprocessors won't accept all the flags the compiler does. Preprocessing is done separately in some configure tests, and in the 'ansi2knr' support for K&R compilers.

C++ Support, '--enable-cxx'

C++ support in GMP can be enabled with '--enable-cxx', in which case a C++ compiler will be required. As a convenience '--enable-cxx=detect' can be used to enable C++ support only if a compiler can be found. The C++ support consists of a library 'libgmpxx.la' and header file 'gmpxx.h'.

A separate 'libgmpxx.la' has been adopted rather than having C++ objects within 'libgmp.la' in order to ensure dynamic linked C programs aren't bloated by a dependency on the C++ standard library, and to avoid any chance that the C++ compiler could be required when linking plain C programs.

'libgmpxx.la' will use certain internals from 'libgmp.la' and can only be expected to work with 'libgmp.la' from the same GMP version. Future changes to the relevant internals will be accompanied by renaming, so a mismatch will cause unresolved symbols rather than perhaps mysterious misbehaviour.

In general 'libgmpxx.la' will be usable only with the C++ compiler that built it, since name mangling and runtime support are usually incompatible between different compilers.

'CXX', 'CXXFLAGS'

When C++ support is enabled, the C++ compiler and its flags can be set with variables 'CXX' and 'CXXFLAGS' in the usual way. The default for 'CXX' is the first compiler that works from a list of likely candidates, with g++ normally preferred when available. The default for 'CXXFLAGS' is to try 'CFLAGS', 'CFLAGS' without '-g', then for g++ either '-g -O2' or '-O2', or for other compilers '-g' or nothing. Trying 'CFLAGS' this way is convenient when using 'gcc' and 'g++' together, since the flags for 'gcc' will usually suit 'g++'.

It's important that the C and C++ compilers match, meaning their startup and runtime support routines are compatible and that they generate code in the same ABI (if there's a choice of ABIs on the system). './configure' isn't currently able to check these things very well itself, so for that reason '--disable-cxx' is the default, to avoid a build failure due to a compiler mismatch. Perhaps this will change in the future.

Incidentally, it's normally not good enough to set 'CXX' to the same as 'CC'. Although gcc for instance recognises 'foo.cc' as C++ code, only g++ will invoke the linker the right way when building an executable or shared library from object files.

Temporary Memory, '--enable-alloca=<choice>'

GMP allocates temporary workspace using one of the following three methods, which can be selected with for instance '--enable-alloca=malloc-reentrant'.

- 'alloca' - C library or compiler builtin.
- 'malloc-reentrant' - the heap, in a re-entrant fashion.
- 'malloc-notreentrant' - the heap, with global variables.

For convenience, the following choices are also available. '--disable-alloca' is the same as '--enable-alloca=no'.

- 'yes' - a synonym for 'alloca'.
- 'no' - a synonym for 'malloc-reentrant'.
- 'reentrant' - alloca if available, otherwise 'malloc-reentrant'. This is the default.
- 'notreentrant' - alloca if available, otherwise 'malloc-notreentrant'.

alloca is reentrant and fast, and is recommended, but when working with large numbers it can overflow the available stack space, in which case one of the two malloc methods will need to be used. Alternately it might be possible to increase available stack with limit, ulimit or setrlimit, or under DJGPP with stubedit or _stklen. Note that depending on the system the only indication of stack overflow might be a segmentation violation.

'malloc-reentrant' is, as the name suggests, reentrant and thread safe, but 'malloc-notreentrant' is faster and should be used if reentrancy is not required.

The two malloc methods in fact use the memory allocation functions selected by mp_set_memory_functions, these being malloc and friends by default. See Chapter 14 [Custom Allocation], page 82.

An additional choice '--enable-alloca=debug' is available, to help when debugging memory related problems (see Section 3.12 [Debugging], page 23).

FFT Multiplication, '--disable-fft'

By default multiplications are done using Karatsuba, 3-way Toom-Cook, and Fermat FFT. The FFT is only used on large to very large operands and can be disabled to save code size if desired.

Berkeley MP, '--enable-mpbsd'

The Berkeley MP compatibility library ('libmp') and header file ('mp.h') are built and installed only if '--enable-mpbsd' is used. See Chapter 13 [BSD Compatible Functions], page 80.

MPFR, '--enable-mpfr'

The optional MPFR functions are built and installed only if '--enable-mpfr' is used. These are in a separate library 'libmpfr.a' and are documented separately too (see section "Introduction to MPFR" in *MPFR*).

Assertion Checking, '--enable-assert'

This option enables some consistency checking within the library. This can be of use while debugging, see Section 3.12 [Debugging], page 23.

Execution Profiling, '--enable-profiling=prof/gprof'

Profiling support can be enabled either for prof or gprof. This adds '-p' or '-pg' respectively to 'CFLAGS', and for some systems adds corresponding mcount calls to the assembler code. See Section 3.13 [Profiling], page 25.

'MPN_PATH'

Various assembler versions of each mpn subroutines are provided. For a given CPU, a search is made though a path to choose a version of each. For example 'sparcv8' has

```
MPN_PATH="sparc32/v8 sparc32 generic"
```

which means look first for v8 code, then plain sparc32 (which is v7), and finally fall back on generic C. Knowledgeable users with special requirements can specify a different path. Normally this is completely unnecessary.

Documentation

The document you're now reading is 'gmp.texi'. The usual automake targets are available to make PostScript 'gmp.ps' and/or DVI 'gmp.dvi'.

HTML can be produced with 'makeinfo --html', see section "Generating HTML" in *Texinfo*. Or alternately 'texi2html', see section "About" in *Texinfo To HTML*.

PDF can be produced with 'texi2dvi --pdf' (see section "PDF Output" in *Texinfo*) or with 'pdftex'.

Some supplementary notes can be found in the 'doc' subdirectory.

2.2 ABI and ISA

ABI (Application Binary Interface) refers to the calling conventions between functions, meaning what registers are used and what sizes the various C data types are. ISA (Instruction Set Architecture) refers to the instructions and registers a CPU has available.

Some 64-bit ISA CPUs have both a 64-bit ABI and a 32-bit ABI defined, the latter for compatibility with older CPUs in the family. GMP supports some CPUs like this in both ABIs. In fact within GMP 'ABI' means a combination of chip ABI, plus how GMP chooses to use it. For example in some 32-bit ABIs, GMP may support a limb as either a 32-bit `long` or a 64-bit `long long`.

By default GMP chooses the best ABI available for a given system, and this generally gives significantly greater speed. But an ABI can be chosen explicitly to make GMP compatible with other libraries, or particular application requirements. For example,

```
./configure ABI=32
```

In all cases it's vital that all object code used in a given program is compiled for the same ABI.

Usually a limb is implemented as a `long`. When a `long long` limb is used this is encoded in the generated 'gmp.h'. This is convenient for applications, but it does mean that 'gmp.h' will vary, and can't be just copied around. 'gmp.h' remains compiler independent though, since all compilers for a particular ABI will be expected to use the same limb type.

Currently no attempt is made to follow whatever conventions a system has for installing library or header files built for a particular ABI. This will probably only matter when installing multiple builds of GMP, and it might be as simple as configuring with a special 'libdir', or it might require more than that. Note that builds for different ABIs need to done separately, with a fresh `./configure` and `make` each.

HPPA 2.0 ('hppa2.0*')

'ABI=2.0w'

The 2.0w ABI uses 64-bit limbs and pointers and is available on HP-UX 11 or up when using `cc`. `gcc` support for this is in progress. Applications must be compiled with

```
cc  +DD64
```

'ABI=2.0n'

The 2.0n ABI means the 32-bit HPPA 1.0 ABI but with a 64-bit limb using `long long`. This is available on HP-UX 10 or up when using `cc`. No `gcc` support is planned for this. Applications must be compiled with

```
cc  +DA2.0 +e
```

'ABI=1.0'

HPPA 2.0 CPUs can run all HPPA 1.0 and 1.1 code in the 32-bit HPPA 1.0 ABI. No special compiler options are needed for applications.

All three ABIs are available for CPUs 'hppa2.0w' and 'hppa2.0', but for CPU 'hppa2.0n' only 2.0n or 1.0 are allowed.

MIPS under IRIX 6 ('mips*-*-irix[6789]')

IRIX 6 supports the n32 and 64 ABIs and always has a 64-bit MIPS 3 or better CPU. In both these ABIs GMP uses a 64-bit limb. A new enough gcc is required (2.95 for instance).

'ABI=n32'

The n32 ABI is 32-bit pointers and integers, but with a 64-bit limb using a long long. Applications must be compiled with

```
gcc  -mabi=n32
cc   -n32
```

'ABI=64'

The 64-bit ABI is 64-bit pointers and integers. Applications must be compiled with

```
gcc  -mabi=64
cc   -64
```

Note that MIPS GNU/Linux, as of kernel version 2.2, doesn't have the necessary support for n32 or 64 and so only gets a 32-bit limb and the MIPS 2 code.

PowerPC 64 ('powerpc64', 'powerpc620', 'powerpc630')

'ABI=aix64'

The AIX 64 ABI uses 64-bit limbs and pointers and is available on systems '*-*-aix*'. Applications must be compiled (and linked) with

```
gcc  -maix64
xlc  -q64
```

'ABI=32'

This is the basic 32-bit PowerPC ABI. No special compiler options are needed for applications.

Sparc V9 ('sparcv9' and 'ultrasparc*')

'ABI=64'

The 64-bit V9 ABI is available on Solaris 2.7 and up and GNU/Linux. GCC 2.95 or up, or Sun cc is required. Applications must be compiled with

```
gcc  -m64 -mptr64 -Wa,-xarch=v9 -mcpu=v9
cc   -xarch=v9
```

'ABI=32'

On Solaris 2.6 and earlier, and on Solaris 2.7 with the kernel in 32-bit mode, only the plain V8 32-bit ABI can be used, since the kernel doesn't save all registers. GMP still uses as much of the V9 ISA as it can in these circumstances. No special compiler options are required for applications, though using something like the following requesting V9 code within the V8 ABI is recommended.

```
gcc   -mv8plus
cc    -xarch=v8plus
```

gcc 2.8 and earlier only supports '-mv8' though.

Don't be confused by the names of these sparc '-m' and '-x' options, they're called 'arch' but they effectively control the ABI.

On Solaris 2.7 with the kernel in 32-bit-mode, a normal native build will reject 'ABI=64' because the resulting executables won't run. 'ABI=64' can still be built if desired by making it look like a cross-compile, for example

```
./configure --build=none --host=sparcv9-sun-solaris2.7 ABI=64
```

2.3 Notes for Package Builds

GMP should present no great difficulties for packaging in a binary distribution.

Libtool is used to build the library and '-version-info' is set appropriately, having started from '3:0:0' in GMP 3.0. The GMP 4 series will be upwardly binary compatible in each release and will be upwardly binary compatible with all of the GMP 3 series. Additional function interfaces may be added in each release, so on systems where libtool versioning is not fully checked by the loader an auxiliary mechanism may be needed to express that a dynamic linked application depends on a new enough GMP.

An auxiliary mechanism may also be needed to express that 'libgmpxx.la' (from '--enable-cxx', see Section 2.1 [Build Options], page 4) requires 'libgmp.la' from the same GMP version, since this is not done by the libtool versioning, nor otherwise. A mismatch will result in unresolved symbols from the linker, or perhaps the loader.

Using 'DESTDIR' or a 'prefix' override with 'make install' and a shared 'libgmpxx' may run into a libtool relinking problem, see Section 2.5 [Known Build Problems], page 13.

When building a package for a CPU family, care should be taken to use '--host' (or '--build') to choose the least common denominator among the CPUs which might use the package. For example this might necessitate 'i386' for x86s, or plain 'sparc' (meaning V7) for SPARCs.

Users who care about speed will want GMP built for their exact CPU type, to make use of the available optimizations. Providing a way to suitably rebuild a package may be useful. This could be as simple as making it possible for a user to omit '--build' (and '--host') so './config.guess' will detect the CPU. But a way to manually specify a '--build' will be wanted for systems where './config.guess' is inexact.

Note that 'gmp.h' is a generated file, and will be architecture and ABI dependent.

2.4 Notes for Particular Systems

AIX 3 and 4

On systems '*-*-aix[34]*' shared libraries are disabled by default, since some versions of the native ar fail on the convenience libraries used. A shared build can be attempted with

```
./configure --enable-shared --disable-static
```

Note that the '--disable-static' is necessary because in a shared build libtool makes 'libgmp.a' a symlink to 'libgmp.so', apparently for the benefit of old versions of ld which only recognise '.a', but unfortunately this is done even if a fully functional ld is available.

ARM

On systems '`arm*-*-*`', versions of GCC up to and including 2.95.3 have a bug in unsigned division, giving wrong results for some operands. GMP '`./configure`' will demand GCC 2.95.4 or later.

Compaq C++
: Compaq C++ on OSF 5.1 has two flavours of `iostream`, a standard one and an old pre-standard one (see '`man iostream_intro`'). GMP can only use the standard one, which unfortunately is not the default but must be selected by defining `__USE_STD_IOSTREAM`. Configure with for instance

```
./configure --enable-cxx CPPFLAGS=-D__USE_STD_IOSTREAM
```

Floating Point Mode
: On some systems, the hardware floating point has a control mode which can set all operations to be done in a particular precision, for instance single, double or extended on x86 systems (x87 floating point). The GMP functions involving a `double` cannot be expected to operate to their full precision when the hardware is in single precision mode. Of course this affects all code, including application code, not just GMP.

Microsoft Windows
: On systems '`*-*-cygwin*`', '`*-*-mingw*`' and '`*-*-pw32*`' by default GMP builds only a static library, but a DLL can be built instead using

```
./configure --disable-static --enable-shared
```

Static and DLL libraries can't both be built, since certain export directives in '`gmp.h`' must be different. '`--enable-cxx`' cannot be used when building a DLL, since libtool doesn't currently support C++ DLLs. This might change in the future.

Microsoft C
: A MINGW DLL build of GMP can be used with Microsoft C. Libtool doesn't install '`.lib`' and '`.exp`' files, but they can be created with the following commands, where '`/my/inst/dir`' is the install directory (with a '`lib`' subdirectory).

```
lib /machine:IX86 /def:.libs/libgmp-3.dll-def
cp libgmp-3.lib /my/inst/dir/lib
cp .libs/libgmp-3.dll-exp /my/inst/dir/lib/libgmp-3.exp
```

MINGW uses the C runtime library '`msvcrt.dll`' for I/O, so applications wanting to use the GMP I/O routines must be compiled with '`cl /MD`' to do the same. If one of the other C runtime library choices provided by MS C is desired then the suggestion is to use the GMP string functions and confine I/O to the application.

Motorola 68k CPU Types
: '`m68k`' is taken to mean 68000. '`m68020`' or higher will give a performance boost on applicable CPUs. '`m68360`' can be used for CPU32 series chips. '`m68302`' can be used for "Dragonball" series chips, though this is merely a synonym for '`m68000`'.

OpenBSD 2.6
: `m4` in this release of OpenBSD has a bug in `eval` that makes it unsuitable for '`.asm`' file processing. '`./configure`' will detect the problem and either abort or choose another m4 in the `PATH`. The bug is fixed in OpenBSD 2.7, so either upgrade or use GNU m4.

Power CPU Types
: In GMP, CPU types '`power*`' and '`powerpc*`' will each use instructions not available on the other, so it's important to choose the right one for the CPU that will be used. Currently GMP has no assembler code support for using just the common instruction subset. To get executables that run on both, the current suggestion is

to use the generic C code (CPU '`none`'), possibly with appropriate compiler options (like '`-mcpu=common`' for `gcc`). CPU '`rs6000`' (which is not a CPU but a family of workstations) is accepted by '`config.sub`', but is currently equivalent to '`none`'.

Sparc CPU Types
'`sparcv8`' or '`supersparc`' on relevant systems will give a significant performance increase over the V7 code.

Sparc App Regs
The GMP assembler code for both 32-bit and 64-bit Sparc clobbers the "application registers" `g2`, `g3` and `g4`, the same way that the GCC default '`-mapp-regs`' does (see section "SPARC Options" in *Using the GNU Compiler Collection (GCC)*).

This makes that code unsuitable for use with the special V9 '`-mcmodel=embmedany`' (which uses `g4` as a data segment pointer), and for applications wanting to use those registers for special purposes. In these cases the only suggestion currently is to build GMP with CPU '`none`' to avoid the assembler code.

SunOS 4
`/usr/bin/m4` lacks various features needed to process '`.asm`' files, and instead '`./configure`' will automatically use `/usr/5bin/m4`, which we believe is always available (if not then use GNU m4).

x86 CPU Types
'`i386`' selects generic code which will run reasonably well on all x86 chips.

'`i586`', '`pentium`' or '`pentiummmx`' code is good for the intended P5 Pentium chips, but quite slow when run on Intel P6 class chips (PPro, P-II, P-III). '`i386`' is a better choice when making binaries that must run on both.

'`pentium4`' and an SSE2 capable assembler are important for best results on Pentium 4. The specific code is for instance roughly a $2\times$ to $3\times$ speedup over the generic '`i386`' code.

x86 MMX and SSE2 Code
If the CPU selected has MMX code but the assembler doesn't support it, a warning is given and non-MMX code is used instead. This will be an inferior build, since the MMX code that's present is there because it's faster than the corresponding plain integer code. The same applies to SSE2.

Old versions of '`gas`' don't support MMX instructions, in particular version 1.92.3 that comes with FreeBSD 2.2.8 doesn't (and unfortunately there's no newer assembler for that system).

Solaris 2.6 and 2.7 `as` generate incorrect object code for register to register `movq` instructions, and so can't be used for MMX code. Install a recent `gas` if MMX code is wanted on these systems.

2.5 Known Build Problems

You might find more up-to-date information at `http://swox.com/gmp/`.

Compiler link options
The version of libtool currently in use rather aggressively strips compiler options when linking a shared library. This will hopefully be relaxed in the future, but for now if this is a problem the suggestion is to create a little script to hide them, and for instance configure with

```
./configure CC=gcc-with-my-options
```

DJGPP
: The DJGPP port of `bash` 2.03 is unable to run the '`configure`' script, it exits silently, having died writing a preamble to '`config.log`'. Use `bash` 2.04 or higher.

'`make all`' was found to run out of memory during the final '`libgmp.la`' link on one system tested, despite having 64Mb available. A separate '`make libgmp.la`' helped, perhaps recursing into the various subdirectories uses up memory.

'`DESTDIR`' and shared '`libgmpxx`'
: '`make install DESTDIR=/my/staging/area`', or the same with a '`prefix`' override, to install to a temporary directory is not fully supported by current versions of libtool when building a shared version of a library which depends on another being built at the same time, like '`libgmpxx`' and '`libgmp`'.

The problem is that '`libgmpxx`' is relinked at the install stage to ensure that if the system puts a hard-coded path to '`libgmp`' within '`libgmpxx`' then that path will be correct. Naturally the linker is directed to look only at the final location, not the staging area, so if '`libgmp`' is not already in that final location then the link will fail.

A workaround for this on SVR4 style systems, such as GNU/Linux, where paths are not hard-coded, is to include the staging area in the linker's search using `LD_LIBRARY_PATH`. For example with '`--prefix=/usr`' but installing under '`/my/staging/area`',

```
LD_LIBRARY_PATH=/my/staging/area/usr/lib \
  make install DESTDIR=/my/staging/area
```

GNU binutils `strip` prior to 2.12
: `strip` from GNU binutils 2.11 and earlier should not be used on the static libraries '`libgmp.a`' and '`libmp.a`' since it will discard all but the last of multiple archive members with the same name, like the three versions of '`init.o`' in '`libgmp.a`'. Binutils 2.12 or higher can be used successfully.

The shared libraries '`libgmp.so`' and '`libmp.so`' are not affected by this and any version of `strip` can be used on them.

`make` syntax error
: On certain versions of SCO OpenServer 5 and IRIX 6.5 the native `make` is unable to handle the long dependencies list for '`libgmp.la`'. The symptom is a "syntax error" on the following line of the top-level '`Makefile`'.

```
libgmp.la: $(libgmp_la_OBJECTS) $(libgmp_la_DEPENDENCIES)
```

Either use GNU Make, or as a workaround remove `$(libgmp_la_DEPENDENCIES)` from that line (which will make the initial build work, but if any recompiling is done '`libgmp.la`' might not be rebuilt).

MacOS X and GCC
: Libtool currently only knows how to create shared libraries on MacOS X using the native `cc` (which is a modified GCC), not a plain GCC. A static-only build should work though ('`--disable-shared`').

Also, libtool currently cannot build C++ shared libraries on MacOS X, so if '`--enable-cxx`' is desired then '`--disable-shared`' must be used. Hopefully this will be fixed in the future.

Motorola 68k ABI
: The GMP assembler code has been written for the SVR4 standard ABI. GCC option '`-mshort`' changes the calling conventions and is not currently supported. We believe the PalmOS calling conventions are similarly different and are likewise not currently supported.

NeXT prior to 3.3
: The system compiler on old versions of NeXT was a massacred and old GCC, even if it called itself 'cc'. This compiler cannot be used to build GMP, you need to get a real GCC, and install that. (NeXT may have fixed this in release 3.3 of their system.)

POWER and PowerPC
: Bugs in GCC 2.7.2 (and 2.6.3) mean it can't be used to compile GMP on POWER or PowerPC. If you want to use GCC for these machines, get GCC 2.7.2.1 (or later).

Sequent Symmetry
: Use the GNU assembler instead of the system assembler, since the latter has serious bugs.

Solaris 2.6
: The system `sed` prints an error "Output line too long" when libtool builds '`libgmp.la`'. This doesn't seem to cause any obvious ill effects, but GNU `sed` is recommended, to avoid any doubt.

Sparc Solaris 2.7 with gcc 2.95.2 in ABI=32
: A shared library build of GMP seems to fail in this combination, it builds but then fails the tests, apparently due to some incorrect data relocations within `gmp_randinit_lc_2exp_size`. The exact cause is unknown, '`--disable-shared`' is recommended.

Windows DLL test programs
: When creating a DLL version of '`libgmp`', libtool creates wrapper scripts like '`t-mul`' for programs that would normally be '`t-mul.exe`', in order to setup the right library paths etc. This works fine, but the absence of '`t-mul.exe`' etc causes `make` to think they need recompiling every time, which is an annoyance when re-running a '`make check`'.

3 GMP Basics

Using functions, macros, data types, etc. not documented in this manual is strongly discouraged. If you do so your application is guaranteed to be incompatible with future versions of GMP.

3.1 Headers and Libraries

All declarations needed to use GMP are collected in the include file 'gmp.h'. It is designed to work with both C and C++ compilers.

```
#include <gmp.h>
```

Note however that prototypes for GMP functions with FILE * parameters are only provided if <stdio.h> is included too.

```
#include <stdio.h>
#include <gmp.h>
```

Likewise <stdarg.h> (or <varargs.h>) is required for prototypes with va_list parameters, such as gmp_vprintf. And <obstack.h> for prototypes with struct obstack parameters, such as gmp_obstack_printf, when available.

All programs using GMP must link against the 'libgmp' library. On a typical Unix-like system this can be done with '-lgmp', for example

```
gcc myprogram.c -lgmp
```

GMP C++ functions are in a separate 'libgmpxx' library. This is built and installed if C++ support has been enabled (see Section 2.1 [Build Options], page 4). For example,

```
g++ mycxxprog.cc -lgmpxx -lgmp
```

GMP is built using Libtool and an application can use that to link if desired, see section "Introduction" in *GNU Libtool*

If GMP has been installed to a non-standard location then it may be necessary to use '-I' and '-L' compiler options to point to the right directories, and some sort of run-time path for a shared library. Consult your compiler documentation, for instance section "Introduction" in *Using and Porting the GNU Compiler Collection*.

3.2 Nomenclature and Types

In this manual, *integer* usually means a multiple precision integer, as defined by the GMP library. The C data type for such integers is mpz_t. Here are some examples of how to declare such integers:

```
mpz_t sum;

struct foo { mpz_t x, y; };

mpz_t vec[20];
```

Rational number means a multiple precision fraction. The C data type for these fractions is mpq_t. For example:

```
mpq_t quotient;
```

Floating point number or *Float* for short, is an arbitrary precision mantissa with a limited precision exponent. The C data type for such objects is mpf_t.

A *limb* means the part of a multi-precision number that fits in a single machine word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb is 32 or 64 bits. The C data type for a limb is `mp_limb_t`.

3.3 Function Classes

There are six classes of functions in the GMP library:

1. Functions for signed integer arithmetic, with names beginning with `mpz_`. The associated type is `mpz_t`. There are about 150 functions in this class.
2. Functions for rational number arithmetic, with names beginning with `mpq_`. The associated type is `mpq_t`. There are about 40 functions in this class, but the integer functions can be used for arithmetic on the numerator and denominator separately.
3. Functions for floating-point arithmetic, with names beginning with `mpf_`. The associated type is `mpf_t`. There are about 60 functions is this class.
4. Functions compatible with Berkeley MP, such as `itom`, `madd`, and `mult`. The associated type is `MINT`.
5. Fast low-level functions that operate on natural numbers. These are used by the functions in the preceding groups, and you can also call them directly from very time-critical user programs. These functions' names begin with `mpn_`. The associated type is array of `mp_limb_t`. There are about 30 (hard-to-use) functions in this class.
6. Miscellaneous functions. Functions for setting up custom allocation and functions for generating random numbers.

3.4 Variable Conventions

GMP functions generally have output arguments before input arguments. This notation is by analogy with the assignment operator. The BSD MP compatibility functions are exceptions, having the output arguments last.

GMP lets you use the same variable for both input and output in one call. For example, the main function for integer multiplication, `mpz_mul`, can be used to square `x` and put the result back in `x` with

```
mpz_mul (x, x, x);
```

Before you can assign to a GMP variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose. Which function to use depends on the type of variable. See the chapters on integer functions, rational number functions, and floating-point functions for details.

A variable should only be initialized once, or at least cleared between each initialization. After a variable has been initialized, it may be assigned to any number of times.

For efficiency reasons, avoid excessive initializing and clearing. In general, initialize near the start of a function and clear near the end. For example,

```
void
foo (void)
{
  mpz_t  n;
  int    i;
  mpz_init (n);
  for (i = 1; i < 100; i++)
```

```
    {
      mpz_mul (n, ...);
      mpz_fdiv_q (n, ...);
      ...
    }
  mpz_clear (n);
}
```

3.5 Parameter Conventions

When a GMP variable is used as a function parameter, it's effectively a call-by-reference, meaning if the function stores a value there it will change the original in the caller. Parameters which are input-only can be designated `const` to provoke a compiler error or warning on attempting to modify them.

When a function is going to return a GMP result, it should designate a parameter that it sets, like the library functions do. More than one value can be returned by having more than one output parameter, again like the library functions. A `return` of an `mpz_t` etc doesn't return the object, only a pointer, and this is almost certainly not what's wanted.

Here's an example accepting an `mpz_t` parameter, doing a calculation, and storing the result to the indicated parameter.

```
void
foo (mpz_t result, const mpz_t param, unsigned long n)
{
  unsigned long  i;
  mpz_mul_ui (result, param, n);
  for (i = 1; i < n; i++)
    mpz_add_ui (result, result, i*7);
}

int
main (void)
{
  mpz_t  r, n;
  mpz_init (r);
  mpz_init_set_str (n, "123456", 0);
  foo (r, n, 20L);
  gmp_printf ("%Zd\n", r);
  return 0;
}
```

`foo` works even if the mainline passes the same variable for `param` and `result`, just like the library functions. But sometimes it's tricky to make that work, and an application might not want to bother supporting that sort of thing.

For interest, the GMP types `mpz_t` etc are implemented as one-element arrays of certain structures. This is why declaring a variable creates an object with the fields GMP needs, but then using it as a parameter passes a pointer to the object. Note that the actual fields in each `mpz_t` etc are for internal use only and should not be accessed directly by code that expects to be compatible with future GMP releases.

3.6 Memory Management

The GMP types like mpz_t are small, containing only a couple of sizes, and pointers to allocated data. Once a variable is initialized, GMP takes care of all space allocation. Additional space is allocated whenever a variable doesn't have enough.

mpz_t and mpq_t variables never reduce their allocated space. Normally this is the best policy, since it avoids frequent reallocation. Applications that need to return memory to the heap at some particular point can use mpz_realloc2, or clear variables no longer needed.

mpf_t variables, in the current implementation, use a fixed amount of space, determined by the chosen precision and allocated at initialization, so their size doesn't change.

All memory is allocated using malloc and friends by default, but this can be changed, see Chapter 14 [Custom Allocation], page 82. Temporary memory on the stack is also used (via alloca), but this can be changed at build-time if desired, see Section 2.1 [Build Options], page 4.

3.7 Reentrancy

GMP is reentrant and thread-safe, with some exceptions:

- If configured with '--enable-alloca=malloc-notreentrant' (or with '--enable-alloca=notreentrant' when alloca is not available), then naturally GMP is not reentrant.
- mpf_set_default_prec and mpf_init use a global variable for the selected precision. mpf_init2 can be used instead, and in the C++ interface an explicit precision to the mpf_class constructor.
- mpz_random and the other old random number functions use a global random state and are hence not reentrant. The newer random number functions that accept a gmp_randstate_t parameter can be used instead.
- gmp_randinit (obsolete) returns an error indication through a global variable, which is not thread safe. Applications are advised to use gmp_randinit_lc_2exp instead.
- mp_set_memory_functions uses global variables to store the selected memory allocation functions.
- If the memory allocation functions set by a call to mp_set_memory_functions (or malloc and friends by default) are not reentrant, then GMP will not be reentrant either.
- If the standard I/O functions such as fwrite are not reentrant then the GMP I/O functions using them will not be reentrant either.
- It's safe for two threads to read from the same GMP variable simultaneously, but it's not safe for one to read while the another might be writing, nor for two threads to write simultaneously. It's not safe for two threads to generate a random number from the same gmp_randstate_t simultaneously, since this involves an update of that variable.

3.8 Useful Macros and Constants

const int mp_bits_per_limb [Global Constant]

The number of bits per limb.

__GNU_MP_VERSION [Macro]
__GNU_MP_VERSION_MINOR [Macro]

__GNU_MP_VERSION_PATCHLEVEL [Macro]

The major and minor GMP version, and patch level, respectively, as integers. For GMP i.j, these numbers will be i, j, and 0, respectively. For GMP i.j.k, these numbers will be i, j, and k, respectively.

const char * const gmp_version [Global Constant]

The GMP version number, as a null-terminated string, in the form "i.j" or "i.j.k". This release is "4.1.3".

3.9 Compatibility with older versions

This version of GMP is upwardly binary compatible with all 4.x and 3.x versions, and upwardly compatible at the source level with all 2.x versions, with the following exceptions.

- mpn_gcd had its source arguments swapped as of GMP 3.0, for consistency with other mpn functions.
- mpf_get_prec counted precision slightly differently in GMP 3.0 and 3.0.1, but in 3.1 reverted to the 2.x style.

There are a number of compatibility issues between GMP 1 and GMP 2 that of course also apply when porting applications from GMP 1 to GMP 4. Please see the GMP 2 manual for details.

The Berkeley MP compatibility library (see Chapter 13 [BSD Compatible Functions], page 80) is source and binary compatible with the standard 'libmp'.

3.10 Demonstration programs

The 'demos' subdirectory has some sample programs using GMP. These aren't built or installed, but there's a 'Makefile' with rules for them. For instance,

```
make pexpr
./pexpr 68^975+10
```

The following programs are provided

- 'pexpr' is an expression evaluator, the program used on the GMP web page.
- The 'calc' subdirectory has a similar but simpler evaluator using lex and yacc.
- The 'expr' subdirectory is yet another expression evaluator, a library designed for ease of use within a C program. See 'demos/expr/README' for more information.
- 'factorize' is a Pollard-Rho factorization program.
- 'isprime' is a command-line interface to the mpz_probab_prime_p function.
- 'primes' counts or lists primes in an interval, using a sieve.
- 'qcn' is an example use of mpz_kronecker_ui to estimate quadratic class numbers.
- The 'perl' subdirectory is a comprehensive perl interface to GMP. See 'demos/perl/INSTALL' for more information. Documentation is in POD format in 'demos/perl/GMP.pm'.

3.11 Efficiency

Small operands

On small operands, the time for function call overheads and memory allocation can be significant in comparison to actual calculation. This is unavoidable in a general purpose variable precision library, although GMP attempts to be as efficient as it can on both large and small operands.

Static Linking

On some CPUs, in particular the x86s, the static 'libgmp.a' should be used for maximum speed, since the PIC code in the shared 'libgmp.so' will have a small overhead on each function call and global data address. For many programs this will be insignificant, but for long calculations there's a gain to be had.

Initializing and clearing

Avoid excessive initializing and clearing of variables, since this can be quite time consuming, especially in comparison to otherwise fast operations like addition.

A language interpreter might want to keep a free list or stack of initialized variables ready for use. It should be possible to integrate something like that with a garbage collector too.

Reallocations

An `mpz_t` or `mpq_t` variable used to hold successively increasing values will have its memory repeatedly `realloc`ed, which could be quite slow or could fragment memory, depending on the C library. If an application can estimate the final size then `mpz_init2` or `mpz_realloc2` can be called to allocate the necessary space from the beginning (see Section 5.1 [Initializing Integers], page 28).

It doesn't matter if a size set with `mpz_init2` or `mpz_realloc2` is too small, since all functions will do a further reallocation if necessary. Badly overestimating memory required will waste space though.

`2exp` functions

It's up to an application to call functions like `mpz_mul_2exp` when appropriate. General purpose functions like `mpz_mul` make no attempt to identify powers of two or other special forms, because such inputs will usually be very rare and testing every time would be wasteful.

`ui` and `si` functions

The `ui` functions and the small number of `si` functions exist for convenience and should be used where applicable. But if for example an `mpz_t` contains a value that fits in an `unsigned long` there's no need extract it and call a `ui` function, just use the regular `mpz` function.

In-Place Operations

`mpz_abs`, `mpq_abs`, `mpf_abs`, `mpz_neg`, `mpq_neg` and `mpf_neg` are fast when used for in-place operations like `mpz_abs(x,x)`, since in the current implementation only a single field of `x` needs changing. On suitable compilers (GCC for instance) this is inlined too.

`mpz_add_ui`, `mpz_sub_ui`, `mpf_add_ui` and `mpf_sub_ui` benefit from an in-place operation like `mpz_add_ui(x,x,y)`, since usually only one or two limbs of `x` will need to be changed. The same applies to the full precision `mpz_add` etc if `y` is small. If `y` is big then cache locality may be helped, but that's all.

`mpz_mul` is currently the opposite, a separate destination is slightly better. A call like `mpz_mul(x,x,y)` will, unless `y` is only one limb, make a temporary copy of `x` before forming the result. Normally that copying will only be a tiny fraction of the time for the multiply, so this is not a particularly important consideration.

`mpz_set`, `mpq_set`, `mpq_set_num`, `mpf_set`, etc, make no attempt to recognise a copy of something to itself, so a call like `mpz_set(x,x)` will be wasteful. Naturally that would never be written deliberately, but if it might arise from two pointers to the same object then a test to avoid it might be desirable.

```
if (x != y)
  mpz_set (x, y);
```

Note that it's never worth introducing extra mpz_set calls just to get in-place operations. If a result should go to a particular variable then just direct it there and let GMP take care of data movement.

Divisibility Testing (Small Integers)

mpz_divisible_ui_p and mpz_congruent_ui_p are the best functions for testing whether an mpz_t is divisible by an individual small integer. They use an algorithm which is faster than mpz_tdiv_ui, but which gives no useful information about the actual remainder, only whether it's zero (or a particular value).

However when testing divisibility by several small integers, it's best to take a remainder modulo their product, to save multi-precision operations. For instance to test whether a number is divisible by any of 23, 29 or 31 take a remainder modulo $23 \times 29 \times 31 = 20677$ and then test that.

The division functions like mpz_tdiv_q_ui which give a quotient as well as a remainder are generally a little slower than the remainder-only functions like mpz_tdiv_ui. If the quotient is only rarely wanted then it's probably best to just take a remainder and then go back and calculate the quotient if and when it's wanted (mpz_divexact_ui can be used if the remainder is zero).

Rational Arithmetic

The mpq functions operate on mpq_t values with no common factors in the numerator and denominator. Common factors are checked-for and cast out as necessary. In general, cancelling factors every time is the best approach since it minimizes the sizes for subsequent operations.

However, applications that know something about the factorization of the values they're working with might be able to avoid some of the GCDs used for canonicalization, or swap them for divisions. For example when multiplying by a prime it's enough to check for factors of it in the denominator instead of doing a full GCD. Or when forming a big product it might be known that very little cancellation will be possible, and so canonicalization can be left to the end.

The mpq_numref and mpq_denref macros give access to the numerator and denominator to do things outside the scope of the supplied mpq functions. See Section 6.5 [Applying Integer Functions], page 44.

The canonical form for rationals allows mixed-type mpq_t and integer additions or subtractions to be done directly with multiples of the denominator. This will be somewhat faster than mpq_add. For example,

```
/* mpq increment */
mpz_add (mpq_numref(q), mpq_numref(q), mpq_denref(q));

/* mpq += unsigned long */
mpz_addmul_ui (mpq_numref(q), mpq_denref(q), 123UL);

/* mpq -= mpz */
mpz_submul (mpq_numref(q), mpq_denref(q), z);
```

Number Sequences

Functions like mpz_fac_ui, mpz_fib_ui and mpz_bin_uiui are designed for calculating isolated values. If a range of values is wanted it's probably best to call to get a starting point and iterate from there.

Text Input/Output

Hexadecimal or octal are suggested for input or output in text form. Power-of-2 bases like these can be converted much more efficiently than other bases, like decimal. For big numbers there's usually nothing of particular interest to be seen in the digits, so the base doesn't matter much.

Maybe we can hope octal will one day become the normal base for everyday use, as proposed by King Charles XII of Sweden and later reformers.

3.12 Debugging

Stack Overflow

Depending on the system, a segmentation violation or bus error might be the only indication of stack overflow. See '`--enable-alloca`' choices in Section 2.1 [Build Options], page 4, for how to address this.

In new enough versions of GCC, '`-fstack-check`' may be able to ensure an overflow is recognised by the system before too much damage is done, or '`-fstack-limit-symbol`' or '`-fstack-limit-register`' may be able to add checking if the system itself doesn't do any (see section "Options for Code Generation" in *Using the GNU Compiler Collection (GCC)*). These options must be added to the '`CFLAGS`' used in the GMP build (see Section 2.1 [Build Options], page 4), adding them just to an application will have no effect. Note also they're a slowdown, adding overhead to each function call and each stack allocation.

Heap Problems

The most likely cause of application problems with GMP is heap corruption. Failing to `init` GMP variables will have unpredictable effects, and corruption arising elsewhere in a program may well affect GMP. Initializing GMP variables more than once or failing to clear them will cause memory leaks.

In all such cases a malloc debugger is recommended. On a GNU or BSD system the standard C library `malloc` has some diagnostic facilities, see section "Allocation Debugging" in *The GNU C Library Reference Manual*, or '`man 3 malloc`'. Other possibilities, in no particular order, include

http://www.inf.ethz.ch/personal/biere/projects/ccmalloc
http://dmalloc.com
http://www.perens.com/FreeSoftware (electric fence)
http://packages.debian.org/fda
http://www.gnupdate.org/components/leakbug
http://people.redhat.com/~otaylor/memprof
http://www.cbmamiga.demon.co.uk/mpatrol

The GMP default allocation routines in '`memory.c`' also have a simple sentinel scheme which can be enabled with `#define DEBUG` in that file. This is mainly designed for detecting buffer overruns during GMP development, but might find other uses.

Stack Backtraces

On some systems the compiler options GMP uses by default can interfere with debugging. In particular on x86 and 68k systems '`-fomit-frame-pointer`' is used and this generally inhibits stack backtracing. Recompiling without such options may help while debugging, though the usual caveats about it potentially moving a memory problem or hiding a compiler bug will apply.

GNU Debugger

A sample '`.gdbinit`' is included in the distribution, showing how to call some undocumented dump functions to print GMP variables from within GDB. Note that these functions shouldn't be used in final application code since they're undocumented and may be subject to incompatible changes in future versions of GMP.

Source File Paths

GMP has multiple source files with the same name, in different directories. For example 'mpz', 'mpq', 'mpf' and 'mpfr' each have an 'init.c'. If the debugger can't already determine the right one it may help to build with absolute paths on each C file. One way to do that is to use a separate object directory with an absolute path to the source directory.

```
cd /my/build/dir
/my/source/dir/gmp-4.1.3/configure
```

This works via VPATH, and might require GNU make. Alternately it might be possible to change the .c.lo rules appropriately.

Assertion Checking

The build option '--enable-assert' is available to add some consistency checks to the library (see Section 2.1 [Build Options], page 4). These are likely to be of limited value to most applications. Assertion failures are just as likely to indicate memory corruption as a library or compiler bug.

Applications using the low-level mpn functions, however, will benefit from '--enable-assert' since it adds checks on the parameters of most such functions, many of which have subtle restrictions on their usage. Note however that only the generic C code has checks, not the assembler code, so CPU 'none' should be used for maximum checking.

Temporary Memory Checking

The build option '--enable-alloca=debug' arranges that each block of temporary memory in GMP is allocated with a separate call to malloc (or the allocation function set with mp_set_memory_functions).

This can help a malloc debugger detect accesses outside the intended bounds, or detect memory not released. In a normal build, on the other hand, temporary memory is allocated in blocks which GMP divides up for its own use, or may be allocated with a compiler builtin alloca which will go nowhere near any malloc debugger hooks.

Maximum Debuggability

To summarize the above, a GMP build for maximum debuggability would be

```
./configure --disable-shared --enable-assert \
  --enable-alloca=debug --host=none CFLAGS=-g
```

For C++, add '--enable-cxx CXXFLAGS=-g'.

Checker

The checker program (http://savannah.gnu.org/projects/checker) can be used with GMP. It contains a stub library which means GMP applications compiled with checker can use a normal GMP build.

A build of GMP with checking within GMP itself can be made. This will run very very slowly. Configure with

```
./configure --host=none-pc-linux-gnu CC=checkergcc
```

'--host=none' must be used, since the GMP assembler code doesn't support the checking scheme. The GMP C++ features cannot be used, since current versions of checker (0.9.9.1) don't yet support the standard C++ library.

Valgrind

The valgrind program (http://valgrind.kde.org/) is a memory checker for x86s. It translates and emulates machine instructions to do strong checks for uninitialized data (at the level of individual bits), memory accesses through bad pointers, and memory leaks.

Recent versions of Valgrind are getting support for MMX and SSE/SSE2 instructions, for past versions GMP will need to be configured not to use those, ie. for an x86 without them (for instance plain 'i486').

Other Problems

Any suspected bug in GMP itself should be isolated to make sure it's not an application problem, see Chapter 4 [Reporting Bugs], page 27.

3.13 Profiling

Running a program under a profiler is a good way to find where it's spending most time and where improvements can be best sought.

Depending on the system, it may be possible to get a flat profile, meaning simple timer sampling of the program counter, with no special GMP build options, just a '-p' when compiling the mainline. This is a good way to ensure minimum interference with normal operation. The necessary symbol type and size information exists in most of the GMP assembler code.

The '--enable-profiling' build option can be used to add suitable compiler flags, either for prof ('-p') or gprof ('-pg'), see Section 2.1 [Build Options], page 4. Which of the two is available and what they do will depend on the system, and possibly on support available in 'libc'. For some systems appropriate corresponding mcount calls are added to the assembler code too.

On x86 systems prof gives call counting, so that average time spent in a function can be determined. gprof, where supported, adds call graph construction, so for instance calls to mpn_add_n from mpz_add and from mpz_mul can be differentiated.

On x86 and 68k systems '-pg' and '-fomit-frame-pointer' are incompatible, so the latter is not used when gprof profiling is selected, which may result in poorer code generation. If prof profiling is selected instead it should still be possible to use gprof, but only the 'gprof -p' flat profile and call counts can be expected to be valid, not the 'gprof -q' call graph.

3.14 Autoconf

Autoconf based applications can easily check whether GMP is installed. The only thing to be noted is that GMP library symbols from version 3 onwards have prefixes like __gmpz. The following therefore would be a simple test,

```
AC_CHECK_LIB(gmp, __gmpz_init)
```

This just uses the default AC_CHECK_LIB actions for found or not found, but an application that must have GMP would want to generate an error if not found. For example,

```
AC_CHECK_LIB(gmp, __gmpz_init, , [AC_MSG_ERROR(
[GNU MP not found, see http://swox.com/gmp])])
```

If functions added in some particular version of GMP are required, then one of those can be used when checking. For example mpz_mul_si was added in GMP 3.1,

```
AC_CHECK_LIB(gmp, __gmpz_mul_si, , [AC_MSG_ERROR(
[GNU MP not found, or not 3.1 or up, see http://swox.com/gmp])])
```

An alternative would be to test the version number in 'gmp.h' using say AC_EGREP_CPP. That would make it possible to test the exact version, if some particular sub-minor release is known to be necessary.

An application that can use either GMP 2 or 3 will need to test for `__gmpz_init` (GMP 3 and up) or `mpz_init` (GMP 2), and it's also worth checking for '`libgmp2`' since Debian GNU/Linux systems used that name in the past. For example,

```
AC_CHECK_LIB(gmp, __gmpz_init, ,
  [AC_CHECK_LIB(gmp, mpz_init, ,
    [AC_CHECK_LIB(gmp2, mpz_init)])])
```

In general it's suggested that applications should simply demand a new enough GMP rather than trying to provide supplements for features not available in past versions.

Occasionally an application will need or want to know the size of a type at configuration or preprocessing time, not just with `sizeof` in the code. This can be done in the normal way with `mp_limb_t` etc, but GMP 4.0 or up is best for this, since prior versions needed certain '`-D`' defines on systems using a `long long` limb. The following would suit Autoconf 2.50 or up,

```
AC_CHECK_SIZEOF(mp_limb_t, , [#include <gmp.h>])
```

The optional `mpfr` functions are provided in a separate '`libmpfr.a`', and this might be from GMP with '`--enable-mpfr`' or from MPFR installed separately. Either way '`libmpfr`' depends on '`libgmp`', it doesn't stand alone. Currently only a static '`libmpfr.a`' will be available, not a shared library, since upward binary compatibility is not guaranteed.

```
AC_CHECK_LIB(mpfr, mpfr_add, , [AC_MSG_ERROR(
[Need MPFR either from GNU MP 4 or separate MPFR package.
See http://www.mpfr.org or http://swox.com/gmp])
```

3.15 Emacs

⟨C-h C-i⟩ (`info-lookup-symbol`) is a good way to find documentation on C functions while editing (see section "Info Documentation Lookup" in *The Emacs Editor*).

The GMP manual can be included in such lookups by putting the following in your '`.emacs`',

```
(eval-after-load "info-look"
  '(let ((mode-value (assoc 'c-mode (assoc 'symbol info-lookup-alist))))
     (setcar (nthcdr 3 mode-value)
             (cons '("(gmp)Function Index" nil "^ -.* " "\\>")
                   (nth 3 mode-value)))))
```

The same can be done for MPFR, with `(mpfr)` in place of `(gmp)`.

4 Reporting Bugs

If you think you have found a bug in the GMP library, please investigate it and report it. We have made this library available to you, and it is not too much to ask you to report the bugs you find.

Before you report a bug, check it's not already addressed in Section 2.5 [Known Build Problems], page 13, or perhaps Section 2.4 [Notes for Particular Systems], page 11. You may also want to check `http://swox.com/gmp/` for patches for this release.

Please include the following in any report,

- The GMP version number, and if pre-packaged or patched then say so.
- A test program that makes it possible for us to reproduce the bug. Include instructions on how to run the program.
- A description of what is wrong. If the results are incorrect, in what way. If you get a crash, say so.
- If you get a crash, include a stack backtrace from the debugger if it's informative ('`where`' in `gdb`, or '`$C`' in `adb`).
- Please do not send core dumps, executables or `strace`s.
- The configuration options you used when building GMP, if any.
- The name of the compiler and its version. For `gcc`, get the version with '`gcc -v`', otherwise perhaps '`what `which cc``', or similar.
- The output from running '`uname -a`'.
- The output from running '`./config.guess`', and from running '`./configfsf.guess`' (might be the same).
- If the bug is related to '`configure`', then the contents of '`config.log`'.
- If the bug is related to an '`asm`' file not assembling, then the contents of '`config.m4`' and the offending line or lines from the temporary '`mpn/tmp-<file>.s`'.

Please make an effort to produce a self-contained report, with something definite that can be tested or debugged. Vague queries or piecemeal messages are difficult to act on and don't help the development effort.

It is not uncommon that an observed problem is actually due to a bug in the compiler; the GMP code tends to explore interesting corners in compilers.

If your bug report is good, we will do our best to help you get a corrected version of the library; if the bug report is poor, we won't do anything about it (except maybe ask you to send a better report).

Send your report to: `bug-gmp@gnu.org`.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

5 Integer Functions

This chapter describes the GMP functions for performing integer arithmetic. These functions start with the prefix `mpz_`.

GMP integers are stored in objects of type `mpz_t`.

5.1 Initialization Functions

The functions for integer arithmetic assume that all integer objects are initialized. You do that by calling the function `mpz_init`. For example,

```
{
  mpz_t integ;
  mpz_init (integ);
  ...
  mpz_add (integ, ...);
  ...
  mpz_sub (integ, ...);

  /* Unless the program is about to exit, do ... */
  mpz_clear (integ);
}
```

As you can see, you can store new values any number of times, once an object is initialized.

`void mpz_init` (*mpz_t* `integer`) [Function]

Initialize *integer*, and set its value to 0.

`void mpz_init2` (*mpz_t* `integer`, *unsigned long* `n`) [Function]

Initialize *integer*, with space for *n* bits, and set its value to 0.

n is only the initial space, *integer* will grow automatically in the normal way, if necessary, for subsequent values stored. `mpz_init2` makes it possible to avoid such reallocations if a maximum size is known in advance.

`void mpz_clear` (*mpz_t* `integer`) [Function]

Free the space occupied by *integer*. Call this function for all `mpz_t` variables when you are done with them.

`void mpz_realloc2` (*mpz_t* `integer`, *unsigned long* `n`) [Function]

Change the space allocated for *integer* to *n* bits. The value in *integer* is preserved if it fits, or is set to 0 if not.

This function can be used to increase the space for a variable in order to avoid repeated automatic reallocations, or to decrease it to give memory back to the heap.

`void mpz_array_init` (*mpz_t* `integer_array`[], *size_t* `array_size`, *mp_size_t* `fixed_num_bits`) [Function]

This is a special type of initialization. **Fixed** space of *fixed_num_bits* bits is allocated to each of the *array_size* integers in *integer_array*.

The space will not be automatically increased, unlike the normal `mpz_init`, but instead an application must ensure it's sufficient for any value stored. The following space requirements apply to various functions,

- `mpz_abs`, `mpz_neg`, `mpz_set`, `mpz_set_si` and `mpz_set_ui` need room for the value they store.
- `mpz_add`, `mpz_add_ui`, `mpz_sub` and `mpz_sub_ui` need room for the larger of the two operands, plus an extra `mp_bits_per_limb`.
- `mpz_mul`, `mpz_mul_ui` and `mpz_mul_ui` need room for the sum of the number of bits in their operands, but each rounded up to a multiple of `mp_bits_per_limb`.
- `mpz_swap` can be used between two array variables, but not between an array and a normal variable.

For other functions, or if in doubt, the suggestion is to calculate in a regular `mpz_init` variable and copy the result to an array variable with `mpz_set`.

`mpz_array_init` can reduce memory usage in algorithms that need large arrays of integers, since it avoids allocating and reallocating lots of small memory blocks. There is no way to free the storage allocated by this function. Don't call `mpz_clear`!

`void * _mpz_realloc` (*mpz_t* **`integer`**, *mp_size_t* **`new_alloc`**) [Function]

Change the space for *integer* to *new_alloc* limbs. The value in *integer* is preserved if it fits, or is set to 0 if not. The return value is not useful to applications and should be ignored.

`mpz_realloc2` is the preferred way to accomplish allocation changes like this. `mpz_realloc2` and `_mpz_realloc` are the same except that `_mpz_realloc` takes the new size in limbs.

5.2 Assignment Functions

These functions assign new values to already initialized integers (see Section 5.1 [Initializing Integers], page 28).

`void mpz_set` (*mpz_t* **`rop`**, *mpz_t* **`op`**) [Function]
`void mpz_set_ui` (*mpz_t* **`rop`**, *unsigned long int* **`op`**) [Function]
`void mpz_set_si` (*mpz_t* **`rop`**, *signed long int* **`op`**) [Function]
`void mpz_set_d` (*mpz_t* **`rop`**, *double* **`op`**) [Function]
`void mpz_set_q` (*mpz_t* **`rop`**, *mpq_t* **`op`**) [Function]
`void mpz_set_f` (*mpz_t* **`rop`**, *mpf_t* **`op`**) [Function]

Set the value of *rop* from *op*.

`mpz_set_d`, `mpz_set_q` and `mpz_set_f` truncate *op* to make it an integer.

`int mpz_set_str` (*mpz_t* **`rop`**, *char* ***`str`**, *int* **`base`**) [Function]

Set the value of *rop* from *str*, a null-terminated C string in base *base*. White space is allowed in the string, and is simply ignored. The base may vary from 2 to 36. If *base* is 0, the actual base is determined from the leading characters: if the first two characters are "0x" or "0X", hexadecimal is assumed, otherwise if the first character is "0", octal is assumed, otherwise decimal is assumed.

This function returns 0 if the entire string is a valid number in base *base*. Otherwise it returns −1.

`void mpz_swap` (*mpz_t* **`rop1`**, *mpz_t* **`rop2`**) [Function]

Swap the values *rop1* and *rop2* efficiently.

5.3 Combined Initialization and Assignment Functions

For convenience, GMP provides a parallel series of initialize-and-set functions which initialize the output and then store the value there. These functions' names have the form `mpz_init_set...`

Here is an example of using one:

```
{
  mpz_t pie;
  mpz_init_set_str (pie, "3141592653589793238462643383279502884", 10);
  ...
  mpz_sub (pie, ...);
  ...
  mpz_clear (pie);
}
```

Once the integer has been initialized by any of the `mpz_init_set...` functions, it can be used as the source or destination operand for the ordinary integer functions. Don't use an initialize-and-set function on a variable already initialized!

`void mpz_init_set` (*mpz_t* **rop**, *mpz_t* **op**) [Function]
`void mpz_init_set_ui` (*mpz_t* **rop**, *unsigned long int* **op**) [Function]
`void mpz_init_set_si` (*mpz_t* **rop**, *signed long int* **op**) [Function]
`void mpz_init_set_d` (*mpz_t* **rop**, *double* **op**) [Function]

Initialize *rop* with limb space and set the initial numeric value from *op*.

`int mpz_init_set_str` (*mpz_t* **rop**, *char* ***str**, *int* **base**) [Function]

Initialize *rop* and set its value like `mpz_set_str` (see its documentation above for details).

If the string is a correct base *base* number, the function returns 0; if an error occurs it returns −1. *rop* is initialized even if an error occurs. (I.e., you have to call `mpz_clear` for it.)

5.4 Conversion Functions

This section describes functions for converting GMP integers to standard C types. Functions for converting *to* GMP integers are described in Section 5.2 [Assigning Integers], page 29 and Section 5.12 [I/O of Integers], page 38.

`unsigned long int mpz_get_ui` (*mpz_t* **op**) [Function]

Return the value of *op* as an `unsigned long`.

If *op* is too big to fit an `unsigned long` then just the least significant bits that do fit are returned. The sign of *op* is ignored, only the absolute value is used.

`signed long int mpz_get_si` (*mpz_t* **op**) [Function]

If *op* fits into a `signed long int` return the value of *op*. Otherwise return the least significant part of *op*, with the same sign as *op*.

If *op* is too big to fit in a `signed long int`, the returned result is probably not very useful. To find out if the value will fit, use the function `mpz_fits_slong_p`.

`double mpz_get_d` (*mpz_t* **op**) [Function]

Convert *op* to a `double`.

`double mpz_get_d_2exp` (*signed long int* ***exp**, *mpz_t* **op**) [Function]

Find *d* and *exp* such that $d \times 2^{exp}$, with $0.5 \le |d| < 1$, is a good approximation to *op*.

`char * mpz_get_str` (*char* ***str**, *int* **base**, *mpz_t* **op**) [Function]

Convert *op* to a string of digits in base *base*. The base may vary from 2 to 36.

If *str* is `NULL`, the result string is allocated using the current allocation function (see Chapter 14 [Custom Allocation], page 82). The block will be `strlen(str)+1` bytes, that being exactly enough for the string and null-terminator.

If *str* is not `NULL`, it should point to a block of storage large enough for the result, that being `mpz_sizeinbase (op, base) + 2`. The two extra bytes are for a possible minus sign, and the null-terminator.

A pointer to the result string is returned, being either the allocated block, or the given *str*.

`mp_limb_t mpz_getlimbn` (*mpz_t* `op`, *mp_size_t* `n`) [Function]

Return limb number *n* from *op*. The sign of *op* is ignored, just the absolute value is used. The least significant limb is number 0.

`mpz_size` can be used to find how many limbs make up *op*. `mpz_getlimbn` returns zero if *n* is outside the range 0 to `mpz_size(op)-1`.

5.5 Arithmetic Functions

`void mpz_add` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_add_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Set *rop* to $op1 + op2$.

`void mpz_sub` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_sub_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]
`void mpz_ui_sub` (*mpz_t* `rop`, *unsigned long int* `op1`, *mpz_t* `op2`) [Function]

Set *rop* to $op1 - op2$.

`void mpz_mul` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_mul_si` (*mpz_t* `rop`, *mpz_t* `op1`, *long int* `op2`) [Function]
`void mpz_mul_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Set *rop* to $op1 \times op2$.

`void mpz_addmul` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_addmul_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Set *rop* to $rop + op1 \times op2$.

`void mpz_submul` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_submul_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Set *rop* to $rop - op1 \times op2$.

`void mpz_mul_2exp` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Set *rop* to $op1 \times 2^{op2}$. This operation can also be defined as a left shift by *op2* bits.

`void mpz_neg` (*mpz_t* `rop`, *mpz_t* `op`) [Function]

Set *rop* to $-op$.

`void mpz_abs` (*mpz_t* `rop`, *mpz_t* `op`) [Function]

Set *rop* to the absolute value of *op*.

5.6 Division Functions

Division is undefined if the divisor is zero. Passing a zero divisor to the division or modulo functions (including the modular powering functions `mpz_powm` and `mpz_powm_ui`), will cause an intentional division by zero. This lets a program handle arithmetic exceptions in these functions the same way as for normal C `int` arithmetic.

`void mpz_cdiv_q` (*mpz_t* `q`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_cdiv_r` (*mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_cdiv_qr` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`unsigned long int mpz_cdiv_q_ui` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_cdiv_r_ui` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_cdiv_qr_ui` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_cdiv_ui` (*mpz_t* `n`, *unsigned long int* `d`) [Function]
`void mpz_cdiv_q_2exp` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `b`) [Function]
`void mpz_cdiv_r_2exp` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `b`) [Function]

`void mpz_fdiv_q` (*mpz_t* `q`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_fdiv_r` (*mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_fdiv_qr` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`unsigned long int mpz_fdiv_q_ui` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_fdiv_r_ui` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_fdiv_qr_ui` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_fdiv_ui` (*mpz_t* `n`, *unsigned long int* `d`) [Function]
`void mpz_fdiv_q_2exp` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `b`) [Function]
`void mpz_fdiv_r_2exp` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `b`) [Function]

`void mpz_tdiv_q` (*mpz_t* `q`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_tdiv_r` (*mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_tdiv_qr` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`unsigned long int mpz_tdiv_q_ui` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_tdiv_r_ui` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_tdiv_qr_ui` (*mpz_t* `q`, *mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]
`unsigned long int mpz_tdiv_ui` (*mpz_t* `n`, *unsigned long int* `d`) [Function]
`void mpz_tdiv_q_2exp` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long int* `b`) [Function]
`void mpz_tdiv_r_2exp` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `b`) [Function]

Divide n by d, forming a quotient q and/or remainder r. For the `2exp` functions, $d = 2^b$. The rounding is in three styles, each suiting different applications.

- `cdiv` rounds q up towards $+\infty$, and r will have the opposite sign to d. The `c` stands for "ceil".
- `fdiv` rounds q down towards $-\infty$, and r will have the same sign as d. The `f` stands for "floor".

- `tdiv` rounds q towards zero, and r will have the same sign as n. The `t` stands for "truncate".

In all cases q and r will satisfy $n = qd + r$, and r will satisfy $0 \leq |r| < |d|$.

The `q` functions calculate only the quotient, the `r` functions only the remainder, and the `qr` functions calculate both. Note that for `qr` the same variable cannot be passed for both q and r, or results will be unpredictable.

For the `ui` variants the return value is the remainder, and in fact returning the remainder is all the `div_ui` functions do. For `tdiv` and `cdiv` the remainder can be negative, so for those the return value is the absolute value of the remainder.

The `2exp` functions are right shifts and bit masks, but of course rounding the same as the other functions. For positive n both `mpz_fdiv_q_2exp` and `mpz_tdiv_q_2exp` are simple bitwise right shifts. For negative n, `mpz_fdiv_q_2exp` is effectively an arithmetic right shift treating n as twos complement the same as the bitwise logical functions do, whereas `mpz_tdiv_q_2exp` effectively treats n as sign and magnitude.

`void mpz_mod` (*mpz_t* `r`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`unsigned long int mpz_mod_ui` (*mpz_t* `r`, *mpz_t* `n`, *unsigned long int* `d`) [Function]

Set r to n `mod` d. The sign of the divisor is ignored; the result is always non-negative.

`mpz_mod_ui` is identical to `mpz_fdiv_r_ui` above, returning the remainder as well as setting r. See `mpz_fdiv_ui` above if only the return value is wanted.

`void mpz_divexact` (*mpz_t* `q`, *mpz_t* `n`, *mpz_t* `d`) [Function]
`void mpz_divexact_ui` (*mpz_t* `q`, *mpz_t* `n`, *unsigned long* `d`) [Function]

Set q to n/d. These functions produce correct results only when it is known in advance that d divides n.

These routines are much faster than the other division functions, and are the best choice when exact division is known to occur, for example reducing a rational to lowest terms.

`int mpz_divisible_p` (*mpz_t* `n`, *mpz_t* `d`) [Function]
`int mpz_divisible_ui_p` (*mpz_t* `n`, *unsigned long int* `d`) [Function]
`int mpz_divisible_2exp_p` (*mpz_t* `n`, *unsigned long int* `b`) [Function]

Return non-zero if n is exactly divisible by d, or in the case of `mpz_divisible_2exp_p` by 2^b.

`int mpz_congruent_p` (*mpz_t* `n`, *mpz_t* `c`, *mpz_t* `d`) [Function]
`int mpz_congruent_ui_p` (*mpz_t* `n`, *unsigned long int* `c`, *unsigned long int* `d`) [Function]
`int mpz_congruent_2exp_p` (*mpz_t* `n`, *mpz_t* `c`, *unsigned long int* `b`) [Function]

Return non-zero if n is congruent to c modulo d, or in the case of `mpz_congruent_2exp_p` modulo 2^b.

5.7 Exponentiation Functions

`void mpz_powm (mpz_t rop, mpz_t base, mpz_t exp, mpz_t mod)` [Function]
`void mpz_powm_ui (mpz_t rop, mpz_t base, unsigned long int exp, mpz_t mod)` [Function]

Set *rop* to $base^{exp}$ mod *mod*.

Negative *exp* is supported if an inverse $base^{-1}$ mod *mod* exists (see `mpz_invert` in Section 5.9 [Number Theoretic Functions], page 34). If an inverse doesn't exist then a divide by zero is raised.

`void mpz_pow_ui (mpz_t rop, mpz_t base, unsigned long int exp)` [Function]
`void mpz_ui_pow_ui (mpz_t rop, unsigned long int base, unsigned long int exp)` [Function]

Set *rop* to $base^{exp}$. The case 0^0 yields 1.

5.8 Root Extraction Functions

`int mpz_root (mpz_t rop, mpz_t op, unsigned long int n)` [Function]

Set *rop* to $\lfloor \sqrt[n]{op} \rfloor$, the truncated integer part of the *n*th root of *op*. Return non-zero if the computation was exact, i.e., if *op* is *rop* to the *n*th power.

`void mpz_sqrt (mpz_t rop, mpz_t op)` [Function]

Set *rop* to $\lfloor \sqrt{op} \rfloor$, the truncated integer part of the square root of *op*.

`void mpz_sqrtrem (mpz_t rop1, mpz_t rop2, mpz_t op)` [Function]

Set *rop1* to $\lfloor \sqrt{op} \rfloor$, like `mpz_sqrt`. Set *rop2* to the remainder $(op - rop1^2)$, which will be zero if *op* is a perfect square.

If *rop1* and *rop2* are the same variable, the results are undefined.

`int mpz_perfect_power_p (mpz_t op)` [Function]

Return non-zero if *op* is a perfect power, i.e., if there exist integers a and b, with $b > 1$, such that $op = a^b$.

Under this definition both 0 and 1 are considered to be perfect powers. Negative values of *op* are accepted, but of course can only be odd perfect powers.

`int mpz_perfect_square_p (mpz_t op)` [Function]

Return non-zero if *op* is a perfect square, i.e., if the square root of *op* is an integer. Under this definition both 0 and 1 are considered to be perfect squares.

5.9 Number Theoretic Functions

`int mpz_probab_prime_p (mpz_t n, int reps)` [Function]

Determine whether *n* is prime. Return 2 if *n* is definitely prime, return 1 if *n* is probably prime (without being certain), or return 0 if *n* is definitely composite.

This function does some trial divisions, then some Miller-Rabin probabilistic primality tests. *reps* controls how many such tests are done, 5 to 10 is a reasonable number, more will reduce the chances of a composite being returned as "probably prime".

Miller-Rabin and similar tests can be more properly called compositeness tests. Numbers which fail are known to be composite but those which pass might be prime or might be composite. Only a few composites pass, hence those which pass are considered probably prime.

`void mpz_nextprime` (*mpz_t* `rop`, *mpz_t* `op`) [Function]

Set *rop* to the next prime greater than *op*.

This function uses a probabilistic algorithm to identify primes. For practical purposes it's adequate, the chance of a composite passing will be extremely small.

`void mpz_gcd` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]

Set *rop* to the greatest common divisor of *op1* and *op2*. The result is always positive even if one or both input operands are negative.

`unsigned long int mpz_gcd_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long int* `op2`) [Function]

Compute the greatest common divisor of *op1* and *op2*. If *rop* is not `NULL`, store the result there.

If the result is small enough to fit in an `unsigned long int`, it is returned. If the result does not fit, 0 is returned, and the result is equal to the argument *op1*. Note that the result will always fit if *op2* is non-zero.

`void mpz_gcdext` (*mpz_t* `g`, *mpz_t* `s`, *mpz_t* `t`, *mpz_t* `a`, *mpz_t* `b`) [Function]

Set *g* to the greatest common divisor of *a* and *b*, and in addition set *s* and *t* to coefficients satisfying $as + bt = g$. *g* is always positive, even if one or both of *a* and *b* are negative.

If *t* is `NULL` then that value is not computed.

`void mpz_lcm` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]
`void mpz_lcm_ui` (*mpz_t* `rop`, *mpz_t* `op1`, *unsigned long* `op2`) [Function]

Set *rop* to the least common multiple of *op1* and *op2*. *rop* is always positive, irrespective of the signs of *op1* and *op2*. *rop* will be zero if either *op1* or *op2* is zero.

`int mpz_invert` (*mpz_t* `rop`, *mpz_t* `op1`, *mpz_t* `op2`) [Function]

Compute the inverse of *op1* modulo *op2* and put the result in *rop*. If the inverse exists, the return value is non-zero and *rop* will satisfy $0 \le rop < op2$. If an inverse doesn't exist the return value is zero and *rop* is undefined.

`int mpz_jacobi` (*mpz_t* `a`, *mpz_t* `b`) [Function]

Calculate the Jacobi symbol $\left(\frac{a}{b}\right)$. This is defined only for *b* odd.

`int mpz_legendre` (*mpz_t* `a`, *mpz_t* `p`) [Function]

Calculate the Legendre symbol $\left(\frac{a}{p}\right)$. This is defined only for *p* an odd positive prime, and for such *p* it's identical to the Jacobi symbol.

`int mpz_kronecker` (*mpz_t* `a`, *mpz_t* `b`) [Function]
`int mpz_kronecker_si` (*mpz_t* `a`, *long* `b`) [Function]
`int mpz_kronecker_ui` (*mpz_t* `a`, *unsigned long* `b`) [Function]
`int mpz_si_kronecker` (*long* `a`, *mpz_t* `b`) [Function]
`int mpz_ui_kronecker` (*unsigned long* `a`, *mpz_t* `b`) [Function]

Calculate the Jacobi symbol $\left(\frac{a}{b}\right)$ with the Kronecker extension $\left(\frac{a}{2}\right) = \left(\frac{2}{a}\right)$ when *a* odd, or $\left(\frac{a}{2}\right) = 0$ when *a* even.

When b is odd the Jacobi symbol and Kronecker symbol are identical, so `mpz_kronecker_ui` etc can be used for mixed precision Jacobi symbols too.

For more information see Henri Cohen section 1.4.2 (see Appendix B [References], page 113), or any number theory textbook. See also the example program 'demos/qcn.c' which uses `mpz_kronecker_ui`.

`unsigned long int mpz_remove` (*mpz_t* **rop**, *mpz_t* **op**, *mpz_t* **f**) [Function]

Remove all occurrences of the factor *f* from *op* and store the result in *rop*. The return value is how many such occurrences were removed.

`void mpz_fac_ui` (*mpz_t* **rop**, *unsigned long int* **op**) [Function]

Set *rop* to *op*!, the factorial of *op*.

`void mpz_bin_ui` (*mpz_t* **rop**, *mpz_t* **n**, *unsigned long int* **k**) [Function]
`void mpz_bin_uiui` (*mpz_t* **rop**, *unsigned long int* **n**, *unsigned long int* **k**) [Function]

Compute the binomial coefficient $\binom{n}{k}$ and store the result in *rop*. Negative values of n are supported by `mpz_bin_ui`, using the identity $\binom{-n}{k} = (-1)^k \binom{n+k-1}{k}$, see Knuth volume 1 section 1.2.6 part G.

`void mpz_fib_ui` (*mpz_t* **fn**, *unsigned long int* **n**) [Function]
`void mpz_fib2_ui` (*mpz_t* **fn**, *mpz_t* **fnsub1**, *unsigned long int* **n**) [Function]

`mpz_fib_ui` sets *fn* to to F_n, the n'th Fibonacci number. `mpz_fib2_ui` sets *fn* to F_n, and *fnsub1* to F_{n-1}.

These functions are designed for calculating isolated Fibonacci numbers. When a sequence of values is wanted it's best to start with `mpz_fib2_ui` and iterate the defining $F_{n+1} = F_n + F_{n-1}$ or similar.

`void mpz_lucnum_ui` (*mpz_t* **ln**, *unsigned long int* **n**) [Function]
`void mpz_lucnum2_ui` (*mpz_t* **ln**, *mpz_t* **lnsub1**, *unsigned long int* **n**) [Function]

`mpz_lucnum_ui` sets *ln* to to L_n, the n'th Lucas number. `mpz_lucnum2_ui` sets *ln* to L_n, and *lnsub1* to L_{n-1}.

These functions are designed for calculating isolated Lucas numbers. When a sequence of values is wanted it's best to start with `mpz_lucnum2_ui` and iterate the defining $L_{n+1} = L_n + L_{n-1}$ or similar.

The Fibonacci numbers and Lucas numbers are related sequences, so it's never necessary to call both `mpz_fib2_ui` and `mpz_lucnum2_ui`. The formulas for going from Fibonacci to Lucas can be found in Section 16.7.4 [Lucas Numbers Algorithm], page 101, the reverse is straightforward too.

5.10 Comparison Functions

`int mpz_cmp` (*mpz_t* **op1**, *mpz_t* **op2**) [Function]
`int mpz_cmp_d` (*mpz_t* **op1**, *double* **op2**) [Function]
`int mpz_cmp_si` (*mpz_t* **op1**, *signed long int* **op2**) [Macro]
`int mpz_cmp_ui` (*mpz_t* **op1**, *unsigned long int* **op2**) [Macro]

Compare *op1* and *op2*. Return a positive value if $op1 > op2$, zero if $op1 = op2$, or a negative value if $op1 < op2$.

Note that `mpz_cmp_ui` and `mpz_cmp_si` are macros and will evaluate their arguments more than once.

`int mpz_cmpabs (mpz_t op1, mpz_t op2)` [Function]
`int mpz_cmpabs_d (mpz_t op1, double op2)` [Function]
`int mpz_cmpabs_ui (mpz_t op1, unsigned long int op2)` [Function]

Compare the absolute values of *op1* and *op2*. Return a positive value if $|op1| > |op2|$, zero if $|op1| = |op2|$, or a negative value if $|op1| < |op2|$.

`int mpz_sgn (mpz_t op)` [Macro]

Return $+1$ if $op > 0$, 0 if $op = 0$, and -1 if $op < 0$.

This function is actually implemented as a macro. It evaluates its argument multiple times.

5.11 Logical and Bit Manipulation Functions

These functions behave as if twos complement arithmetic were used (although sign-magnitude is the actual implementation). The least significant bit is number 0.

`void mpz_and (mpz_t rop, mpz_t op1, mpz_t op2)` [Function]

Set *rop* to *op1* bitwise-and *op2*.

`void mpz_ior (mpz_t rop, mpz_t op1, mpz_t op2)` [Function]

Set *rop* to *op1* bitwise inclusive-or *op2*.

`void mpz_xor (mpz_t rop, mpz_t op1, mpz_t op2)` [Function]

Set *rop* to *op1* bitwise exclusive-or *op2*.

`void mpz_com (mpz_t rop, mpz_t op)` [Function]

Set *rop* to the one's complement of *op*.

`unsigned long int mpz_popcount (mpz_t op)` [Function]

If $op \geq 0$, return the population count of *op*, which is the number of 1 bits in the binary representation. If $op < 0$, the number of 1s is infinite, and the return value is *ULONG_MAX*, the largest possible `unsigned long`.

`unsigned long int mpz_hamdist (mpz_t op1, mpz_t op2)` [Function]

If *op1* and *op2* are both ≥ 0 or both < 0, return the hamming distance between the two operands, which is the number of bit positions where *op1* and *op2* have different bit values. If one operand is ≥ 0 and the other < 0 then the number of bits different is infinite, and the return value is *ULONG_MAX*, the largest possible `unsigned long`.

`unsigned long int mpz_scan0 (mpz_t op, unsigned long int starting_bit)` [Function]
`unsigned long int mpz_scan1 (mpz_t op, unsigned long int starting_bit)` [Function]

Scan *op*, starting from bit *starting_bit*, towards more significant bits, until the first 0 or 1 bit (respectively) is found. Return the index of the found bit.

If the bit at *starting_bit* is already what's sought, then *starting_bit* is returned.

If there's no bit found, then *ULONG_MAX* is returned. This will happen in `mpz_scan0` past the end of a positive number, or `mpz_scan1` past the end of a negative.

`void mpz_setbit (mpz_t rop, unsigned long int bit_index)` [Function]

Set bit *bit_index* in *rop*.

`void mpz_clrbit` (*mpz_t* **rop**, *unsigned long int* **bit_index**) [Function]

Clear bit *bit_index* in *rop*.

`int mpz_tstbit` (*mpz_t* **op**, *unsigned long int* **bit_index**) [Function]

Test bit *bit_index* in *op* and return 0 or 1 accordingly.

5.12 Input and Output Functions

Functions that perform input from a stdio stream, and functions that output to a stdio stream. Passing a `NULL` pointer for a *stream* argument to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, it is a good idea to include '`stdio.h`' before '`gmp.h`', since that will allow '`gmp.h`' to define prototypes for these functions.

`size_t mpz_out_str` (*FILE* ***stream**, *int* **base**, *mpz_t* **op**) [Function]

Output *op* on stdio stream *stream*, as a string of digits in base *base*. The base may vary from 2 to 36.

Return the number of bytes written, or if an error occurred, return 0.

`size_t mpz_inp_str` (*mpz_t* **rop**, *FILE* ***stream**, *int* **base**) [Function]

Input a possibly white-space preceded string in base *base* from stdio stream *stream*, and put the read integer in *rop*. The base may vary from 2 to 36. If *base* is 0, the actual base is determined from the leading characters: if the first two characters are '0x' or '0X', hexadecimal is assumed, otherwise if the first character is '0', octal is assumed, otherwise decimal is assumed.

Return the number of bytes read, or if an error occurred, return 0.

`size_t mpz_out_raw` (*FILE* ***stream**, *mpz_t* **op**) [Function]

Output *op* on stdio stream *stream*, in raw binary format. The integer is written in a portable format, with 4 bytes of size information, and that many bytes of limbs. Both the size and the limbs are written in decreasing significance order (i.e., in big-endian).

The output can be read with `mpz_inp_raw`.

Return the number of bytes written, or if an error occurred, return 0.

The output of this can not be read by `mpz_inp_raw` from GMP 1, because of changes necessary for compatibility between 32-bit and 64-bit machines.

`size_t mpz_inp_raw` (*mpz_t* **rop**, *FILE* ***stream**) [Function]

Input from stdio stream *stream* in the format written by `mpz_out_raw`, and put the result in *rop*. Return the number of bytes read, or if an error occurred, return 0.

This routine can read the output from `mpz_out_raw` also from GMP 1, in spite of changes necessary for compatibility between 32-bit and 64-bit machines.

5.13 Random Number Functions

The random number functions of GMP come in two groups; older function that rely on a global state, and newer functions that accept a state parameter that is read and modified. Please see the Chapter 9 [Random Number Functions], page 61 for more information on how to use and not to use random number functions.

`void mpz_urandomb (mpz_t rop, gmp_randstate_t state, unsigned long int n)` [Function]

Generate a uniformly distributed random integer in the range 0 to $2^n - 1$, inclusive.

The variable *state* must be initialized by calling one of the `gmp_randinit` functions (Section 9.1 [Random State Initialization], page 61) before invoking this function.

`void mpz_urandomm (mpz_t rop, gmp_randstate_t state, mpz_t n)` [Function]

Generate a uniform random integer in the range 0 to $n - 1$, inclusive.

The variable *state* must be initialized by calling one of the `gmp_randinit` functions (Section 9.1 [Random State Initialization], page 61) before invoking this function.

`void mpz_rrandomb (mpz_t rop, gmp_randstate_t state, unsigned long int n)` [Function]

Generate a random integer with long strings of zeros and ones in the binary representation. Useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger corner-case bugs. The random number will be in the range 0 to $2^n - 1$, inclusive.

The variable *state* must be initialized by calling one of the `gmp_randinit` functions (Section 9.1 [Random State Initialization], page 61) before invoking this function.

`void mpz_random (mpz_t rop, mp_size_t max_size)` [Function]

Generate a random integer of at most *max_size* limbs. The generated random number doesn't satisfy any particular requirements of randomness. Negative random numbers are generated when *max_size* is negative.

This function is obsolete. Use `mpz_urandomb` or `mpz_urandomm` instead.

`void mpz_random2 (mpz_t rop, mp_size_t max_size)` [Function]

Generate a random integer of at most *max_size* limbs, with long strings of zeros and ones in the binary representation. Useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when *max_size* is negative.

This function is obsolete. Use `mpz_rrandomb` instead.

5.14 Integer Import and Export

`mpz_t` variables can be converted to and from arbitrary words of binary data with the following functions.

`void mpz_import (mpz_t rop, size_t count, int order, int size, int endian, size_t nails, const void *op)` [Function]

Set *rop* from an array of word data at *op*.

The parameters specify the format of the data. *count* many words are read, each *size* bytes. *order* can be 1 for most significant word first or -1 for least significant first. Within each word *endian* can be 1 for most significant byte first, -1 for least significant first, or 0 for the native endianness of the host CPU. The most significant *nails* bits of each word are skipped, this can be 0 to use the full words.

There is no sign taken from the data, *rop* will simply be a positive integer. An application can handle any sign itself, and apply it for instance with `mpz_neg`.

There are no data alignment restrictions on *op*, any address is allowed.

Here's an example converting an array of `unsigned long` data, most significant element first, and host byte order within each value.

```
unsigned long  a[20];
mpz_t          z;
mpz_import (z, 20, 1, sizeof(a[0]), 0, 0, a);
```

This example assumes the full `sizeof` bytes are used for data in the given type, which is usually true, and certainly true for `unsigned long` everywhere we know of. However on Cray vector systems it may be noted that `short` and `int` are always stored in 8 bytes (and with `sizeof` indicating that) but use only 32 or 46 bits. The *nails* feature can account for this, by passing for instance `8*sizeof(int)-INT_BIT`.

`void * mpz_export` (*void* ***rop***, *size_t* ***countp***, *int* **order**, *int* **size**, *int* **endian**, *size_t* **nails**, *mpz_t* **op**) [Function]

Fill *rop* with word data from *op*.

The parameters specify the format of the data produced. Each word will be *size* bytes and *order* can be 1 for most significant word first or -1 for least significant first. Within each word *endian* can be 1 for most significant byte first, -1 for least significant first, or 0 for the native endianness of the host CPU. The most significant *nails* bits of each word are unused and set to zero, this can be 0 to produce full words.

The number of words produced is written to `*countp`, or *countp* can be `NULL` to discard the count. *rop* must have enough space for the data, or if *rop* is `NULL` then a result array of the necessary size is allocated using the current GMP allocation function (see Chapter 14 [Custom Allocation], page 82). In either case the return value is the destination used, either *rop* or the allocated block.

If *op* is non-zero then the most significant word produced will be non-zero. If *op* is zero then the count returned will be zero and nothing written to *rop*. If *rop* is `NULL` in this case, no block is allocated, just `NULL` is returned.

The sign of *op* is ignored, just the absolute value is exported. An application can use `mpz_sgn` to get the sign and handle it as desired. (see Section 5.10 [Integer Comparisons], page 36)

There are no data alignment restrictions on *rop*, any address is allowed.

When an application is allocating space itself the required size can be determined with a calculation like the following. Since `mpz_sizeinbase` always returns at least 1, `count` here will be at least one, which avoids any portability problems with `malloc(0)`, though if `z` is zero no space at all is actually needed (or written).

```
numb = 8*size - nail;
count = (mpz_sizeinbase (z, 2) + numb-1) / numb;
p = malloc (count * size);
```

5.15 Miscellaneous Functions

`int mpz_fits_ulong_p (mpz_t op)` [Function]
`int mpz_fits_slong_p (mpz_t op)` [Function]
`int mpz_fits_uint_p (mpz_t op)` [Function]
`int mpz_fits_sint_p (mpz_t op)` [Function]
`int mpz_fits_ushort_p (mpz_t op)` [Function]
`int mpz_fits_sshort_p (mpz_t op)` [Function]

Return non-zero iff the value of *op* fits in an `unsigned long int`, `signed long int`, `unsigned int`, `signed int`, `unsigned short int`, or `signed short int`, respectively. Otherwise, return zero.

`int mpz_odd_p (mpz_t op)` [Macro]
`int mpz_even_p (mpz_t op)` [Macro]

Determine whether *op* is odd or even, respectively. Return non-zero if yes, zero if no. These macros evaluate their argument more than once.

`size_t mpz_size (mpz_t op)` [Function]

Return the size of *op* measured in number of limbs. If *op* is zero, the returned value will be zero.

`size_t mpz_sizeinbase (mpz_t op, int base)` [Function]

Return the size of *op* measured in number of digits in the given *base*. *base* can vary from 2 to 36. The sign of *op* is ignored, just the absolute value is used. The result will be either exact or 1 too big. If *base* is a power of 2, the result is always exact. If *op* is zero the return value is always 1.

This function can be used to determine the space required when converting *op* to a string. The right amount of allocation is normally two more than the value returned by `mpz_sizeinbase`, one extra for a minus sign and one for the null-terminator.

It will be noted that `mpz_sizeinbase(op,2)` can be used to locate the most significant 1 bit in *op*, counting from 1. (Unlike the bitwise functions which start from 0, See Section 5.11 [Logical and Bit Manipulation Functions], page 37.)

6 Rational Number Functions

This chapter describes the GMP functions for performing arithmetic on rational numbers. These functions start with the prefix `mpq_`.

Rational numbers are stored in objects of type `mpq_t`.

All rational arithmetic functions assume operands have a canonical form, and canonicalize their result. The canonical from means that the denominator and the numerator have no common factors, and that the denominator is positive. Zero has the unique representation 0/1.

Pure assignment functions do not canonicalize the assigned variable. It is the responsibility of the user to canonicalize the assigned variable before any arithmetic operations are performed on that variable.

`void mpq_canonicalize` (*mpq_t* **op**) [Function]

Remove any factors that are common to the numerator and denominator of *op*, and make the denominator positive.

6.1 Initialization and Assignment Functions

`void mpq_init` (*mpq_t* **dest_rational**) [Function]

Initialize *dest_rational* and set it to 0/1. Each variable should normally only be initialized once, or at least cleared out (using the function `mpq_clear`) between each initialization.

`void mpq_clear` (*mpq_t* **rational_number**) [Function]

Free the space occupied by *rational_number*. Make sure to call this function for all `mpq_t` variables when you are done with them.

`void mpq_set` (*mpq_t* **rop**, *mpq_t* **op**) [Function]
`void mpq_set_z` (*mpq_t* **rop**, *mpz_t* **op**) [Function]

Assign *rop* from *op*.

`void mpq_set_ui` (*mpq_t* **rop**, *unsigned long int* **op1**, *unsigned long int* **op2**) [Function]
`void mpq_set_si` (*mpq_t* **rop**, *signed long int* **op1**, *unsigned long int* **op2**) [Function]

Set the value of *rop* to *op1*/*op2*. Note that if *op1* and *op2* have common factors, *rop* has to be passed to `mpq_canonicalize` before any operations are performed on *rop*.

`int mpq_set_str` (*mpq_t* **rop**, *char* ***str**, *int* **base**) [Function]

Set *rop* from a null-terminated string *str* in the given *base*.

The string can be an integer like “41” or a fraction like “41/152”. The fraction must be in canonical form (see Chapter 6 [Rational Number Functions], page 42), or if not then `mpq_canonicalize` must be called.

The numerator and optional denominator are parsed the same as in `mpz_set_str` (see Section 5.2 [Assigning Integers], page 29). White space is allowed in the string, and is simply ignored. The *base* can vary from 2 to 36, or if *base* is 0 then the leading characters are used: `0x` for hex, `0` for octal, or decimal otherwise. Note that this is done separately for the numerator and denominator, so for instance `0xEF/100` is 239/100, whereas `0xEF/0x100` is 239/256.

The return value is 0 if the entire string is a valid number, or −1 if not.

`void mpq_swap (mpq_t rop1, mpq_t rop2)` [Function]
Swap the values *rop1* and *rop2* efficiently.

6.2 Conversion Functions

`double mpq_get_d (mpq_t op)` [Function]
Convert *op* to a `double`.

`void mpq_set_d (mpq_t rop, double op)` [Function]
`void mpq_set_f (mpq_t rop, mpf_t op)` [Function]
Set *rop* to the value of *op*, without rounding.

`char * mpq_get_str (char *str, int base, mpq_t op)` [Function]
Convert *op* to a string of digits in base *base*. The base may vary from 2 to 36. The string will be of the form '`num/den`', or if the denominator is 1 then just '`num`'.

If *str* is `NULL`, the result string is allocated using the current allocation function (see Chapter 14 [Custom Allocation], page 82). The block will be `strlen(str)+1` bytes, that being exactly enough for the string and null-terminator.

If *str* is not `NULL`, it should point to a block of storage large enough for the result, that being

```
mpz_sizeinbase (mpq_numref(op), base)
+ mpz_sizeinbase (mpq_denref(op), base) + 3
```

The three extra bytes are for a possible minus sign, possible slash, and the null-terminator.

A pointer to the result string is returned, being either the allocated block, or the given *str*.

6.3 Arithmetic Functions

`void mpq_add (mpq_t sum, mpq_t addend1, mpq_t addend2)` [Function]
Set *sum* to *addend1* + *addend2*.

`void mpq_sub (mpq_t difference, mpq_t minuend, mpq_t subtrahend)` [Function]
Set *difference* to *minuend* − *subtrahend*.

`void mpq_mul (mpq_t product, mpq_t multiplier, mpq_t multiplicand)` [Function]
Set *product* to *multiplier* × *multiplicand*.

`void mpq_mul_2exp (mpq_t rop, mpq_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1 \times 2^{op2}$.

`void mpq_div (mpq_t quotient, mpq_t dividend, mpq_t divisor)` [Function]
Set *quotient* to *dividend*/*divisor*.

`void mpq_div_2exp (mpq_t rop, mpq_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1/2^{op2}$.

`void mpq_neg (mpq_t negated_operand, mpq_t operand)` [Function]
Set *negated_operand* to −*operand*.

`void mpq_abs (mpq_t rop, mpq_t op)` [Function]
Set *rop* to the absolute value of *op*.

`void mpq_inv (mpq_t inverted_number, mpq_t number)` [Function]

Set *inverted_number* to 1/*number*. If the new denominator is zero, this routine will divide by zero.

6.4 Comparison Functions

`int mpq_cmp (mpq_t op1, mpq_t op2)` [Function]

Compare *op1* and *op2*. Return a positive value if $op1 > op2$, zero if $op1 = op2$, and a negative value if $op1 < op2$.

To determine if two rationals are equal, `mpq_equal` is faster than `mpq_cmp`.

`int mpq_cmp_ui (mpq_t op1, unsigned long int num2, unsigned long int den2)` [Macro]
`int mpq_cmp_si (mpq_t op1, long int num2, unsigned long int den2)` [Macro]

Compare *op1* and *num2*/*den2*. Return a positive value if $op1 > num2/den2$, zero if $op1 = num2/den2$, and a negative value if $op1 < num2/den2$.

num2 and *den2* are allowed to have common factors.

These functions are implemented as a macros and evaluate their arguments multiple times.

`int mpq_sgn (mpq_t op)` [Macro]

Return $+1$ if $op > 0$, 0 if $op = 0$, and -1 if $op < 0$.

This function is actually implemented as a macro. It evaluates its arguments multiple times.

`int mpq_equal (mpq_t op1, mpq_t op2)` [Function]

Return non-zero if *op1* and *op2* are equal, zero if they are non-equal. Although `mpq_cmp` can be used for the same purpose, this function is much faster.

6.5 Applying Integer Functions to Rationals

The set of `mpq` functions is quite small. In particular, there are few functions for either input or output. The following functions give direct access to the numerator and denominator of an `mpq_t`.

Note that if an assignment to the numerator and/or denominator could take an `mpq_t` out of the canonical form described at the start of this chapter (see Chapter 6 [Rational Number Functions], page 42) then `mpq_canonicalize` must be called before any other `mpq` functions are applied to that `mpq_t`.

`mpz_t mpq_numref (mpq_t op)` [Macro]
`mpz_t mpq_denref (mpq_t op)` [Macro]

Return a reference to the numerator and denominator of *op*, respectively. The `mpz` functions can be used on the result of these macros.

`void mpq_get_num (mpz_t numerator, mpq_t rational)` [Function]
`void mpq_get_den (mpz_t denominator, mpq_t rational)` [Function]
`void mpq_set_num (mpq_t rational, mpz_t numerator)` [Function]
`void mpq_set_den (mpq_t rational, mpz_t denominator)` [Function]

Get or set the numerator or denominator of a rational. These functions are equivalent to calling `mpz_set` with an appropriate `mpq_numref` or `mpq_denref`. Direct use of `mpq_numref` or `mpq_denref` is recommended instead of these functions.

6.6 Input and Output Functions

When using any of these functions, it's a good idea to include 'stdio.h' before 'gmp.h', since that will allow 'gmp.h' to define prototypes for these functions.

Passing a NULL pointer for a *stream* argument to any of these functions will make them read from stdin and write to stdout, respectively.

size_t mpq_out_str (*FILE* **stream*, *int* *base*, *mpq_t* *op*) [Function]

Output *op* on stdio stream *stream*, as a string of digits in base *base*. The base may vary from 2 to 36. Output is in the form 'num/den' or if the denominator is 1 then just 'num'.

Return the number of bytes written, or if an error occurred, return 0.

size_t mpq_inp_str (*mpq_t* *rop*, *FILE* **stream*, *int* *base*) [Function]

Read a string of digits from *stream* and convert them to a rational in *rop*. Any initial white-space characters are read and discarded. Return the number of characters read (including white space), or 0 if a rational could not be read.

The input can be a fraction like '17/63' or just an integer like '123'. Reading stops at the first character not in this form, and white space is not permitted within the string. If the input might not be in canonical form, then mpq_canonicalize must be called (see Chapter 6 [Rational Number Functions], page 42).

The *base* can be between 2 and 36, or can be 0 in which case the leading characters of the string determine the base, '0x' or '0X' for hexadecimal, '0' for octal, or decimal otherwise. The leading characters are examined separately for the numerator and denominator of a fraction, so for instance '0x10/11' is 16/11, whereas '0x10/0x11' is 16/17.

7 Floating-point Functions

GMP floating point numbers are stored in objects of type `mpf_t` and functions operating on them have an `mpf_` prefix.

The mantissa of each float has a user-selectable precision, limited only by available memory. Each variable has its own precision, and that can be increased or decreased at any time.

The exponent of each float is a fixed precision, one machine word on most systems. In the current implementation the exponent is a count of limbs, so for example on a 32-bit system this means a range of roughly $2^{-68719476768}$ to $2^{68719476736}$, or on a 64-bit system this will be greater. Note however `mpf_get_str` can only return an exponent which fits an `mp_exp_t` and currently `mpf_set_str` doesn't accept exponents bigger than a `long`.

Each variable keeps a size for the mantissa data actually in use. This means that if a float is exactly represented in only a few bits then only those bits will be used in a calculation, even if the selected precision is high.

All calculations are performed to the precision of the destination variable. Each function is defined to calculate with "infinite precision" followed by a truncation to the destination precision, but of course the work done is only what's needed to determine a result under that definition.

The precision selected for a variable is a minimum value, GMP may increase it a little to facilitate efficient calculation. Currently this means rounding up to a whole limb, and then sometimes having a further partial limb, depending on the high limb of the mantissa. But applications shouldn't be concerned by such details.

The mantissa in stored in binary, as might be imagined from the fact precisions are expressed in bits. One consequence of this is that decimal fractions like 0.1 cannot be represented exactly. The same is true of plain IEEE `double` floats. This makes both highly unsuitable for calculations involving money or other values that should be exact decimal fractions. (Suitably scaled integers, or perhaps rationals, are better choices.)

`mpf` functions and variables have no special notion of infinity or not-a-number, and applications must take care not to overflow the exponent or results will be unpredictable. This might change in a future release.

Note that the `mpf` functions are *not* intended as a smooth extension to IEEE P754 arithmetic. In particular results obtained on one computer often differ from the results on a computer with a different word size.

7.1 Initialization Functions

`void mpf_set_default_prec` (*unsigned long int* `prec`) [Function]

Set the default precision to be **at least** *prec* bits. All subsequent calls to `mpf_init` will use this precision, but previously initialized variables are unaffected.

`unsigned long int mpf_get_default_prec` (*void*) [Function]

Return the default default precision actually used.

An `mpf_t` object must be initialized before storing the first value in it. The functions `mpf_init` and `mpf_init2` are used for that purpose.

`void mpf_init (mpf_t x)` [Function]

Initialize *x* to 0. Normally, a variable should be initialized once only or at least be cleared, using `mpf_clear`, between initializations. The precision of *x* is undefined unless a default precision has already been established by a call to `mpf_set_default_prec`.

`void mpf_init2 (mpf_t x, unsigned long int prec)` [Function]

Initialize *x* to 0 and set its precision to be **at least** *prec* bits. Normally, a variable should be initialized once only or at least be cleared, using `mpf_clear`, between initializations.

`void mpf_clear (mpf_t x)` [Function]

Free the space occupied by *x*. Make sure to call this function for all `mpf_t` variables when you are done with them.

Here is an example on how to initialize floating-point variables:

```
{
  mpf_t x, y;
  mpf_init (x);          /* use default precision */
  mpf_init2 (y, 256);    /* precision at least 256 bits */
  ...
  /* Unless the program is about to exit, do ... */
  mpf_clear (x);
  mpf_clear (y);
}
```

The following three functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

`unsigned long int mpf_get_prec (mpf_t op)` [Function]

Return the current precision of *op*, in bits.

`void mpf_set_prec (mpf_t rop, unsigned long int prec)` [Function]

Set the precision of *rop* to be **at least** *prec* bits. The value in *rop* will be truncated to the new precision.

This function requires a call to `realloc`, and so should not be used in a tight loop.

`void mpf_set_prec_raw (mpf_t rop, unsigned long int prec)` [Function]

Set the precision of *rop* to be **at least** *prec* bits, without changing the memory allocated.

prec must be no more than the allocated precision for *rop*, that being the precision when *rop* was initialized, or in the most recent `mpf_set_prec`.

The value in *rop* is unchanged, and in particular if it had a higher precision than *prec* it will retain that higher precision. New values written to *rop* will use the new *prec*.

Before calling `mpf_clear` or the full `mpf_set_prec`, another `mpf_set_prec_raw` call must be made to restore *rop* to its original allocated precision. Failing to do so will have unpredictable results.

`mpf_get_prec` can be used before `mpf_set_prec_raw` to get the original allocated precision. After `mpf_set_prec_raw` it reflects the *prec* value set.

`mpf_set_prec_raw` is an efficient way to use an `mpf_t` variable at different precisions during a calculation, perhaps to gradually increase precision in an iteration, or just to use various different precisions for different purposes during a calculation.

7.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 7.1 [Initializing Floats], page 46).

`void mpf_set` (*mpf_t* **rop**, *mpf_t* **op**) [Function]
`void mpf_set_ui` (*mpf_t* **rop**, *unsigned long int* **op**) [Function]
`void mpf_set_si` (*mpf_t* **rop**, *signed long int* **op**) [Function]
`void mpf_set_d` (*mpf_t* **rop**, *double* **op**) [Function]
`void mpf_set_z` (*mpf_t* **rop**, *mpz_t* **op**) [Function]
`void mpf_set_q` (*mpf_t* **rop**, *mpq_t* **op**) [Function]

Set the value of *rop* from *op*.

`int mpf_set_str` (*mpf_t* **rop**, *char* ***str**, *int* **base**) [Function]

Set the value of *rop* from the string in *str*. The string is of the form '`M@N`' or, if the base is 10 or less, alternatively '`MeN`'. '`M`' is the mantissa and '`N`' is the exponent. The mantissa is always in the specified base. The exponent is either in the specified base or, if *base* is negative, in decimal. The decimal point expected is taken from the current locale, on systems providing `localeconv`.

The argument *base* may be in the ranges 2 to 36, or −36 to −2. Negative values are used to specify that the exponent is in decimal.

Unlike the corresponding `mpz` function, the base will not be determined from the leading characters of the string if *base* is 0. This is so that numbers like '`0.23`' are not interpreted as octal.

White space is allowed in the string, and is simply ignored. [This is not really true; white-space is ignored in the beginning of the string and within the mantissa, but not in other places, such as after a minus sign or in the exponent. We are considering changing the definition of this function, making it fail when there is any white-space in the input, since that makes a lot of sense. Please tell us your opinion about this change. Do you really want it to accept `"3 14"` as meaning 314 as it does now?]

This function returns 0 if the entire string is a valid number in base *base*. Otherwise it returns −1.

`void mpf_swap` (*mpf_t* **rop1**, *mpf_t* **rop2**) [Function]

Swap *rop1* and *rop2* efficiently. Both the values and the precisions of the two variables are swapped.

7.3 Combined Initialization and Assignment Functions

For convenience, GMP provides a parallel series of initialize-and-set functions which initialize the output and then store the value there. These functions' names have the form `mpf_init_set...`

Once the float has been initialized by any of the `mpf_init_set...` functions, it can be used as the source or destination operand for the ordinary float functions. Don't use an initialize-and-set function on a variable already initialized!

`void mpf_init_set` (*mpf_t* **rop**, *mpf_t* **op**) [Function]

`void mpf_init_set_ui` (*mpf_t* `rop`, *unsigned long int* `op`) [Function]
`void mpf_init_set_si` (*mpf_t* `rop`, *signed long int* `op`) [Function]
`void mpf_init_set_d` (*mpf_t* `rop`, *double* `op`) [Function]

Initialize *rop* and set its value from *op*.

The precision of *rop* will be taken from the active default precision, as set by `mpf_set_default_prec`.

`int mpf_init_set_str` (*mpf_t* `rop`, *char* *`str`, *int* `base`) [Function]

Initialize *rop* and set its value from the string in *str*. See `mpf_set_str` above for details on the assignment operation.

Note that *rop* is initialized even if an error occurs. (I.e., you have to call `mpf_clear` for it.)

The precision of *rop* will be taken from the active default precision, as set by `mpf_set_default_prec`.

7.4 Conversion Functions

`double mpf_get_d` (*mpf_t* `op`) [Function]

Convert *op* to a `double`.

`double mpf_get_d_2exp` (*signed long int* *`exp`, *mpf_t* `op`) [Function]

Find d and exp such that $d \times 2^{exp}$, with $0.5 \le |d| < 1$, is a good approximation to *op*. This is similar to the standard C function `frexp`.

`long mpf_get_si` (*mpf_t* `op`) [Function]
`unsigned long mpf_get_ui` (*mpf_t* `op`) [Function]

Convert *op* to a `long` or `unsigned long`, truncating any fraction part. If *op* is too big for the return type, the result is undefined.

See also `mpf_fits_slong_p` and `mpf_fits_ulong_p` (see Section 7.8 [Miscellaneous Float Functions], page 51).

`char * mpf_get_str` (*char* *`str`, *mp_exp_t* *`expptr`, *int* `base`, *size_t* `n_digits`, *mpf_t* `op`) [Function]

Convert *op* to a string of digits in base *base*. *base* can be 2 to 36. Up to *n_digits* digits will be generated. Trailing zeros are not returned. No more digits than can be accurately represented by *op* are ever generated. If *n_digits* is 0 then that accurate maximum number of digits are generated.

If *str* is `NULL`, the result string is allocated using the current allocation function (see Chapter 14 [Custom Allocation], page 82). The block will be `strlen(str)+1` bytes, that being exactly enough for the string and null-terminator.

If *str* is not `NULL`, it should point to a block of $n\_digits + 2$ bytes, that being enough for the mantissa, a possible minus sign, and a null-terminator. When *n_digits* is 0 to get all significant digits, an application won't be able to know the space required, and *str* should be `NULL` in that case.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. The applicable exponent is written through the *expptr* pointer. For example, the number 3.1416 would be returned as string `"31416"` and exponent 1.

When *op* is zero, an empty string is produced and the exponent returned is 0.

A pointer to the result string is returned, being either the allocated block or the given *str*.

7.5 Arithmetic Functions

`void mpf_add (mpf_t rop, mpf_t op1, mpf_t op2)` [Function]
`void mpf_add_ui (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1 + op2$.

`void mpf_sub (mpf_t rop, mpf_t op1, mpf_t op2)` [Function]
`void mpf_ui_sub (mpf_t rop, unsigned long int op1, mpf_t op2)` [Function]
`void mpf_sub_ui (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1 - op2$.

`void mpf_mul (mpf_t rop, mpf_t op1, mpf_t op2)` [Function]
`void mpf_mul_ui (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1 \times op2$.

Division is undefined if the divisor is zero, and passing a zero divisor to the divide functions will make these functions intentionally divide by zero. This lets the user handle arithmetic exceptions in these functions in the same manner as other arithmetic exceptions.

`void mpf_div (mpf_t rop, mpf_t op1, mpf_t op2)` [Function]
`void mpf_ui_div (mpf_t rop, unsigned long int op1, mpf_t op2)` [Function]
`void mpf_div_ui (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1/op2$.

`void mpf_sqrt (mpf_t rop, mpf_t op)` [Function]
`void mpf_sqrt_ui (mpf_t rop, unsigned long int op)` [Function]
Set *rop* to $\sqrt{op}$.

`void mpf_pow_ui (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1^{op2}$.

`void mpf_neg (mpf_t rop, mpf_t op)` [Function]
Set *rop* to $-op$.

`void mpf_abs (mpf_t rop, mpf_t op)` [Function]
Set *rop* to the absolute value of *op*.

`void mpf_mul_2exp (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1 \times 2^{op2}$.

`void mpf_div_2exp (mpf_t rop, mpf_t op1, unsigned long int op2)` [Function]
Set *rop* to $op1/2^{op2}$.

7.6 Comparison Functions

`int mpf_cmp (mpf_t op1, mpf_t op2)` [Function]
`int mpf_cmp_d (mpf_t op1, double op2)` [Function]
`int mpf_cmp_ui (mpf_t op1, unsigned long int op2)` [Function]
`int mpf_cmp_si (mpf_t op1, signed long int op2)` [Function]
Compare *op1* and *op2*. Return a positive value if $op1 > op2$, zero if $op1 = op2$, and a negative value if $op1 < op2$.

`int mpf_eq (`*mpf_t* `op1`, *mpf_t* `op2`, *unsigned long int* *op3*`)` [Function]

Return non-zero if the first *op3* bits of *op1* and *op2* are equal, zero otherwise. I.e., test of *op1* and *op2* are approximately equal.

Caution: Currently only whole limbs are compared, and only in an exact fashion. In the future values like 1000 and 0111 may be considered the same to 3 bits (on the basis that their difference is that small).

`void mpf_reldiff (`*mpf_t* `rop`, *mpf_t* `op1`, *mpf_t* `op2``)` [Function]

Compute the relative difference between *op1* and *op2* and store the result in *rop*. This is $|op1 - op2|/op1$.

`int mpf_sgn (`*mpf_t* `op``)` [Macro]

Return $+1$ if $op > 0$, 0 if $op = 0$, and -1 if $op < 0$.

This function is actually implemented as a macro. It evaluates its arguments multiple times.

7.7 Input and Output Functions

Functions that perform input from a stdio stream, and functions that output to a stdio stream. Passing a `NULL` pointer for a *stream* argument to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, it is a good idea to include '`stdio.h`' before '`gmp.h`', since that will allow '`gmp.h`' to define prototypes for these functions.

`size_t mpf_out_str (`*FILE* `*stream`, *int* `base`, *size_t* `n_digits`, *mpf_t* `op``)` [Function]

Print *op* to *stream*, as a string of digits. Return the number of bytes written, or if an error occurred, return 0.

The mantissa is prefixed with an '`0.`' and is in the given *base*, which may vary from 2 to 36. An exponent then printed, separated by an '`e`', or if *base* is greater than 10 then by an '`@`'. The exponent is always in decimal. The decimal point follows the current locale, on systems providing `localeconv`.

Up to *n_digits* will be printed from the mantissa, except that no more digits than are accurately representable by *op* will be printed. *n_digits* can be 0 to select that accurate maximum.

`size_t mpf_inp_str (`*mpf_t* `rop`, *FILE* `*stream`, *int* `base``)` [Function]

Read a string in base *base* from *stream*, and put the read float in *rop*. The string is of the form '`M@N`' or, if the base is 10 or less, alternatively '`MeN`'. '`M`' is the mantissa and '`N`' is the exponent. The mantissa is always in the specified base. The exponent is either in the specified base or, if *base* is negative, in decimal. The decimal point expected is taken from the current locale, on systems providing `localeconv`.

The argument *base* may be in the ranges 2 to 36, or -36 to -2. Negative values are used to specify that the exponent is in decimal.

Unlike the corresponding `mpz` function, the base will not be determined from the leading characters of the string if *base* is 0. This is so that numbers like '`0.23`' are not interpreted as octal.

Return the number of bytes read, or if an error occurred, return 0.

7.8 Miscellaneous Functions

`void mpf_ceil (`*mpf_t* `rop`, *mpf_t* `op``)` [Function]

`void mpf_floor` (*mpf_t* **rop**, *mpf_t* **op**) [Function]
`void mpf_trunc` (*mpf_t* **rop**, *mpf_t* **op**) [Function]

Set *rop* to *op* rounded to an integer. `mpf_ceil` rounds to the next higher integer, `mpf_floor` to the next lower, and `mpf_trunc` to the integer towards zero.

`int mpf_integer_p` (*mpf_t* **op**) [Function]

Return non-zero if *op* is an integer.

`int mpf_fits_ulong_p` (*mpf_t* **op**) [Function]
`int mpf_fits_slong_p` (*mpf_t* **op**) [Function]
`int mpf_fits_uint_p` (*mpf_t* **op**) [Function]
`int mpf_fits_sint_p` (*mpf_t* **op**) [Function]
`int mpf_fits_ushort_p` (*mpf_t* **op**) [Function]
`int mpf_fits_sshort_p` (*mpf_t* **op**) [Function]

Return non-zero if *op* would fit in the respective C data type, when truncated to an integer.

`void mpf_urandomb` (*mpf_t* **rop**, *gmp_randstate_t* **state**, *unsigned long int* **nbits**) [Function]

Generate a uniformly distributed random float in *rop*, such that $0 \le rop < 1$, with *nbits* significant bits in the mantissa.

The variable *state* must be initialized by calling one of the `gmp_randinit` functions (Section 9.1 [Random State Initialization], page 61) before invoking this function.

`void mpf_random2` (*mpf_t* **rop**, *mp_size_t* **max_size**, *mp_exp_t* **exp**) [Function]

Generate a random float of at most *max_size* limbs, with long strings of zeros and ones in the binary representation. The exponent of the number is in the interval $-exp$ to *exp* (in limbs). This function is useful for testing functions and algorithms, since these kind of random numbers have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when *max_size* is negative.

8 Low-level Functions

This chapter describes low-level GMP functions, used to implement the high-level GMP functions, but also intended for time-critical user code.

These functions start with the prefix `mpn_`.

The `mpn` functions are designed to be as fast as possible, **not** to provide a coherent calling interface. The different functions have somewhat similar interfaces, but there are variations that make them hard to use. These functions do as little as possible apart from the real multiple precision computation, so that no time is spent on things that not all callers need.

A source operand is specified by a pointer to the least significant limb and a limb count. A destination operand is specified by just a pointer. It is the responsibility of the caller to ensure that the destination has enough space for storing the result.

With this way of specifying operands, it is possible to perform computations on subranges of an argument, and store the result into a subrange of a destination.

A common requirement for all functions is that each source area needs at least one limb. No size argument may be zero. Unless otherwise stated, in-place operations are allowed where source and destination are the same, but not where they only partly overlap.

The `mpn` functions are the base for the implementation of the `mpz_`, `mpf_`, and `mpq_` functions.

This example adds the number beginning at *s1p* and the number beginning at *s2p* and writes the sum at *destp*. All areas have *n* limbs.

```
cy = mpn_add_n (destp, s1p, s2p, n)
```

In the notation used here, a source operand is identified by the pointer to the least significant limb, and the limb count in braces. For example, {*s1p*, *s1n*}.

`mp_limb_t mpn_add_n` (*mp_limb_t* ***rp**, *const mp_limb_t* ***s1p**, *const mp_limb_t* ***s2p**, *mp_size_t* **n**) [Function]

Add {*s1p*, *n*} and {*s2p*, *n*}, and write the *n* least significant limbs of the result to *rp*. Return carry, either 0 or 1.

This is the lowest-level function for addition. It is the preferred function for addition, since it is written in assembly for most CPUs. For addition of a variable to itself (i.e., *s1p* equals *s2p*, use `mpn_lshift` with a count of 1 for optimal speed.

`mp_limb_t mpn_add_1` (*mp_limb_t* ***rp**, *const mp_limb_t* ***s1p**, *mp_size_t* **n**, *mp_limb_t* **s2limb**) [Function]

Add {*s1p*, *n*} and *s2limb*, and write the *n* least significant limbs of the result to *rp*. Return carry, either 0 or 1.

`mp_limb_t mpn_add` (*mp_limb_t* ***rp**, *const mp_limb_t* ***s1p**, *mp_size_t* **s1n**, *const mp_limb_t* ***s2p**, *mp_size_t* **s2n**) [Function]

Add {*s1p*, *s1n*} and {*s2p*, *s2n*}, and write the *s1n* least significant limbs of the result to *rp*. Return carry, either 0 or 1.

This function requires that *s1n* is greater than or equal to *s2n*.

`mp_limb_t mpn_sub_n` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *const mp_limb_t* *`s2p`, *mp_size_t* `n`) [Function]

Subtract {*s2p*, *n*} from {*s1p*, *n*}, and write the *n* least significant limbs of the result to *rp*. Return borrow, either 0 or 1.

This is the lowest-level function for subtraction. It is the preferred function for subtraction, since it is written in assembly for most CPUs.

`mp_limb_t mpn_sub_1` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *mp_size_t* `n`, *mp_limb_t* `s2limb`) [Function]

Subtract *s2limb* from {*s1p*, *n*}, and write the *n* least significant limbs of the result to *rp*. Return borrow, either 0 or 1.

`mp_limb_t mpn_sub` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *mp_size_t* `s1n`, *const mp_limb_t* *`s2p`, *mp_size_t* `s2n`) [Function]

Subtract {*s2p*, *s2n*} from {*s1p*, *s1n*}, and write the *s1n* least significant limbs of the result to *rp*. Return borrow, either 0 or 1.

This function requires that *s1n* is greater than or equal to *s2n*.

`void mpn_mul_n` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *const mp_limb_t* *`s2p`, *mp_size_t* `n`) [Function]

Multiply {*s1p*, *n*} and {*s2p*, *n*}, and write the 2**n*-limb result to *rp*.

The destination has to have space for 2**n* limbs, even if the product's most significant limb is zero. No overlap is permitted between the destination and either source.

`mp_limb_t mpn_mul_1` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *mp_size_t* `n`, *mp_limb_t* `s2limb`) [Function]

Multiply {*s1p*, *n*} by *s2limb*, and write the *n* least significant limbs of the product to *rp*. Return the most significant limb of the product. {*s1p*, *n*} and {*rp*, *n*} are allowed to overlap provided $rp \leq s1p$.

This is a low-level function that is a building block for general multiplication as well as other operations in GMP. It is written in assembly for most CPUs.

Don't call this function if *s2limb* is a power of 2; use `mpn_lshift` with a count equal to the logarithm of *s2limb* instead, for optimal speed.

`mp_limb_t mpn_addmul_1` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *mp_size_t* `n`, *mp_limb_t* `s2limb`) [Function]

Multiply {*s1p*, *n*} and *s2limb*, and add the *n* least significant limbs of the product to {*rp*, *n*} and write the result to *rp*. Return the most significant limb of the product, plus carry-out from the addition.

This is a low-level function that is a building block for general multiplication as well as other operations in GMP. It is written in assembly for most CPUs.

`mp_limb_t mpn_submul_1` (*mp_limb_t* *`rp`, *const mp_limb_t* *`s1p`, *mp_size_t* `n`, *mp_limb_t* `s2limb`) [Function]

Multiply {*s1p*, *n*} and *s2limb*, and subtract the *n* least significant limbs of the product from {*rp*, *n*} and write the result to *rp*. Return the most significant limb of the product, minus borrow-out from the subtraction.

This is a low-level function that is a building block for general multiplication and division as well as other operations in GMP. It is written in assembly for most CPUs.

`mp_limb_t mpn_mul (mp_limb_t *rp, const mp_limb_t *s1p, mp_size_t s1n, const mp_limb_t *s2p, mp_size_t s2n)` [Function]

Multiply {*s1p*, *s1n*} and {*s2p*, *s2n*}, and write the result to *rp*. Return the most significant limb of the result.

The destination has to have space for *s1n* + *s2n* limbs, even if the result might be one limb smaller.

This function requires that *s1n* is greater than or equal to *s2n*. The destination must be distinct from both input operands.

`void mpn_tdiv_qr (mp_limb_t *qp, mp_limb_t *rp, mp_size_t qxn, const mp_limb_t *np, mp_size_t nn, const mp_limb_t *dp, mp_size_t dn)` [Function]

Divide {*np*, *nn*} by {*dp*, *dn*} and put the quotient at {*qp*, *nn*−*dn*+1} and the remainder at {*rp*, *dn*}. The quotient is rounded towards 0.

No overlap is permitted between arguments. *nn* must be greater than or equal to *dn*. The most significant limb of *dp* must be non-zero. The *qxn* operand must be zero.

`mp_limb_t mpn_divrem (mp_limb_t *r1p, mp_size_t qxn, mp_limb_t *rs2p, mp_size_t rs2n, const mp_limb_t *s3p, mp_size_t s3n)` [Function]

[This function is obsolete. Please call `mpn_tdiv_qr` instead for best performance.]

Divide {*rs2p*, *rs2n*} by {*s3p*, *s3n*}, and write the quotient at *r1p*, with the exception of the most significant limb, which is returned. The remainder replaces the dividend at *rs2p*; it will be *s3n* limbs long (i.e., as many limbs as the divisor).

In addition to an integer quotient, *qxn* fraction limbs are developed, and stored after the integral limbs. For most usages, *qxn* will be zero.

It is required that *rs2n* is greater than or equal to *s3n*. It is required that the most significant bit of the divisor is set.

If the quotient is not needed, pass *rs2p* + *s3n* as *r1p*. Aside from that special case, no overlap between arguments is permitted.

Return the most significant limb of the quotient, either 0 or 1.

The area at *r1p* needs to be *rs2n* − *s3n* + *qxn* limbs large.

`mp_limb_t mpn_divrem_1 (mp_limb_t *r1p, mp_size_t qxn, mp_limb_t *s2p, mp_size_t s2n, mp_limb_t s3limb)` [Function]

`mp_limb_t mpn_divmod_1 (mp_limb_t *r1p, mp_limb_t *s2p, mp_size_t s2n, mp_limb_t s3limb)` [Macro]

Divide {*s2p*, *s2n*} by *s3limb*, and write the quotient at *r1p*. Return the remainder.

The integer quotient is written to {*r1p*+*qxn*, *s2n*} and in addition *qxn* fraction limbs are developed and written to {*r1p*, *qxn*}. Either or both *s2n* and *qxn* can be zero. For most usages, *qxn* will be zero.

`mpn_divmod_1` exists for upward source compatibility and is simply a macro calling `mpn_divrem_1` with a *qxn* of 0.

The areas at *r1p* and *s2p* have to be identical or completely separate, not partially overlapping.

`mp_limb_t mpn_divmod` (*mp_limb_t *`r1p`, mp_limb_t *`rs2p`, mp_size_t `rs2n`, const mp_limb_t *`s3p`, mp_size_t `s3n`*) [Function]

[This function is obsolete. Please call `mpn_tdiv_qr` instead for best performance.]

`mp_limb_t mpn_divexact_by3` (*mp_limb_t *`rp`, mp_limb_t *`sp`, mp_size_t `n`*) [Macro]

`mp_limb_t mpn_divexact_by3c` (*mp_limb_t *`rp`, mp_limb_t *`sp`, mp_size_t `n`, mp_limb_t `carry`*) [Function]

Divide {*sp*, *n*} by 3, expecting it to divide exactly, and writing the result to {*rp*, *n*}. If 3 divides exactly, the return value is zero and the result is the quotient. If not, the return value is non-zero and the result won't be anything useful.

`mpn_divexact_by3c` takes an initial carry parameter, which can be the return value from a previous call, so a large calculation can be done piece by piece from low to high. `mpn_divexact_by3` is simply a macro calling `mpn_divexact_by3c` with a 0 carry parameter.

These routines use a multiply-by-inverse and will be faster than `mpn_divrem_1` on CPUs with fast multiplication but slow division.

The source a, result q, size n, initial carry i, and return value c satisfy $cb^n + a - i = 3q$, where $b = 2^{\texttt{mp\_bits\_per\_limb}}$. The return c is always 0, 1 or 2, and the initial carry i must also be 0, 1 or 2 (these are both borrows really). When $c = 0$ clearly $q = (a - i)/3$. When $c \neq 0$, the remainder $(a - i) \bmod 3$ is given by $3 - c$, because $b \equiv 1 \bmod 3$ (when `mp_bits_per_limb` is even, which is always so currently).

`mp_limb_t mpn_mod_1` (*mp_limb_t *`s1p`, mp_size_t `s1n`, mp_limb_t `s2limb`*) [Function]

Divide {*s1p*, *s1n*} by *s2limb*, and return the remainder. *s1n* can be zero.

`mp_limb_t mpn_bdivmod` (*mp_limb_t *`rp`, mp_limb_t *`s1p`, mp_size_t `s1n`, const mp_limb_t *`s2p`, mp_size_t `s2n`, unsigned long int `d`*) [Function]

This function puts the low $\lfloor d/\texttt{mp\_bits\_per\_limb} \rfloor$ limbs of q = {*s1p*, *s1n*}/{*s2p*, *s2n*} mod 2^d at *rp*, and returns the high d mod `mp_bits_per_limb` bits of q.

{*s1p*, *s1n*} - *q* * {*s2p*, *s2n*} mod $2^{s1n * \texttt{mp\_bits\_per\_limb}}$ is placed at *s1p*. Since the low $\lfloor d/\texttt{mp\_bits\_per\_limb} \rfloor$ limbs of this difference are zero, it is possible to overwrite the low limbs at *s1p* with this difference, provided $rp \leq s1p$.

This function requires that $s1n * \texttt{mp\_bits\_per\_limb} \geq D$, and that {*s2p*, *s2n*} is odd.

This interface is preliminary. It might change incompatibly in future revisions.

`mp_limb_t mpn_lshift` (*mp_limb_t *`rp`, const mp_limb_t *`sp`, mp_size_t `n`, unsigned int `count`*) [Function]

Shift {*sp*, *n*} left by *count* bits, and write the result to {*rp*, *n*}. The bits shifted out at the left are returned in the least significant *count* bits of the return value (the rest of the return value is zero).

count must be in the range 1 to $\texttt{mp\_bits\_per\_limb} - 1$. The regions {*sp*, *n*} and {*rp*, *n*} may overlap, provided $rp \geq sp$.

This function is written in assembly for most CPUs.

`mp_limb_t mpn_rshift` (*mp_limb_t *rp*, *const mp_limb_t *sp*, *mp_size_t n*, *unsigned int count*) [Function]

Shift {*sp*, *n*} right by *count* bits, and write the result to {*rp*, *n*}. The bits shifted out at the right are returned in the most significant *count* bits of the return value (the rest of the return value is zero).

count must be in the range 1 to `mp_bits_per_limb`−1. The regions {*sp*, *n*} and {*rp*, *n*} may overlap, provided $rp \le sp$.

This function is written in assembly for most CPUs.

`int mpn_cmp` (*const mp_limb_t *s1p*, *const mp_limb_t *s2p*, *mp_size_t n*) [Function]

Compare {*s1p*, *n*} and {*s2p*, *n*} and return a positive value if $s1 > s2$, 0 if they are equal, or a negative value if $s1 < s2$.

`mp_size_t mpn_gcd` (*mp_limb_t *rp*, *mp_limb_t *s1p*, *mp_size_t s1n*, *mp_limb_t *s2p*, *mp_size_t s2n*) [Function]

Set {*rp*, *retval*} to the greatest common divisor of {*s1p*, *s1n*} and {*s2p*, *s2n*}. The result can be up to *s2n* limbs, the return value is the actual number produced. Both source operands are destroyed.

{*s1p*, *s1n*} must have at least as many bits as {*s2p*, *s2n*}. {*s2p*, *s2n*} must be odd. Both operands must have non-zero most significant limbs. No overlap is permitted between {*s1p*, *s1n*} and {*s2p*, *s2n*}.

`mp_limb_t mpn_gcd_1` (*const mp_limb_t *s1p*, *mp_size_t s1n*, *mp_limb_t s2limb*) [Function]

Return the greatest common divisor of {*s1p*, *s1n*} and *s2limb*. Both operands must be non-zero.

`mp_size_t mpn_gcdext` (*mp_limb_t *r1p*, *mp_limb_t *r2p*, *mp_size_t *r2n*, *mp_limb_t *s1p*, *mp_size_t s1n*, *mp_limb_t *s2p*, *mp_size_t s2n*) [Function]

Calculate the greatest common divisor of {*s1p*, *s1n*} and {*s2p*, *s2n*}. Store the gcd at {*r1p*, *retval*} and the first cofactor at {*r2p*, **r2n*}, with **r2n* negative if the cofactor is negative. *r1p* and *r2p* should each have room for $s1n + 1$ limbs, but the return value and value stored through *r2n* indicate the actual number produced.

$\{s1p, s1n\} \ge \{s2p, s2n\}$ is required, and both must be non-zero. The regions {*s1p*, $s1n + 1$} and {*s2p*, $s2n + 1$} are destroyed (i.e. the operands plus an extra limb past the end of each).

The cofactor *r1* will satisfy $r_2 s_1 + k s_2 = r_1$. The second cofactor *k* is not calculated but can easily be obtained from $(r_1 - r_2 s_1)/s_2$.

`mp_size_t mpn_sqrtrem` (*mp_limb_t *r1p*, *mp_limb_t *r2p*, *const mp_limb_t *sp*, *mp_size_t n*) [Function]

Compute the square root of {*sp*, *n*} and put the result at {*r1p*, $\lceil n/2 \rceil$} and the remainder at {*r2p*, *retval*}. *r2p* needs space for *n* limbs, but the return value indicates how many are produced.

The most significant limb of {*sp*, *n*} must be non-zero. The areas {*r1p*, $\lceil n/2 \rceil$} and {*sp*, *n*} must be completely separate. The areas {*r2p*, *n*} and {*sp*, *n*} must be either identical or completely separate.

If the remainder is not wanted then *r2p* can be `NULL`, and in this case the return value is zero or non-zero according to whether the remainder would have been zero or non-zero.

A return value of zero indicates a perfect square. See also `mpz_perfect_square_p`.

`mp_size_t mpn_get_str` (*unsigned char* ***str***, *int* **base**, *mp_limb_t* ***s1p***, *mp_size_t* **s1n**) [Function]

Convert {*s1p*, *s1n*} to a raw unsigned char array at *str* in base *base*, and return the number of characters produced. There may be leading zeros in the string. The string is not in ASCII; to convert it to printable format, add the ASCII codes for '`0`' or '`A`', depending on the base and range. *base* can vary from 2 to 256.

The most significant limb of the input {*s1p*, *s1n*} must be non-zero. The input {*s1p*, *s1n*} is clobbered, except when *base* is a power of 2, in which case it's unchanged.

The area at *str* has to have space for the largest possible number represented by a *s1n* long limb array, plus one extra character.

`mp_size_t mpn_set_str` (*mp_limb_t* ***rp***, *const unsigned char* ***str***, *size_t* **strsize**, *int* **base**) [Function]

Convert bytes {*str*,*strsize*} in the given *base* to limbs at *rp*.

str[0] is the most significant byte and *str*[*strsize* − 1] is the least significant. Each byte should be a value in the range 0 to *base* − 1, not an ASCII character. *base* can vary from 2 to 256.

The return value is the number of limbs written to *rp*. If the most significant input byte is non-zero then the high limb at *rp* will be non-zero, and only that exact number of limbs will be required there.

If the most significant input byte is zero then there may be high zero limbs written to *rp* and included in the return value.

strsize must be at least 1, and no overlap is permitted between {*str*,*strsize*} and the result at *rp*.

`unsigned long int mpn_scan0` (*const mp_limb_t* ***s1p***, *unsigned long int* **bit**) [Function]

Scan *s1p* from bit position *bit* for the next clear bit.

It is required that there be a clear bit within the area at *s1p* at or beyond bit position *bit*, so that the function has something to return.

`unsigned long int mpn_scan1` (*const mp_limb_t* ***s1p***, *unsigned long int* **bit**) [Function]

Scan *s1p* from bit position *bit* for the next set bit.

It is required that there be a set bit within the area at *s1p* at or beyond bit position *bit*, so that the function has something to return.

`void mpn_random` (*mp_limb_t* ***r1p***, *mp_size_t* **r1n**) [Function]
`void mpn_random2` (*mp_limb_t* ***r1p***, *mp_size_t* **r1n**) [Function]

Generate a random number of length *r1n* and store it at *r1p*. The most significant limb is always non-zero. `mpn_random` generates uniformly distributed limb data, `mpn_random2` generates long strings of zeros and ones in the binary representation.

`mpn_random2` is intended for testing the correctness of the `mpn` routines.

`unsigned long int mpn_popcount` (*const mp_limb_t* ***s1p***, *mp_size_t* **n**) [Function]

Count the number of set bits in {*s1p*, *n*}.

`unsigned long int mpn_hamdist` (*const mp_limb_t* `*s1p`, *const mp_limb_t* `*s2p`, *mp_size_t* `n`) [Function]

Compute the hamming distance between {*s1p*, *n*} and {*s2p*, *n*}, which is the number of bit positions where the two operands have different bit values.

`int mpn_perfect_square_p` (*const mp_limb_t* `*s1p`, *mp_size_t* `n`) [Function]

Return non-zero iff {*s1p*, *n*} is a perfect square.

8.1 Nails

Everything in this section is highly experimental and may disappear or be subject to incompatible changes in a future version of GMP.

Nails are an experimental feature whereby a few bits are left unused at the top of each `mp_limb_t`. This can significantly improve carry handling on some processors.

All the `mpn` functions accepting limb data will expect the nail bits to be zero on entry, and will return data with the nails similarly all zero. This applies both to limb vectors and to single limb arguments.

Nails can be enabled by configuring with '`--enable-nails`'. By default the number of bits will be chosen according to what suits the host processor, but a particular number can be selected with '`--enable-nails=N`'.

At the mpn level, a nail build is neither source nor binary compatible with a non-nail build, strictly speaking. But programs acting on limbs only through the mpn functions are likely to work equally well with either build, and judicious use of the definitions below should make any program compatible with either build, at the source level.

For the higher level routines, meaning `mpz` etc, a nail build should be fully source and binary compatible with a non-nail build.

`GMP_NAIL_BITS` [Macro]
`GMP_NUMB_BITS` [Macro]
`GMP_LIMB_BITS` [Macro]

`GMP_NAIL_BITS` is the number of nail bits, or 0 when nails are not in use. `GMP_NUMB_BITS` is the number of data bits in a limb. `GMP_LIMB_BITS` is the total number of bits in an `mp_limb_t`. In all cases

```
GMP_LIMB_BITS == GMP_NAIL_BITS + GMP_NUMB_BITS
```

`GMP_NAIL_MASK` [Macro]
`GMP_NUMB_MASK` [Macro]

Bit masks for the nail and number parts of a limb. `GMP_NAIL_MASK` is 0 when nails are not in use.

`GMP_NAIL_MASK` is not often needed, since the nail part can be obtained with `x >> GMP_NUMB_BITS`, and that means one less large constant, which can help various RISC chips.

`GMP_NUMB_MAX` [Macro]

The maximum value that can be stored in the number part of a limb. This is the same as `GMP_NUMB_MASK`, but can be used for clarity when doing comparisons rather than bit-wise operations.

The term “nails” comes from finger or toe nails, which are at the ends of a limb (arm or leg). “numb” is short for number, but is also how the developers felt after trying for a long time to come up with sensible names for these things.

In the future (the distant future most likely) a non-zero nail might be permitted, giving non-unique representations for numbers in a limb vector. This would help vector processors since carries would only ever need to propagate one or two limbs.

9 Random Number Functions

Sequences of pseudo-random numbers in GMP are generated using a variable of type `gmp_randstate_t`, which holds an algorithm selection and a current state. Such a variable must be initialized by a call to one of the `gmp_randinit` functions, and can be seeded with one of the `gmp_randseed` functions.

The functions actually generating random numbers are described in Section 5.13 [Integer Random Numbers], page 39, and Section 7.8 [Miscellaneous Float Functions], page 51.

The older style random number functions don't accept a `gmp_randstate_t` parameter but instead share a global variable of that type. They use a default algorithm and are currently not seeded (though perhaps that will change in the future). The new functions accepting a `gmp_randstate_t` are recommended for applications that care about randomness.

9.1 Random State Initialization

`void gmp_randinit_default` (*gmp_randstate_t* `state`) [Function]

Initialize *state* with a default algorithm. This will be a compromise between speed and randomness, and is recommended for applications with no special requirements.

`void gmp_randinit_lc_2exp` (*gmp_randstate_t* `state`, *mpz_t* `a`, *unsigned long* `c`, *unsigned long* `m2exp`) [Function]

Initialize *state* with a linear congruential algorithm $X = (aX + c) \bmod 2^{m2exp}$.

The low bits of X in this algorithm are not very random. The least significant bit will have a period no more than 2, and the second bit no more than 4, etc. For this reason only the high half of each X is actually used.

When a random number of more than *m2exp*/2 bits is to be generated, multiple iterations of the recurrence are used and the results concatenated.

`int gmp_randinit_lc_2exp_size` (*gmp_randstate_t* `state`, *unsigned long* `size`) [Function]

Initialize *state* for a linear congruential algorithm as per `gmp_randinit_lc_2exp`. *a*, *c* and *m2exp* are selected from a table, chosen so that *size* bits (or more) of each X will be used, ie. $m2exp/2 \geq size$.

If successful the return value is non-zero. If *size* is bigger than the table data provides then the return value is zero. The maximum *size* currently supported is 128.

`void gmp_randinit` (*gmp_randstate_t* `state`, *gmp_randalg_t* `alg`, ...) [Function]

This function is obsolete.

Initialize *state* with an algorithm selected by *alg*. The only choice is `GMP_RAND_ALG_LC`, which is `gmp_randinit_lc_2exp_size` described above. A third parameter of type `unsigned long` is required, this is the *size* for that function. `GMP_RAND_ALG_DEFAULT` or 0 are the same as `GMP_RAND_ALG_LC`.

`gmp_randinit` sets bits in the global variable `gmp_errno` to indicate an error. `GMP_ERROR_UNSUPPORTED_ARGUMENT` if *alg* is unsupported, or `GMP_ERROR_INVALID_ARGUMENT` if the *size* parameter is too big. It may be noted this error reporting is not thread safe (a good reason to use `gmp_randinit_lc_2exp_size` instead).

`void gmp_randclear` (*gmp_randstate_t* `state`) [Function]

Free all memory occupied by *state*.

9.2 Random State Seeding

`void gmp_randseed` (*gmp_randstate_t* `state`, *mpz_t* `seed`) [Function]

`void gmp_randseed_ui` (*gmp_randstate_t* `state`, *unsigned long int* `seed`) [Function]

Set an initial seed value into *state*.

The size of a seed determines how many different sequences of random numbers that it's possible to generate. The "quality" of the seed is the randomness of a given seed compared to the previous seed used, and this affects the randomness of separate number sequences. The method for choosing a seed is critical if the generated numbers are to be used for important applications, such as generating cryptographic keys.

Traditionally the system time has been used to seed, but care needs to be taken with this. If an application seeds often and the resolution of the system clock is low, then the same sequence of numbers might be repeated. Also, the system time is quite easy to guess, so if unpredictability is required then it should definitely not be the only source for the seed value. On some systems there's a special device '`/dev/random`' which provides random data better suited for use as a seed.

10 Formatted Output

10.1 Format Strings

`gmp_printf` and friends accept format strings similar to the standard C `printf` (see section "Formatted Output" in *The GNU C Library Reference Manual*). A format specification is of the form

```
% [flags] [width] [.[precision]] [type] conv
```

GMP adds types 'Z', 'Q' and 'F' for `mpz_t`, `mpq_t` and `mpf_t` respectively, and 'N' for an `mp_limb_t` array. 'Z', 'Q' and 'N' behave like integers. 'Q' will print a '/' and a denominator, if needed. 'F' behaves like a float. For example,

```
mpz_t z;
gmp_printf ("%s is an mpz %Zd\n", "here", z);

mpq_t q;
gmp_printf ("a hex rational: %#40Qx\n", q);

mpf_t f;
int   n;
gmp_printf ("fixed point mpf %.*Ff with %d digits\n", n, f, n);

const mp_limb_t *ptr;
mp_size_t       size;
gmp_printf ("limb array %Nx\n", ptr, size);
```

For 'N' the limbs are expected least significant first, as per the `mpn` functions (see Chapter 8 [Low-level Functions], page 53). A negative size can be given to print the value as a negative.

All the standard C `printf` types behave the same as the C library `printf`, and can be freely intermixed with the GMP extensions. In the current implementation the standard parts of the format string are simply handed to `printf` and only the GMP extensions handled directly.

The flags accepted are as follows. GLIBC style ''' is only for the standard C types (not the GMP types), and only if the C library supports it.

`0`	pad with zeros (rather than spaces)
`#`	show the base with '0x', '0X' or '0'
`+`	always show a sign
(space)	show a space or a '-' sign
`'`	group digits, GLIBC style (not GMP types)

The optional width and precision can be given as a number within the format string, or as a '*' to take an extra parameter of type `int`, the same as the standard `printf`.

The standard types accepted are as follows. 'h' and 'l' are portable, the rest will depend on the compiler (or include files) for the type and the C library for the output.

`h`	`short`
`hh`	`char`
`j`	`intmax_t` or `uintmax_t`
`l`	`long` or `wchar_t`
`ll`	`long long`
`L`	`long double`

q	quad_t or u_quad_t
t	ptrdiff_t
z	size_t

The GMP types are

F	mpf_t, float conversions
Q	mpq_t, integer conversions
N	mp_limb_t array, integer conversions
Z	mpz_t, integer conversions

The conversions accepted are as follows. 'a' and 'A' are always supported for mpf_t but depend on the C library for standard C float types. 'm' and 'p' depend on the C library.

a A	hex floats, C99 style
c	character
d	decimal integer
e E	scientific format float
f	fixed point float
i	same as d
g G	fixed or scientific float
m	strerror string, GLIBC style
n	store characters written so far
o	octal integer
p	pointer
s	string
u	unsigned integer
x X	hex integer

'o', 'x' and 'X' are unsigned for the standard C types, but for types 'Z', 'Q' and 'N' they are signed. 'u' is not meaningful for 'Z', 'Q' and 'N'.

'n' can be used with any type, even the GMP types.

Other types or conversions that might be accepted by the C library printf cannot be used through gmp_printf, this includes for instance extensions registered with GLIBC register_printf_function. Also currently there's no support for POSIX '$' style numbered arguments (perhaps this will be added in the future).

The precision field has it's usual meaning for integer 'Z' and float 'F' types, but is currently undefined for 'Q' and should not be used with that.

mpf_t conversions only ever generate as many digits as can be accurately represented by the operand, the same as mpf_get_str does. Zeros will be used if necessary to pad to the requested precision. This happens even for an 'f' conversion of an mpf_t which is an integer, for instance 2^{1024} in an mpf_t of 128 bits precision will only produce about 40 digits, then pad with zeros to the decimal point. An empty precision field like '%.Fe' or '%.Ff' can be used to specifically request just the significant digits.

The decimal point character (or string) is taken from the current locale settings on systems which provide localeconv (see section "Locales and Internationalization" in *The GNU C Library Reference Manual*). The C library will normally do the same for standard float output.

The format string is only interpreted as plain chars, multibyte characters are not recognised. Perhaps this will change in the future.

10.2 Functions

Each of the following functions is similar to the corresponding C library function. The basic `printf` forms take a variable argument list. The `vprintf` forms take an argument pointer, see section "Variadic Functions" in *The GNU C Library Reference Manual*, or '`man 3 va_start`'.

It should be emphasised that if a format string is invalid, or the arguments don't match what the format specifies, then the behaviour of any of these functions will be unpredictable. GCC format string checking is not available, since it doesn't recognise the GMP extensions.

The file based functions `gmp_printf` and `gmp_fprintf` will return −1 to indicate a write error. All the functions can return −1 if the C library `printf` variant in use returns −1, but this shouldn't normally occur.

`int gmp_printf` (*const char* *`fmt`, ...) [Function]
`int gmp_vprintf` (*const char* *`fmt`, *va_list* `ap`) [Function]

Print to the standard output `stdout`. Return the number of characters written, or −1 if an error occurred.

`int gmp_fprintf` (*FILE* *`fp`, *const char* *`fmt`, ...) [Function]
`int gmp_vfprintf` (*FILE* *`fp`, *const char* *`fmt`, *va_list* `ap`) [Function]

Print to the stream *fp*. Return the number of characters written, or −1 if an error occurred.

`int gmp_sprintf` (*char* *`buf`, *const char* *`fmt`, ...) [Function]
`int gmp_vsprintf` (*char* *`buf`, *const char* *`fmt`, *va_list* `ap`) [Function]

Form a null-terminated string in *buf*. Return the number of characters written, excluding the terminating null.

No overlap is permitted between the space at *buf* and the string *fmt*.

These functions are not recommended, since there's no protection against exceeding the space available at *buf*.

`int gmp_snprintf` (*char* *`buf`, *size_t* `size`, *const char* *`fmt`, ...) [Function]
`int gmp_vsnprintf` (*char* *`buf`, *size_t* `size`, *const char* *`fmt`, *va_list* `ap`) [Function]

Form a null-terminated string in *buf*. No more than *size* bytes will be written. To get the full output, *size* must be enough for the string and null-terminator.

The return value is the total number of characters which ought to have been produced, excluding the terminating null. If $retval \geq size$ then the actual output has been truncated to the first $size - 1$ characters, and a null appended.

No overlap is permitted between the region {*buf*,*size*} and the *fmt* string.

Notice the return value is in ISO C99 `snprintf` style. This is so even if the C library `vsnprintf` is the older GLIBC 2.0.x style.

`int gmp_asprintf` (*char* **`pp`, *const char* *`fmt`, ...) [Function]
`int gmp_vasprintf` (*char* *`pp`, *const char* *`fmt`, *va_list* `ap`) [Function]

Form a null-terminated string in a block of memory obtained from the current memory allocation function (see Chapter 14 [Custom Allocation], page 82). The block will be the size of the string and null-terminator. Put the address of the block in **pp*. Return the number of characters produced, excluding the null-terminator.

Unlike the C library `asprintf`, `gmp_asprintf` doesn't return −1 if there's no more memory available, it lets the current allocation function handle that.

`int gmp_obstack_printf` (*struct obstack *ob, const char *fmt, ...*) [Function]
`int gmp_obstack_vprintf` (*struct obstack *ob, const char *fmt, va_list ap*) [Function]

Append to the current obstack object, in the same style as `obstack_printf`. Return the number of characters written. A null-terminator is not written.

fmt cannot be within the current obstack object, since the object might move as it grows.

These functions are available only when the C library provides the obstack feature, which probably means only on GNU systems, see section "Obstacks" in *The GNU C Library Reference Manual*.

10.3 C++ Formatted Output

The following functions are provided in '`libgmpxx`', which is built if C++ support is enabled (see Section 2.1 [Build Options], page 4). Prototypes are available from `<gmp.h>`.

`ostream& operator<<` (*ostream& stream, mpz_t op*) [Function]

Print *op* to *stream*, using its `ios` formatting settings. `ios::width` is reset to 0 after output, the same as the standard `ostream operator<<` routines do.

In hex or octal, *op* is printed as a signed number, the same as for decimal. This is unlike the standard `operator<<` routines on `int` etc, which instead give twos complement.

`ostream& operator<<` (*ostream& stream, mpq_t op*) [Function]

Print *op* to *stream*, using its `ios` formatting settings. `ios::width` is reset to 0 after output, the same as the standard `ostream operator<<` routines do.

Output will be a fraction like '`5/9`', or if the denominator is 1 then just a plain integer like '`123`'.

In hex or octal, *op* is printed as a signed value, the same as for decimal. If `ios::showbase` is set then a base indicator is shown on both the numerator and denominator (if the denominator is required).

`ostream& operator<<` (*ostream& stream, mpf_t op*) [Function]

Print *op* to *stream*, using its `ios` formatting settings. `ios::width` is reset to 0 after output, the same as the standard `ostream operator<<` routines do. The decimal point follows the current locale, on systems providing `localeconv`.

Hex and octal are supported, unlike the standard `operator<<` on `double`. The mantissa will be in hex or octal, the exponent will be in decimal. For hex the exponent delimiter is an '`@`'. This is as per `mpf_out_str`.

`ios::showbase` is supported, and will put a base on the mantissa, for example hex '`0x1.8`' or '`0x0.8`', or octal '`01.4`' or '`00.4`'. This last form is slightly strange, but at least differentiates itself from decimal.

These operators mean that GMP types can be printed in the usual C++ way, for example,

```
mpz_t  z;
int    n;
...
cout << "iteration " << n << " value " << z << "\n";
```

But note that `ostream` output (and `istream` input, see Section 11.3 [C++ Formatted Input], page 70) is the only overloading available for the GMP types and that for instance using `+` with an mpz_t will have unpredictable results. For classes with overloading, see Chapter 12 [C++ Class Interface], page 72.

11 Formatted Input

11.1 Formatted Input Strings

`gmp_scanf` and friends accept format strings similar to the standard C `scanf` (see section "Formatted Input" in *The GNU C Library Reference Manual*). A format specification is of the form

```
% [flags] [width] [type] conv
```

GMP adds types 'Z', 'Q' and 'F' for `mpz_t`, `mpq_t` and `mpf_t` respectively. 'Z' and 'Q' behave like integers. 'Q' will read a '/' and a denominator, if present. 'F' behaves like a float.

GMP variables don't require an `&` when passed to `gmp_scanf`, since they're already "call-by-reference". For example,

```
/* to read say "a(5) = 1234" */
int   n;
mpz_t z;
gmp_scanf ("a(%d) = %Zd\n", &n, z);

mpq_t q1, q2;
gmp_sscanf ("0377 + 0x10/0x11", "%Qi + %Qi", q1, q2);

/* to read say "topleft (1.55,-2.66)" */
mpf_t x, y;
char  buf[32];
gmp_scanf ("%31s (%Ff,%Ff)", buf, x, y);
```

All the standard C `scanf` types behave the same as in the C library `scanf`, and can be freely intermixed with the GMP extensions. In the current implementation the standard parts of the format string are simply handed to `scanf` and only the GMP extensions handled directly.

The flags accepted are as follows. 'a' and ''' will depend on support from the C library, and ''' cannot be used with GMP types.

`*`	read but don't store
`a`	allocate a buffer (string conversions)
`'`	group digits, GLIBC style (not GMP types)

The standard types accepted are as follows. 'h' and 'l' are portable, the rest will depend on the compiler (or include files) for the type and the C library for the input.

`h`	`short`
`hh`	`char`
`j`	`intmax_t` or `uintmax_t`
`l`	`long int`, `double` or `wchar_t`
`ll`	`long long`
`L`	`long double`
`q`	`quad_t` or `u_quad_t`
`t`	`ptrdiff_t`
`z`	`size_t`

The GMP types are

`F`	`mpf_t`, float conversions

Q	mpq_t, integer conversions
Z	mpz_t, integer conversions

The conversions accepted are as follows. 'p' and '[' will depend on support from the C library, the rest are standard.

c	character or characters
d	decimal integer
e E f g G	float
i	integer with base indicator
n	characters read so far
o	octal integer
p	pointer
s	string of non-whitespace characters
u	decimal integer
x X	hex integer
[	string of characters in a set

'e', 'E', 'f', 'g' and 'G' are identical, they all read either fixed point or scientific format, and either 'e' or 'E' for the exponent in scientific format.

'x' and 'X' are identical, both accept both upper and lower case hexadecimal.

'o', 'u', 'x' and 'X' all read positive or negative values. For the standard C types these are described as "unsigned" conversions, but that merely affects certain overflow handling, negatives are still allowed (per `strtoul`, see section "Parsing of Integers" in *The GNU C Library Reference Manual*). For GMP types there are no overflows, so 'd' and 'u' are identical.

'Q' type reads the numerator and (optional) denominator as given. If the value might not be in canonical form then `mpq_canonicalize` must be called before using it in any calculations (see Chapter 6 [Rational Number Functions], page 42).

'Qi' will read a base specification separately for the numerator and denominator. For example '0x10/11' would be 16/11, whereas '0x10/0x11' would be 16/17.

'n' can be used with any of the types above, even the GMP types. '*' to suppress assignment is allowed, though the field would then do nothing at all.

Other conversions or types that might be accepted by the C library `scanf` cannot be used through `gmp_scanf`.

Whitespace is read and discarded before a field, except for 'c' and '[' conversions.

For float conversions, the decimal point character (or string) expected is taken from the current locale settings on systems which provide `localeconv` (see section "Locales and Internationalization" in *The GNU C Library Reference Manual*). The C library will normally do the same for standard float input.

The format string is only interpreted as plain `char`s, multibyte characters are not recognised. Perhaps this will change in the future.

11.2 Formatted Input Functions

Each of the following functions is similar to the corresponding C library function. The plain `scanf` forms take a variable argument list. The `vscanf` forms take an argument pointer, see section "Variadic Functions" in *The GNU C Library Reference Manual*, or 'man 3 va_start'.

It should be emphasised that if a format string is invalid, or the arguments don't match what the format specifies, then the behaviour of any of these functions will be unpredictable. GCC format string checking is not available, since it doesn't recognise the GMP extensions.

No overlap is permitted between the *fmt* string and any of the results produced.

`int gmp_scanf` (*const char* ***fmt***, *...*) [Function]
`int gmp_vscanf` (*const char* ***fmt***, *va_list* **ap**) [Function]
Read from the standard input `stdin`.

`int gmp_fscanf` (*FILE* ***fp***, *const char* ***fmt***, *...*) [Function]
`int gmp_vfscanf` (*FILE* ***fp***, *const char* ***fmt***, *va_list* **ap**) [Function]
Read from the stream *fp*.

`int gmp_sscanf` (*const char* ***s***, *const char* ***fmt***, *...*) [Function]
`int gmp_vsscanf` (*const char* ***s***, *const char* ***fmt***, *va_list* **ap**) [Function]
Read from a null-terminated string *s*.

The return value from each of these functions is the same as the standard C99 `scanf`, namely the number of fields successfully parsed and stored. '`%n`' fields and fields read but suppressed by '`*`' don't count towards the return value.

If end of file or file error, or end of string, is reached when a match is required, and when no previous non-suppressed fields have matched, then the return value is EOF instead of 0. A match is required for a literal character in the format string or a field other than '`%n`'. Whitespace in the format string is only an optional match and won't induce an EOF in this fashion. Leading whitespace read and discarded for a field doesn't count as a match.

11.3 C++ Formatted Input

The following functions are provided in '`libgmpxx`', which is built only if C++ support is enabled (see Section 2.1 [Build Options], page 4). Prototypes are available from `<gmp.h>`.

`istream& operator>>` (*istream&* **stream**, *mpz_t* **rop**) [Function]
Read *rop* from *stream*, using its `ios` formatting settings.

`istream& operator>>` (*istream&* **stream**, *mpq_t* **rop**) [Function]
Read *rop* from *stream*, using its `ios` formatting settings.

An integer like '`123`' will be read, or a fraction like '`5/9`'. If the fraction is not in canonical form then `mpq_canonicalize` must be called (see Chapter 6 [Rational Number Functions], page 42).

`istream& operator>>` (*istream&* **stream**, *mpf_t* **rop**) [Function]
Read *rop* from *stream*, using its `ios` formatting settings.

Hex or octal floats are not supported, but might be in the future.

These operators mean that GMP types can be read in the usual C++ way, for example,

```
mpz_t  z;
...
cin >> z;
```

But note that `istream` input (and `ostream` output, see Section 10.3 [C++ Formatted Output], page 66) is the only overloading available for the GMP types and that for instance using `+` with an mpz_t will have unpredictable results. For classes with overloading, see Chapter 12 [C++ Class Interface], page 72.

12 C++ Class Interface

This chapter describes the C++ class based interface to GMP.

All GMP C language types and functions can be used in C++ programs, since 'gmp.h' has extern "C" qualifiers, but the class interface offers overloaded functions and operators which may be more convenient.

Due to the implementation of this interface, a reasonably recent C++ compiler is required, one supporting namespaces, partial specialization of templates and member templates. For GCC this means version 2.91 or later.

Everything described in this chapter is to be considered preliminary and might be subject to incompatible changes if some unforeseen difficulty reveals itself.

12.1 C++ Interface General

All the C++ classes and functions are available with

```
#include <gmpxx.h>
```

Programs should be linked with the 'libgmpxx' and 'libgmp' libraries. For example,

```
g++ mycxxprog.cc -lgmpxx -lgmp
```

The classes defined are

mpz_class [Class]
mpq_class [Class]
mpf_class [Class]

The standard operators and various standard functions are overloaded to allow arithmetic with these classes. For example,

```
int
main (void)
{
  mpz_class a, b, c;

  a = 1234;
  b = "-5678";
  c = a+b;
  cout << "sum is " << c << "\n";
  cout << "absolute value is " << abs(c) << "\n";

  return 0;
}
```

An important feature of the implementation is that an expression like a=b+c results in a single call to the corresponding mpz_add, without using a temporary for the b+c part. Expressions which by their nature imply intermediate values, like a=b*c+d*e, still use temporaries though.

The classes can be freely intermixed in expressions, as can the classes and the standard types long, unsigned long and double. Smaller types like int or float can also be intermixed, since C++ will promote them.

Note that bool is not accepted directly, but must be explicitly cast to an int first. This is because C++ will automatically convert any pointer to a bool, so if GMP accepted bool it would make all sorts of invalid class and pointer combinations compile but almost certainly not do anything sensible.

Conversions back from the classes to standard C++ types aren't done automatically, instead member functions like `get_si` are provided (see the following sections for details).

Also there are no automatic conversions from the classes to the corresponding GMP C types, instead a reference to the underlying C object can be obtained with the following functions,

`mpz_t mpz_class::get_mpz_t ()` [Function]
`mpq_t mpq_class::get_mpq_t ()` [Function]
`mpf_t mpf_class::get_mpf_t ()` [Function]
These can be used to call a C function which doesn't have a C++ class interface. For example to set `a` to the GCD of `b` and `c`,

```
mpz_class a, b, c;
...
mpz_gcd (a.get_mpz_t(), b.get_mpz_t(), c.get_mpz_t());
```

In the other direction, a class can be initialized from the corresponding GMP C type, or assigned to if an explicit constructor is used. In both cases this makes a copy of the value, it doesn't create any sort of association. For example,

```
mpz_t z;
// ... init and calculate z ...
mpz_class x(z);
mpz_class y;
y = mpz_class (z);
```

There are no namespace setups in '`gmpxx.h`', all types and functions are simply put into the global namespace. This is what '`gmp.h`' has done in the past, and continues to do for compatibility. The extras provided by '`gmpxx.h`' follow GMP naming conventions and are unlikely to clash with anything.

12.2 C++ Interface Integers

`void mpz_class::mpz_class` (*type* `n`) [Function]
Construct an `mpz_class`. All the standard C++ types may be used, except `long long` and `long double`, and all the GMP C++ classes can be used. Any necessary conversion follows the corresponding C function, for example `double` follows `mpz_set_d` (see Section 5.2 [Assigning Integers], page 29).

`void mpz_class::mpz_class` (*mpz_t* `z`) [Function]
Construct an `mpz_class` from an `mpz_t`. The value in *z* is copied into the new `mpz_class`, there won't be any permanent association between it and *z*.

`void mpz_class::mpz_class` (*const char* *`s`) [Function]
`void mpz_class::mpz_class` (*const char* *`s`, *int base*) [Function]
`void mpz_class::mpz_class` (*const string&* `s`) [Function]
`void mpz_class::mpz_class` (*const string&* `s`, *int base*) [Function]
Construct an `mpz_class` converted from a string using `mpz_set_str`, (see Section 5.2 [Assigning Integers], page 29). If the *base* is not given then 0 is used.

`mpz_class operator/` (*mpz_class* `a`, *mpz_class* `d`) [Function] `mpz_class operator% (`*mpz_class* `a`, *mpz_class* `d`) [Function]

Divisions involving `mpz_class` round towards zero, as per the `mpz_tdiv_q` and `mpz_tdiv_r` functions (see Section 5.6 [Integer Division], page 32). This is the same as the C99 `/` and `%` operators.

The `mpz_fdiv...` or `mpz_cdiv...` functions can always be called directly if desired. For example,

```
mpz_class q, a, d;
...
mpz_fdiv_q (q.get_mpz_t(), a.get_mpz_t(), d.get_mpz_t());
```

`mpz_class abs (`*mpz_class* `op1`) [Function]
`int cmp (`*mpz_class* `op1`, *type* `op2`) [Function]
`int cmp (`*type* `op1`, *mpz_class* `op2`) [Function]
`double mpz_class::get_d (`*void*`)` [Function]
`long mpz_class::get_si (`*void*`)` [Function]
`unsigned long mpz_class::get_ui (`*void*`)` [Function]
`bool mpz_class::fits_sint_p (`*void*`)` [Function]
`bool mpz_class::fits_slong_p (`*void*`)` [Function]
`bool mpz_class::fits_sshort_p (`*void*`)` [Function]
`bool mpz_class::fits_uint_p (`*void*`)` [Function]
`bool mpz_class::fits_ulong_p (`*void*`)` [Function]
`bool mpz_class::fits_ushort_p (`*void*`)` [Function]
`int sgn (`*mpz_class* `op`) [Function]
`mpz_class sqrt (`*mpz_class* `op`) [Function]

These functions provide a C++ class interface to the corresponding GMP C routines.

`cmp` can be used with any of the classes or the standard C++ types, except `long long` and `long double`.

Overloaded operators for combinations of `mpz_class` and `double` are provided for completeness, but it should be noted that if the given `double` is not an integer then the way any rounding is done is currently unspecified. The rounding might take place at the start, in the middle, or at the end of the operation, and it might change in the future.

Conversions between `mpz_class` and `double`, however, are defined to follow the corresponding C functions `mpz_get_d` and `mpz_set_d`. And comparisons are always made exactly, as per `mpz_cmp_d`.

12.3 C++ Interface Rationals

In all the following constructors, if a fraction is given then it should be in canonical form, or if not then `mpq_class::canonicalize` called.

`void mpq_class::mpq_class (`*type* `op`) [Function]
`void mpq_class::mpq_class (`*integer* `num`, *integer* `den`) [Function]

Construct an `mpq_class`. The initial value can be a single value of any type, or a pair of integers (`mpz_class` or standard C++ integer types) representing a fraction, except that `long long` and `long double` are not supported. For example,

```
mpq_class q (99);
mpq_class q (1.75);
mpq_class q (1, 3);
```

`void mpq_class::mpq_class (mpq_t q)` [Function]

Construct an `mpq_class` from an `mpq_t`. The value in *q* is copied into the new `mpq_class`, there won't be any permanent association between it and *q*.

`void mpq_class::mpq_class (const char *s)` [Function]
`void mpq_class::mpq_class (const char *s, int base)` [Function]
`void mpq_class::mpq_class (const string& s)` [Function]
`void mpq_class::mpq_class (const string& s, int base)` [Function]

Construct an `mpq_class` converted from a string using `mpq_set_str`, (see Section 6.1 [Initializing Rationals], page 42). If the *base* is not given then 0 is used.

`void mpq_class::canonicalize ()` [Function]

Put an `mpq_class` into canonical form, as per Chapter 6 [Rational Number Functions], page 42. All arithmetic operators require their operands in canonical form, and will return results in canonical form.

`mpq_class abs (mpq_class op)` [Function]
`int cmp (mpq_class op1, type op2)` [Function]
`int cmp (type op1, mpq_class op2)` [Function]
`double mpq_class::get_d (void)` [Function]
`int sgn (mpq_class op)` [Function]

These functions provide a C++ class interface to the corresponding GMP C routines.

`cmp` can be used with any of the classes or the standard C++ types, except `long long` and `long double`.

`mpz_class& mpq_class::get_num ()` [Function]
`mpz_class& mpq_class::get_den ()` [Function]

Get a reference to an `mpz_class` which is the numerator or denominator of an `mpq_class`. This can be used both for read and write access. If the object returned is modified, it modifies the original `mpq_class`.

If direct manipulation might produce a non-canonical value, then `mpq_class::canonicalize` must be called before further operations.

`mpz_t mpq_class::get_num_mpz_t ()` [Function]
`mpz_t mpq_class::get_den_mpz_t ()` [Function]

Get a reference to the underlying `mpz_t` numerator or denominator of an `mpq_class`. This can be passed to C functions expecting an `mpz_t`. Any modifications made to the `mpz_t` will modify the original `mpq_class`.

If direct manipulation might produce a non-canonical value, then `mpq_class::canonicalize` must be called before further operations.

`istream& operator>> (istream& stream, mpq_class& rop);` [Function]

Read *rop* from *stream*, using its `ios` formatting settings, the same as `mpq_t operator>>` (see Section 11.3 [C++ Formatted Input], page 70).

If the *rop* read might not be in canonical form then `mpq_class::canonicalize` must be called.

12.4 C++ Interface Floats

When an expression requires the use of temporary intermediate `mpf_class` values, like `f=g*h+x*y`, those temporaries will have the same precision as the destination `f`. Explicit constructors can be used if this doesn't suit.

`mpf_class::mpf_class` (*type* `op`) [Function]
`mpf_class::mpf_class` (*type* `op`, *unsigned long* `prec`) [Function]

Construct an `mpf_class`. Any standard C++ type can be used, except `long long` and `long double`, and any of the GMP C++ classes can be used.

If *prec* is given, the initial precision is that value, in bits. If *prec* is not given, then the initial precision is determined by the type of *op* given. An `mpz_class`, `mpq_class`, string, or C++ builtin type will give the default `mpf` precision (see Section 7.1 [Initializing Floats], page 46). An `mpf_class` or expression will give the precision of that value. The precision of a binary expression is the higher of the two operands.

```
mpf_class f(1.5);         // default precision
mpf_class f(1.5, 500);    // 500 bits (at least)
mpf_class f(x);           // precision of x
mpf_class f(abs(x));      // precision of x
mpf_class f(-g, 1000);    // 1000 bits (at least)
mpf_class f(x+y);         // greater of precisions of x and y
```

`mpf_class& mpf_class::operator=` (*type* `op`) [Function]

Convert and store the given *op* value to an `mpf_class` object. The same types are accepted as for the constructors above.

Note that `operator=` only stores a new value, it doesn't copy or change the precision of the destination, instead the value is truncated if necessary. This is the same as `mpf_set` etc. Note in particular this means for `mpf_class` a copy constructor is not the same as a default constructor plus assignment.

```
mpf_class x (y);   // x created with precision of y

mpf_class x;       // x created with default precision
x = y;             // value truncated to that precision
```

Applications using templated code may need to be careful about the assumptions the code makes in this area, when working with `mpf_class` values of various different or non-default precisions. For instance implementations of the standard `complex` template have been seen in both styles above, though of course `complex` is normally only actually specified for use with the builtin float types.

`mpf_class abs` (*mpf_class* `op`) [Function]
`mpf_class ceil` (*mpf_class* `op`) [Function]
`int cmp` (*mpf_class* `op1`, *type* `op2`) [Function]
`int cmp` (*type* `op1`, *mpf_class* `op2`) [Function]
`mpf_class floor` (*mpf_class* `op`) [Function]
`mpf_class hypot` (*mpf_class* `op1`, *mpf_class* `op2`) [Function]
`double mpf_class::get_d` (*void*) [Function]
`long mpf_class::get_si` (*void*) [Function]
`unsigned long mpf_class::get_ui` (*void*) [Function]
`bool mpf_class::fits_sint_p` (*void*) [Function]
`bool mpf_class::fits_slong_p` (*void*) [Function]

`bool mpf_class::fits_sshort_p` (*void*) [Function]
`bool mpf_class::fits_uint_p` (*void*) [Function]
`bool mpf_class::fits_ulong_p` (*void*) [Function]
`bool mpf_class::fits_ushort_p` (*void*) [Function]
`int sgn` (*mpf_class* `op`) [Function]
`mpf_class sqrt` (*mpf_class* `op`) [Function]
`mpf_class trunc` (*mpf_class* `op`) [Function]

These functions provide a C++ class interface to the corresponding GMP C routines.

`cmp` can be used with any of the classes or the standard C++ types, except `long long` and `long double`.

The accuracy provided by `hypot` is not currently guaranteed.

`unsigned long int mpf_class::get_prec` () [Function]
`void mpf_class::set_prec` (*unsigned long* `prec`) [Function]
`void mpf_class::set_prec_raw` (*unsigned long* `prec`) [Function]

Get or set the current precision of an `mpf_class`.

The restrictions described for `mpf_set_prec_raw` (see Section 7.1 [Initializing Floats], page 46) apply to `mpf_class::set_prec_raw`. Note in particular that the `mpf_class` must be restored to it's allocated precision before being destroyed. This must be done by application code, there's no automatic mechanism for it.

12.5 C++ Interface MPFR

The C++ class interface to MPFR is provided if MPFR is enabled (see Section 2.1 [Build Options], page 4). This interface must be regarded as preliminary and possibly subject to incompatible changes in the future, since MPFR itself is preliminary. All definitions can be obtained with

```
#include <mpfrxx.h>
```

This defines

`mpfr_class` [Class]

which behaves similarly to `mpf_class` (see Section 12.4 [C++ Interface Floats], page 75).

12.6 C++ Interface Random Numbers

`gmp_randclass` [Class]

The C++ class interface to the GMP random number functions uses `gmp_randclass` to hold an algorithm selection and current state, as per `gmp_randstate_t`.

`gmp_randclass::gmp_randclass` (*void* (*`randinit`) (*gmp_randstate_t, ...*), *...*) [Function]

Construct a `gmp_randclass`, using a call to the given *randinit* function (see Section 9.1 [Random State Initialization], page 61). The arguments expected are the same as *randinit*, but with `mpz_class` instead of `mpz_t`. For example,

```
gmp_randclass r1 (gmp_randinit_default);
gmp_randclass r2 (gmp_randinit_lc_2exp_size, 32);
gmp_randclass r3 (gmp_randinit_lc_2exp, a, c, m2exp);
```

`gmp_randinit_lc_2exp_size` can fail if the size requested is too big, the behaviour of `gmp_randclass::gmp_randclass` is undefined in this case (perhaps this will change in the future).

`gmp_randclass::gmp_randclass` (*gmp_randalg_t* **alg**, ...) [Function]

Construct a `gmp_randclass` using the same parameters as `gmp_randinit` (see Section 9.1 [Random State Initialization], page 61). This function is obsolete and the above *randinit* style should be preferred.

`void gmp_randclass::seed` (*unsigned long int* **s**) [Function]
`void gmp_randclass::seed` (*mpz_class* **s**) [Function]

Seed a random number generator. See see Chapter 9 [Random Number Functions], page 61, for how to choose a good seed.

`mpz_class gmp_randclass::get_z_bits` (*unsigned long* **bits**) [Function]
`mpz_class gmp_randclass::get_z_bits` (*mpz_class* **bits**) [Function]

Generate a random integer with a specified number of bits.

`mpz_class gmp_randclass::get_z_range` (*mpz_class* **n**) [Function]

Generate a random integer in the range 0 to $n-1$ inclusive.

`mpf_class gmp_randclass::get_f` () [Function]
`mpf_class gmp_randclass::get_f` (*unsigned long* **prec**) [Function]

Generate a random float f in the range $0 <= f < 1$. f will be to *prec* bits precision, or if *prec* is not given then to the precision of the destination. For example,

```
gmp_randclass  r;
...
mpf_class  f (0, 512);   // 512 bits precision
f = r.get_f();           // random number, 512 bits
```

12.7 C++ Interface Limitations

`mpq_class` and Templated Reading

A generic piece of template code probably won't know that `mpq_class` requires a `canonicalize` call if inputs read with `operator>>` might be non-canonical. This can lead to incorrect results.

`operator>>` behaves as it does for reasons of efficiency. A canonicalize can be quite time consuming on large operands, and is best avoided if it's not necessary.

But this potential difficulty reduces the usefulness of `mpq_class`. Perhaps a mechanism to tell `operator>>` what to do will be adopted in the future, maybe a preprocessor define, a global flag, or an `ios` flag pressed into service. Or maybe, at the risk of inconsistency, the `mpq_class operator>>` could canonicalize and leave `mpq_t operator>>` not doing so, for use on those occasions when that's acceptable. Send feedback or alternate ideas to bug-gmp@gnu.org.

Subclassing

Subclassing the GMP C++ classes works, but is not currently recommended.

Expressions involving subclasses resolve correctly (or seem to), but in normal C++ fashion the subclass doesn't inherit constructors and assignments. There's many of those in the GMP classes, and a good way to reestablish them in a subclass is not yet provided.

Templated Expressions

A subtle difficulty exists when using expressions together with application-defined template functions. Consider the following, with `T` intended to be some numeric type,

```
template <class T>
T fun (const T &, const T &);
```

When used with, say, plain mpz_class variables, it works fine: T is resolved as mpz_class.

```
mpz_class f(1), g(2);
fun (f, g);    // Good
```

But when one of the arguments is an expression, it doesn't work.

```
mpz_class f(1), g(2), h(3);
fun (f, g+h);  // Bad
```

This is because g+h ends up being a certain expression template type internal to gmpxx.h, which the C++ template resolution rules are unable to automatically convert to mpz_class. The workaround is simply to add an explicit cast.

```
mpz_class f(1), g(2), h(3);
fun (f, mpz_class(g+h));  // Good
```

Similarly, within fun it may be necessary to cast an expression to type T when calling a templated fun2.

```
template <class T>
void fun (T f, T g)
{
  fun2 (f, f+g);     // Bad
}

template <class T>
void fun (T f, T g)
{
  fun2 (f, T(f+g));  // Good
}
```

13 Berkeley MP Compatible Functions

These functions are intended to be fully compatible with the Berkeley MP library which is available on many BSD derived U*ix systems. The '--enable-mpbsd' option must be used when building GNU MP to make these available (see Chapter 2 [Installing GMP], page 4).

The original Berkeley MP library has a usage restriction: you cannot use the same variable as both source and destination in a single function call. The compatible functions in GNU MP do not share this restriction—inputs and outputs may overlap.

It is not recommended that new programs are written using these functions. Apart from the incomplete set of functions, the interface for initializing `MINT` objects is more error prone, and the `pow` function collides with `pow` in 'libm.a'.

Include the header 'mp.h' to get the definition of the necessary types and functions. If you are on a BSD derived system, make sure to include GNU 'mp.h' if you are going to link the GNU 'libmp.a' to your program. This means that you probably need to give the '-I<dir>' option to the compiler, where '<dir>' is the directory where you have GNU 'mp.h'.

`MINT * itom` (*signed short int* **initial_value**) [Function]

Allocate an integer consisting of a `MINT` object and dynamic limb space. Initialize the integer to *initial_value*. Return a pointer to the `MINT` object.

`MINT * xtom` (*char* ***initial_value**) [Function]

Allocate an integer consisting of a `MINT` object and dynamic limb space. Initialize the integer from *initial_value*, a hexadecimal, null-terminated C string. Return a pointer to the `MINT` object.

`void move` (*MINT* ***src**, *MINT* ***dest**) [Function]

Set *dest* to *src* by copying. Both variables must be previously initialized.

`void madd` (*MINT* ***src_1**, *MINT* ***src_2**, *MINT* ***destination**) [Function]

Add *src_1* and *src_2* and put the sum in *destination*.

`void msub` (*MINT* ***src_1**, *MINT* ***src_2**, *MINT* ***destination**) [Function]

Subtract *src_2* from *src_1* and put the difference in *destination*.

`void mult` (*MINT* ***src_1**, *MINT* ***src_2**, *MINT* ***destination**) [Function]

Multiply *src_1* and *src_2* and put the product in *destination*.

`void mdiv` (*MINT* ***dividend**, *MINT* ***divisor**, *MINT* ***quotient**, *MINT* ***remainder**) [Function]

`void sdiv` (*MINT* ***dividend**, *signed short int* **divisor**, *MINT* ***quotient**, *signed short int* ***remainder**) [Function]

Set *quotient* to *dividend*/*divisor*, and *remainder* to *dividend* mod *divisor*. The quotient is rounded towards zero; the remainder has the same sign as the dividend unless it is zero.

Some implementations of these functions work differently—or not at all—for negative arguments.

`void msqrt` (*MINT* ***op**, *MINT* ***root**, *MINT* ***remainder**) [Function]

Set *root* to $\lfloor\sqrt{op}\rfloor$, like `mpz_sqrt`. Set *remainder* to $(op - root^2)$, i.e. zero if *op* is a perfect square.

If *root* and *remainder* are the same variable, the results are undefined.

`void pow (MINT *base, MINT *exp, MINT *mod, MINT *dest)` [Function]

Set *dest* to (*base* raised to *exp*) modulo *mod*.

Note that the name `pow` clashes with `pow` from the standard C math library (see section "Exponentiation and Logarithms" in *The GNU C Library Reference Manual*). An application will only be able to use one or the other.

`void rpow (MINT *base, signed short int exp, MINT *dest)` [Function]

Set *dest* to *base* raised to *exp*.

`void gcd (MINT *op1, MINT *op2, MINT *res)` [Function]

Set *res* to the greatest common divisor of *op1* and *op2*.

`int mcmp (MINT *op1, MINT *op2)` [Function]

Compare *op1* and *op2*. Return a positive value if *op1* > *op2*, zero if *op1* = *op2*, and a negative value if *op1* < *op2*.

`void min (MINT *dest)` [Function]

Input a decimal string from `stdin`, and put the read integer in *dest*. SPC and TAB are allowed in the number string, and are ignored.

`void mout (MINT *src)` [Function]

Output *src* to `stdout`, as a decimal string. Also output a newline.

`char * mtox (MINT *op)` [Function]

Convert *op* to a hexadecimal string, and return a pointer to the string. The returned string is allocated using the default memory allocation function, `malloc` by default. It will be `strlen(str)+1` bytes, that being exactly enough for the string and null-terminator.

`void mfree (MINT *op)` [Function]

De-allocate, the space used by *op*. **This function should only be passed a value returned by `itom` or `xtom`.**

14 Custom Allocation

By default GMP uses `malloc`, `realloc` and `free` for memory allocation, and if they fail GMP prints a message to the standard error output and terminates the program.

Alternate functions can be specified to allocate memory in a different way or to have a different error action on running out of memory.

This feature is available in the Berkeley compatibility library (see Chapter 13 [BSD Compatible Functions], page 80) as well as the main GMP library.

`void mp_set_memory_functions (` *void *(****alloc_func_ptr***) (size_t),* *void *(****realloc_func_ptr***) (void *, size_t, size_t),* *void (****free_func_ptr***) (void *, size_t))* [Function]

Replace the current allocation functions from the arguments. If an argument is `NULL`, the corresponding default function is used.

These functions will be used for all memory allocation done by GMP, apart from temporary space from `alloca` if that function is available and GMP is configured to use it (see Section 2.1 [Build Options], page 4).

Be sure to call `mp_set_memory_functions` only when there are no active GMP objects allocated using the previous memory functions! Usually that means calling it before any other GMP function.

The functions supplied should fit the following declarations:

`void * allocate_function (`*size_t* ***alloc_size***`)` [Function]

Return a pointer to newly allocated space with at least *alloc_size* bytes.

`void * reallocate_function (`*void ****ptr***, *size_t* ***old_size***, *size_t* ***new_size***`)` [Function]

Resize a previously allocated block *ptr* of *old_size* bytes to be *new_size* bytes.

The block may be moved if necessary or if desired, and in that case the smaller of *old_size* and *new_size* bytes must be copied to the new location. The return value is a pointer to the resized block, that being the new location if moved or just *ptr* if not.

ptr is never `NULL`, it's always a previously allocated block. *new_size* may be bigger or smaller than *old_size*.

`void deallocate_function (`*void ****ptr***, *size_t* ***size***`)` [Function]

De-allocate the space pointed to by *ptr*.

ptr is never `NULL`, it's always a previously allocated block of *size* bytes.

A *byte* here means the unit used by the `sizeof` operator.

The *old_size* parameters to *reallocate_function* and *deallocate_function* are passed for convenience, but of course can be ignored if not needed. The default functions using `malloc` and friends for instance don't use them.

No error return is allowed from any of these functions, if they return then they must have performed the specified operation. In particular note that *allocate_function* or *reallocate_function* mustn't return `NULL`.

Getting a different fatal error action is a good use for custom allocation functions, for example giving a graphical dialog rather than the default print to `stderr`. How much is possible when genuinely out of memory is another question though.

There's currently no defined way for the allocation functions to recover from an error such as out of memory, they must terminate program execution. A `longjmp` or throwing a C++ exception will have undefined results. This may change in the future.

GMP may use allocated blocks to hold pointers to other allocated blocks. This will limit the assumptions a conservative garbage collection scheme can make.

Since the default GMP allocation uses `malloc` and friends, those functions will be linked in even if the first thing a program does is an `mp_set_memory_functions`. It's necessary to change the GMP sources if this is a problem.

15 Language Bindings

The following packages and projects offer access to GMP from languages other than C, though perhaps with varying levels of functionality and efficiency.

C++

- GMP C++ class interface, see Chapter 12 [C++ Class Interface], page 72
  Straightforward interface, expression templates to eliminate temporaries.
- ALP `http://www.inria.fr/saga/logiciels/ALP`
  Linear algebra and polynomials using templates.
- Arithmos `http://win-www.uia.ac.be/u/cant/arithmos`
  Rationals with infinities and square roots.
- CLN `http://www.ginac.de/CLN/`
  High level classes for arithmetic.
- LiDIA `http://www.informatik.tu-darmstadt.de/TI/LiDIA`
  A C++ library for computational number theory.
- Linbox `http://www.linalg.org`
  Sparse vectors and matrices.
- NTL `http://www.shoup.net/ntl`
  A C++ number theory library.

Fortran

- Omni F77 `http://phase.hpcc.jp/Omni/home.html`
  Arbitrary precision floats.

Haskell

- Glasgow Haskell Compiler `http://www.haskell.org/ghc`

Java

- Kaffe `http://www.kaffe.org`
- Kissme `http://kissme.sourceforge.net`

Lisp

- GNU Common Lisp `http://www.gnu.org/software/gcl/gcl.html`
  In the process of switching to GMP for bignums.
- Librep `http://librep.sourceforge.net`
- XEmacs (21.5.18 beta and up) `http://www.xemacs.org`
  Optional big integers, rationals and floats using GMP.

M4

- GNU m4 betas `http://www.seindal.dk/rene/gnu`
  Optionally provides an arbitrary precision `mpeval`.

ML

- MLton compiler `http://www.mlton.org`

Objective Caml

- MLGMP `http://www.di.ens.fr/~monniaux/programmes.html.en`
- Numerix `http://pauillac.inria.fr/~quercia/`
  Optionally using GMP.

Oz

- Mozart http://www.mozart-oz.org

Pascal

- GNU Pascal Compiler http://www.gnu-pascal.de
  GMP unit.
- Numerix http://pauillac.inria.fr/~quercia/
  For Free Pascal, optionally using GMP.

Perl

- GMP module, see 'demos/perl' in the GMP sources.
- Math::GMP http://www.cpan.org
  Compatible with Math::BigInt, but not as many functions as the GMP module above.
- Math::BigInt::GMP http://www.cpan.org
  Plug Math::GMP into normal Math::BigInt operations.

Pike

- mpz module in the standard distribution, http://pike.ida.liu.se/

Prolog

- SWI Prolog http://www.swi.psy.uva.nl/projects/SWI-Prolog
  Arbitrary precision floats.

Python

- mpz module in the standard distribution, http://www.python.org
- GMPY http://gmpy.sourceforge.net

Scheme

- GNU Guile (upcoming 1.8) http://www.gnu.org/software/guile/guile.html
- RScheme http://www.rscheme.org
- STklos http://kaolin.unice.fr/STklos

Smalltalk

- GNU Smalltalk http://www.smalltalk.org/versions/GNUSmalltalk.html

Other

- Axiom http://savannah.nongnu.org/projects/axiom
  Computer algebra using GCL.
- DrGenius http://drgenius.seul.org
  Geometry system and mathematical programming language.
- GiNaC http://www.ginac.de
  C++ computer algebra using CLN.
- GOO http://www.googoogaga.org/
  Dynamic object oriented language.
- Maxima http://www.ma.utexas.edu/users/wfs/maxima.html
  Macsyma computer algebra using GCL.
- Q http://www.musikwissenschaft.uni-mainz.de/~ag/q
  Equational programming system.
- Regina http://regina.sourceforge.net
  Topological calculator.
- Yacas http://www.xs4all.nl/~apinkus/yacas.html
  Yet another computer algebra system.

16 Algorithms

This chapter is an introduction to some of the algorithms used for various GMP operations. The code is likely to be hard to understand without knowing something about the algorithms.

Some GMP internals are mentioned, but applications that expect to be compatible with future GMP releases should take care to use only the documented functions.

16.1 Multiplication

N×N limb multiplications and squares are done using one of four algorithms, as the size N increases.

Algorithm	Threshold
Basecase	(none)
Karatsuba	`MUL_KARATSUBA_THRESHOLD`
Toom-3	`MUL_TOOM3_THRESHOLD`
FFT	`MUL_FFT_THRESHOLD`

Similarly for squaring, with the `SQR` thresholds. Note though that the FFT is only used if GMP is configured with '`--enable-fft`', see Section 2.1 [Build Options], page 4.

N×M multiplications of operands with different sizes above `MUL_KARATSUBA_THRESHOLD` are currently done by splitting into M×M pieces. The Karatsuba and Toom-3 routines then operate only on equal size operands. This is not very efficient, and is slated for improvement in the future.

16.1.1 Basecase Multiplication

Basecase N×M multiplication is a straightforward rectangular set of cross-products, the same as long multiplication done by hand and for that reason sometimes known as the schoolbook or grammar school method. This is an $O(NM)$ algorithm. See Knuth section 4.3.1 algorithm M (see Appendix B [References], page 113), and the '`mpn/generic/mul_basecase.c`' code.

Assembler implementations of `mpn_mul_basecase` are essentially the same as the generic C code, but have all the usual assembler tricks and obscurities introduced for speed.

A square can be done in roughly half the time of a multiply, by using the fact that the cross products above and below the diagonal are the same. A triangle of products below the diagonal is formed, doubled (left shift by one bit), and then the products on the diagonal added. This can be seen in '`mpn/generic/sqr_basecase.c`'. Again the assembler implementations take essentially the same approach.

	u0	u1	u2	u3	u4
u0	d				
u1		d			
u2			d		
u3				d	
u4					d

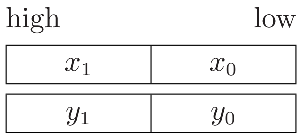
In practice squaring isn't a full 2× faster than multiplying, it's usually around 1.5×. Less than 1.5× probably indicates `mpn_sqr_basecase` wants improving on that CPU.

On some CPUs `mpn_mul_basecase` can be faster than the generic C `mpn_sqr_basecase`. `SQR_BASECASE_THRESHOLD` is the size at which to use `mpn_sqr_basecase`, this will be zero if that routine should be used always.

16.1.2 Karatsuba Multiplication

The Karatsuba multiplication algorithm is described in Knuth section 4.3.3 part A, and various other textbooks. A brief description is given here.

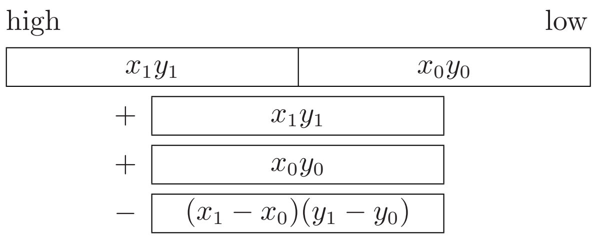
The inputs x and y are treated as each split into two parts of equal length (or the most significant part one limb shorter if N is odd).



Let b be the power of 2 where the split occurs, ie. if x_0 is k limbs (y_0 the same) then $b = 2^{k*\texttt{mp\_bits\_per\_limb}}$. With that $x = x_1b + x_0$ and $y = y_1b + y_0$, and the following holds,

$$xy = (b^2 + b)x_1y_1 - b(x_1 - x_0)(y_1 - y_0) + (b + 1)x_0y_0$$

This formula means doing only three multiplies of (N/2)×(N/2) limbs, whereas a basecase multiply of N×N limbs is equivalent to four multiplies of (N/2)×(N/2). The factors $(b^2 + b)$ etc represent the positions where the three products must be added.



The term $(x_1 - x_0)(y_1 - y_0)$ is best calculated as an absolute value, and the sign used to choose to add or subtract. Notice the sum $\mathrm{high}(x_0y_0) + \mathrm{low}(x_1y_1)$ occurs twice, so it's possible to do $5k$ limb additions, rather than $6k$, but in GMP extra function call overheads outweigh the saving.

Squaring is similar to multiplying, but with $x = y$ the formula reduces to an equivalent with three squares,

$$x^2 = (b^2 + b)x_1^2 - b(x_1 - x_0)^2 + (b + 1)x_0^2$$

The final result is accumulated from those three squares the same way as for the three multiplies above. The middle term $(x_1 - x_0)^2$ is now always positive.

A similar formula for both multiplying and squaring can be constructed with a middle term $(x_1 + x_0)(y_1 + y_0)$. But those sums can exceed k limbs, leading to more carry handling and additions than the form above.

Karatsuba multiplication is asymptotically an $O(N^{1.585})$ algorithm, the exponent being $\log 3/\log 2$, representing 3 multiplies each 1/2 the size of the inputs. This is a big improvement over the basecase multiply at $O(N^2)$ and the advantage soon overcomes the extra additions Karatsuba performs. `MUL_KARATSUBA_THRESHOLD` can be as little as 10 limbs. The `SQR` threshold is usually about twice the `MUL`.

The basecase algorithm will take a time of the form $M(N) = aN^2 + bN + c$ and the Karatsuba algorithm $K(N) = 3M(N/2) + dN + e$, which expands to $K(N) = \frac{3}{4}aN^2 + \frac{3}{2}bN + 3c + dN + e$. The factor $\frac{3}{4}$ for a means per-crossproduct speedups in the basecase code will increase the threshold since they benefit $M(N)$ more than $K(N)$. And conversely the $\frac{3}{2}$ for b means linear style speedups of b will increase the threshold since they benefit $K(N)$ more than $M(N)$. The latter can be seen for instance when adding an optimized `mpn_sqr_diagonal` to `mpn_sqr_basecase`. Of course all speedups reduce total time, and in that sense the algorithm thresholds are merely of academic interest.

16.1.3 Toom-Cook 3-Way Multiplication

The Karatsuba formula is the simplest case of a general approach to splitting inputs that leads to both Toom-Cook and FFT algorithms. A description of Toom-Cook can be found in Knuth section 4.3.3, with an example 3-way calculation after Theorem A. The 3-way form used in GMP is described here.

The operands are each considered split into 3 pieces of equal length (or the most significant part 1 or 2 limbs shorter than the others).

high		low
x_2	x_1	x_0
y_2	y_1	y_0

These parts are treated as the coefficients of two polynomials

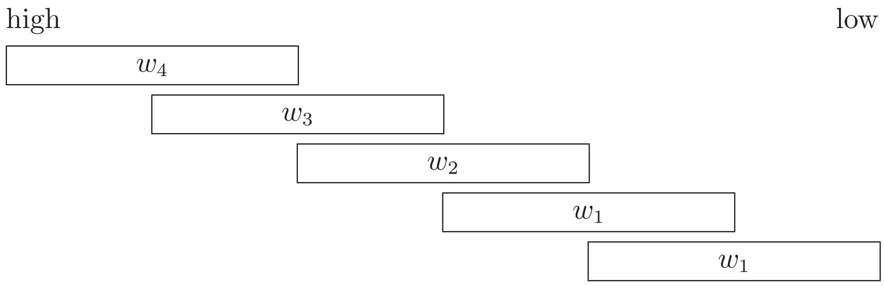
$$X(t) = x_2t^2 + x_1t + x_0$$
$$Y(t) = y_2t^2 + y_1t + y_0$$

Again let b equal the power of 2 which is the size of the x_0, x_1, y_0 and y_1 pieces, ie. if they're k limbs each then $b = 2^{k*\texttt{mp\_bits\_per\_limb}}$. With this $x = X(b)$ and $y = Y(b)$.

Let a polynomial $W(t) = X(t)Y(t)$ and suppose its coefficients are

$$W(t) = w_4t^4 + w_3t^3 + w_2t^2 + w_1t + w_0$$

The w_i are going to be determined, and when they are they'll give the final result using $w = W(b)$, since $xy = X(b)Y(b)$. The coefficients will be roughly b^2 each, and the final $W(b)$ will be an addition like,



The w_i coefficients could be formed by a simple set of cross products, like $w_4 = x_2y_2$, $w_3 = x_2y_1 + x_1y_2$, $w_2 = x_2y_0 + x_1y_1 + x_0y_2$ etc, but this would need all nine x_iy_j for $i, j = 0, 1, 2$, and would be equivalent merely to a basecase multiply. Instead the following approach is used.

$X(t)$ and $Y(t)$ are evaluated and multiplied at 5 points, giving values of $W(t)$ at those points. The points used can be chosen in various ways, but in GMP the following are used

Point	Value
$t = 0$	x_0y_0, which gives w_0 immediately
$t = 2$	$(4x_2 + 2x_1 + x_0)(4y_2 + 2y_1 + y_0)$

$$\begin{array}{ll} t=1 & (x_2+x_1+x_0)(y_2+y_1+y_0) \\ t=\frac{1}{2} & (x_2+2x_1+4x_0)(y_2+2y_1+4y_0) \\ t=\infty & x_2y_2, \text{ which gives } w_4 \text{ immediately} \end{array}$$

At $t = \frac{1}{2}$ the value calculated is actually $16X(\frac{1}{2})Y(\frac{1}{2})$, giving a value for $16W(\frac{1}{2})$, and this is always an integer. At $t = \infty$ the value is actually $\lim_{t\to\infty} \frac{X(t)Y(t)}{t^4}$, but it's much easier to think of as simply x_2y_2 giving w_4 immediately (much like x_0y_0 at $t = 0$ gives w_0 immediately).

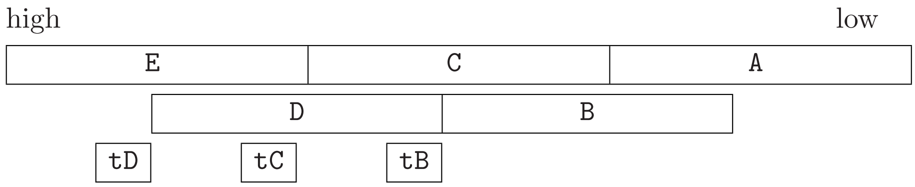
Now each of the points substituted into $W(t) = w_4t^4 + \cdots + w_0$ gives a linear combination of the w_i coefficients, and the value of those combinations has just been calculated.

$$\begin{array}{rcccccccccc} W(0) & = & & & & & & & & & w_0 \\ 16W(\frac{1}{2}) & = & w_4 & + & 2w_3 & + & 4w_2 & + & 8w_1 & + & 16w_0 \\ W(1) & = & w_4 & + & w_3 & + & w_2 & + & w_1 & + & w_0 \\ W(2) & = & 16w_4 & + & 8w_3 & + & 4w_2 & + & 2w_1 & + & w_0 \\ W(\infty) & = & w_4 & & & & & & & & \end{array}$$

This is a set of five equations in five unknowns, and some elementary linear algebra quickly isolates each w_i, by subtracting multiples of one equation from another.

In the code the set of five values $W(0)$,...,$W(\infty)$ will represent those certain linear combinations. By adding or subtracting one from another as necessary, values which are each w_i alone are arrived at. This involves only a few subtractions of small multiples (some of which are powers of 2), and so is fast. A couple of divisions remain by powers of 2 and one division by 3 (or by 6 rather), and that last uses the special `mpn_divexact_by3` (see Section 16.2.4 [Exact Division], page 93).

In the code the values w_4, w_2 and w_0 are formed in the destination with pointers `E`, `C` and `A`, and w_3 and w_1 in temporary space `D` and `B` are added to them. There are extra limbs `tD`, `tC` and `tB` at the high end of w_3, w_2 and w_1 which are handled separately. The final addition then is as follows.



The conversion of $W(t)$ values to the coefficients is interpolation. A polynomial of degree 4 like $W(t)$ is uniquely determined by values known at 5 different points. The points can be chosen to make the linear equations come out with a convenient set of steps for isolating the w_i.

In '`mpn/generic/mul_n.c`' the `interpolate3` routine performs the interpolation. The open-coded one-pass version may be a bit hard to understand, the steps performed can be better seen in the `USE_MORE_MPN` version.

Squaring follows the same procedure as multiplication, but there's only one $X(t)$ and it's evaluated at 5 points, and those values squared to give values of $W(t)$. The interpolation is then identical, and in fact the same `interpolate3` subroutine is used for both squaring and multiplying.

Toom-3 is asymptotically $O(N^{1.465})$, the exponent being $\log 5/\log 3$, representing 5 recursive multiplies of 1/3 the original size. This is an improvement over Karatsuba at $O(N^{1.585})$, though Toom-Cook does more work in the evaluation and interpolation and so it only realizes its advantage above a certain size.

Near the crossover between Toom-3 and Karatsuba there's generally a range of sizes where the difference between the two is small. `MUL_TOOM3_THRESHOLD` is a somewhat arbitrary point in that range and successive runs of the tune program can give different values due to small variations in measuring. A graph of time versus size for the two shows the effect, see '`tune/README`'.

At the fairly small sizes where the Toom-3 thresholds occur it's worth remembering that the asymptotic behaviour for Karatsuba and Toom-3 can't be expected to make accurate predictions, due of course to the big influence of all sorts of overheads, and the fact that only a few recursions of each are being performed. Even at large sizes there's a good chance machine dependent effects like cache architecture will mean actual performance deviates from what might be predicted.

The formula given above for the Karatsuba algorithm has an equivalent for Toom-3 involving only five multiplies, but this would be complicated and unenlightening.

An alternate view of Toom-3 can be found in Zuras (see Appendix B [References], page 113), using a vector to represent the x and y splits and a matrix multiplication for the evaluation and interpolation stages. The matrix inverses are not meant to be actually used, and they have elements with values much greater than in fact arise in the interpolation steps. The diagram shown for the 3-way is attractive, but again doesn't have to be implemented that way and for example with a bit of rearrangement just one division by 6 can be done.

16.1.4 FFT Multiplication

At large to very large sizes a Fermat style FFT multiplication is used, following Schönhage and Strassen (see Appendix B [References], page 113). Descriptions of FFTs in various forms can be found in many textbooks, for instance Knuth section 4.3.3 part C or Lipson chapter IX. A brief description of the form used in GMP is given here.

The multiplication done is $xy \bmod 2^N + 1$, for a given N. A full product xy is obtained by choosing $N \ge \text{bits}(x) + \text{bits}(y)$ and padding x and y with high zero limbs. The modular product is the native form for the algorithm, so padding to get a full product is unavoidable.

The algorithm follows a split, evaluate, pointwise multiply, interpolate and combine similar to that described above for Karatsuba and Toom-3. A k parameter controls the split, with an FFT-k splitting into 2^k pieces of $M = N/2^k$ bits each. N must be a multiple of $2^k \times$`mp_bits_per_limb` so the split falls on limb boundaries, avoiding bit shifts in the split and combine stages.

The evaluations, pointwise multiplications, and interpolation, are all done modulo $2^{N'}+1$ where N' is $2M + k + 3$ rounded up to a multiple of 2^k and of `mp_bits_per_limb`. The results of interpolation will be the following negacyclic convolution of the input pieces, and the choice of N' ensures these sums aren't truncated.

$$w_n = \sum_{\substack{i+j=b2^k+n \\ b=0,1}} (-1)^b x_i y_j$$

The points used for the evaluation are g^i for $i = 0$ to $2^k - 1$ where $g = 2^{2N'/2^k}$. g is a 2^kth root of unity mod $2^{N'}+1$, which produces necessary cancellations at the interpolation stage, and it's also a power of 2 so the fast fourier transforms used for the evaluation and interpolation do only shifts, adds and negations.

The pointwise multiplications are done modulo $2^{N'} + 1$ and either recurse into a further FFT or use a plain multiplication (Toom-3, Karatsuba or basecase), whichever is optimal at the size N'. The interpolation is an inverse fast fourier transform. The resulting set of sums of $x_i y_j$ are added at appropriate offsets to give the final result.

Squaring is the same, but x is the only input so it's one transform at the evaluate stage and the pointwise multiplies are squares. The interpolation is the same.

For a mod $2^N + 1$ product, an FFT-k is an $O(N^{k/(k-1)})$ algorithm, the exponent representing 2^k recursed modular multiplies each $1/2^{k-1}$ the size of the original. Each successive k is an asymptotic improvement, but overheads mean each is only faster at bigger and bigger sizes. In the code, `MUL_FFT_TABLE` and `SQR_FFT_TABLE` are the thresholds where each k is used. Each new k effectively swaps some multiplying for some shifts, adds and overheads.

A mod 2^N+1 product can be formed with a normal $N \times N \rightarrow 2N$ bit multiply plus a subtraction, so an FFT and Toom-3 etc can be compared directly. A $k = 4$ FFT at $O(N^{1.333})$ can be expected to be the first faster than Toom-3 at $O(N^{1.465})$. In practice this is what's found, with `MUL_FFT_MODF_THRESHOLD` and `SQR_FFT_MODF_THRESHOLD` being between 300 and 1000 limbs, depending on the CPU. So far it's been found that only very large FFTs recurse into pointwise multiplies above these sizes.

When an FFT is to give a full product, the change of N to $2N$ doesn't alter the theoretical complexity for a given k, but for the purposes of considering where an FFT might be first used it can be assumed that the FFT is recursing into a normal multiply and that on that basis it's doing 2^k recursed multiplies each $1/2^{k-2}$ the size of the inputs, making it $O(N^{k/(k-2)})$. This would mean $k = 7$ at $O(N^{1.4})$ would be the first FFT faster than Toom-3. In practice `MUL_FFT_THRESHOLD` and `SQR_FFT_THRESHOLD` have been found to be in the $k = 8$ range, somewhere between 3000 and 10000 limbs.

The way N is split into 2^k pieces and then $2M + k + 3$ is rounded up to a multiple of 2^k and `mp_bits_per_limb` means that when $2^k \geq$ `mp_bits_per_limb` the effective N is a multiple of 2^{2k-1} bits. The $+k + 3$ means some values of N just under such a multiple will be rounded to the next. The complexity calculations above assume that a favourable size is used, meaning one which isn't padded through rounding, and it's also assumed that the extra $+k + 3$ bits are negligible at typical FFT sizes.

The practical effect of the 2^{2k-1} constraint is to introduce a step-effect into measured speeds. For example $k = 8$ will round N up to a multiple of 32768 bits, so for a 32-bit limb there'll be 512 limb groups of sizes for which `mpn_mul_n` runs at the same speed. Or for $k = 9$ groups of 2048 limbs, $k = 10$ groups of 8192 limbs, etc. In practice it's been found each k is used at quite small multiples of its size constraint and so the step effect is quite noticeable in a time versus size graph.

The threshold determinations currently measure at the mid-points of size steps, but this is sub-optimal since at the start of a new step it can happen that it's better to go back to the previous k for a while. Something more sophisticated for `MUL_FFT_TABLE` and `SQR_FFT_TABLE` will be needed.

16.1.5 Other Multiplication

The 3-way Toom-Cook algorithm described above (see Section 16.1.3 [Toom-Cook 3-Way Multiplication], page 88) generalizes to split into an arbitrary number of pieces, as per Knuth section 4.3.3 algorithm C. This is not currently used, though it's possible a Toom-4 might fit in between Toom-3 and the FFTs. The notes here are merely for interest.

In general a split into $r + 1$ pieces is made, and evaluations and pointwise multiplications done at $2r+1$ points. A 4-way split does 7 pointwise multiplies, 5-way does 9, etc. Asymptotically an $(r + 1)$-way algorithm is $O(N^{log(2r+1)/log(r+1)}$. Only the pointwise multiplications count towards big-O complexity, but the time spent in the evaluate and interpolate stages grows with r and has a significant practical impact, with the asymptotic advantage of each r realized only at bigger and bigger sizes. The overheads grow as $O(Nr)$, whereas in an $r = 2^k$ FFT they grow only as $O(N \log r)$.

Knuth algorithm C evaluates at points 0,1,2,...,$2r$, but exercise 4 uses $-r$,...,0,...,r and the latter saves some small multiplies in the evaluate stage (or rather trades them for additions), and has a further saving of nearly half the interpolate steps. The idea is to separate odd and even final coefficients and then perform algorithm C steps C7 and C8 on them separately. The divisors at step C7 become j^2 and the multipliers at C8 become $2tj - j^2$.

Splitting odd and even parts through positive and negative points can be thought of as using -1 as a square root of unity. If a 4th root of unity was available then a further split and speedup would be possible, but no such root exists for plain integers. Going to complex integers with $i = \sqrt{-1}$ doesn't help, essentially because in cartesian form it takes three real multiplies to do a complex multiply. The existence of 2^kth roots of unity in a suitable ring or field lets the fast fourier transform keep splitting and get to $O(N \log r)$.

Floating point FFTs use complex numbers approximating Nth roots of unity. Some processors have special support for such FFTs. But these are not used in GMP since it's very difficult to guarantee an exact result (to some number of bits). An occasional difference of 1 in the last bit might not matter to a typical signal processing algorithm, but is of course of vital importance to GMP.

16.2 Division Algorithms

16.2.1 Single Limb Division

N×1 division is implemented using repeated 2×1 divisions from high to low, either with a hardware divide instruction or a multiplication by inverse, whichever is best on a given CPU.

The multiply by inverse follows section 8 of "Division by Invariant Integers using Multiplication" by Granlund and Montgomery (see Appendix B [References], page 113) and is implemented as `udiv_qrnnd_preinv` in '`gmp-impl.h`'. The idea is to have a fixed-point approximation to $1/d$ (see `invert_limb`) and then multiply by the high limb (plus one bit) of the dividend to get a quotient q. With d normalized (high bit set), q is no more than 1 too small. Subtracting qd from the dividend gives a remainder, and reveals whether q or $q - 1$ is correct.

The result is a division done with two multiplications and four or five arithmetic operations. On CPUs with low latency multipliers this can be much faster than a hardware divide, though the cost of calculating the inverse at the start may mean it's only better on inputs bigger than say 4 or 5 limbs.

When a divisor must be normalized, either for the generic C `__udiv_qrnnd_c` or the multiply by inverse, the division performed is actually $a2^k$ by $d2^k$ where a is the dividend and k is the power necessary to have the high bit of $d2^k$ set. The bit shifts for the dividend are usually accomplished "on the fly" meaning by extracting the appropriate bits at each step. Done this way the quotient limbs come out aligned ready to store. When only the remainder is wanted, an alternative is to take the dividend limbs unshifted and calculate $r = a \bmod d2^k$ followed by an extra final step $r2^k \bmod d2^k$. This can help on CPUs with poor bit shifts or few registers.

The multiply by inverse can be done two limbs at a time. The calculation is basically the same, but the inverse is two limbs and the divisor treated as if padded with a low zero limb. This means more work, since the inverse will need a 2×2 multiply, but the four 1×1s to do that are independent and can therefore be done partly or wholly in parallel. Likewise for a 2×1 calculating qd. The net effect is to process two limbs with roughly the same two multiplies worth of latency that one limb at a time gives. This extends to 3 or 4 limbs at a time, though the extra work to apply the inverse will almost certainly soon reach the limits of multiplier throughput.

A similar approach in reverse can be taken to process just half a limb at a time if the divisor is only a half limb. In this case the 1×1 multiply for the inverse effectively becomes two $\frac{1}{2} \times 1$ for each limb, which can be a saving on CPUs with a fast half limb multiply, or in fact if the only multiply is a half limb, and especially if it's not pipelined.

16.2.2 Basecase Division

Basecase N×M division is like long division done by hand, but in base $2^{\texttt{mp\_bits\_per\_limb}}$. See Knuth section 4.3.1 algorithm D, and '`mpn/generic/sb_divrem_mn.c`'.

Briefly stated, while the dividend remains larger than the divisor, a high quotient limb is formed and the N×1 product *qd* subtracted at the top end of the dividend. With a normalized divisor (most significant bit set), each quotient limb can be formed with a 2×1 division and a 1×1 multiplication plus some subtractions. The 2×1 division is by the high limb of the divisor and is done either with a hardware divide or a multiply by inverse (the same as in Section 16.2.1 [Single Limb Division], page 92) whichever is faster. Such a quotient is sometimes one too big, requiring an addback of the divisor, but that happens rarely.

With Q=N−M being the number of quotient limbs, this is an $O(QM)$ algorithm and will run at a speed similar to a basecase Q×M multiplication, differing in fact only in the extra multiply and divide for each of the Q quotient limbs.

16.2.3 Divide and Conquer Division

For divisors larger than `DIV_DC_THRESHOLD`, division is done by dividing. Or to be precise by a recursive divide and conquer algorithm based on work by Moenck and Borodin, Jebelean, and Burnikel and Ziegler (see Appendix B [References], page 113).

The algorithm consists essentially of recognising that a 2N×N division can be done with the basecase division algorithm (see Section 16.2.2 [Basecase Division], page 93), but using N/2 limbs as a base, not just a single limb. This way the multiplications that arise are (N/2)×(N/2) and can take advantage of Karatsuba and higher multiplication algorithms (see Section 16.1 [Multiplication Algorithms], page 86). The "digits" of the quotient are formed by recursive N×(N/2) divisions.

If the (N/2)×(N/2) multiplies are done with a basecase multiplication then the work is about the same as a basecase division, but with more function call overheads and with some subtractions separated from the multiplies. These overheads mean that it's only when N/2 is above `MUL_KARATSUBA_THRESHOLD` that divide and conquer is of use.

`DIV_DC_THRESHOLD` is based on the divisor size N, so it will be somewhere above twice `MUL_KARATSUBA_THRESHOLD`, but how much above depends on the CPU. An optimized `mpn_mul_basecase` can lower `DIV_DC_THRESHOLD` a little by offering a ready-made advantage over repeated `mpn_submul_1` calls.

Divide and conquer is asymptotically $O(M(N) \log N)$ where $M(N)$ is the time for an N×N multiplication done with FFTs. The actual time is a sum over multiplications of the recursed sizes, as can be seen near the end of section 2.2 of Burnikel and Ziegler. For example, within the Toom-3 range, divide and conquer is $2.63M(N)$. With higher algorithms the $M(N)$ term improves and the multiplier tends to $\log N$. In practice, at moderate to large sizes, a 2N×N division is about 2 to 4 times slower than an N×N multiplication.

Newton's method used for division is asymptotically $O(M(N))$ and should therefore be superior to divide and conquer, but it's believed this would only be for large to very large N.

16.2.4 Exact Division

A so-called exact division is when the dividend is known to be an exact multiple of the divisor. Jebelean's exact division algorithm uses this knowledge to make some significant optimizations (see Appendix B [References], page 113).

The idea can be illustrated in decimal for example with 368154 divided by 543. Because the low digit of the dividend is 4, the low digit of the quotient must be 8. This is arrived at from $4{\times}7 \bmod 10$, using the fact 7 is the modular inverse of 3 (the low digit of the divisor), since $3{\times}7 \equiv 1 \bmod 10$. So $8{\times}543 = 4344$ can be subtracted from the dividend leaving 363810. Notice the low digit has become zero.

The procedure is repeated at the second digit, with the next quotient digit 7 ($1{\times}7 \bmod 10$), subtracting $7{\times}543 = 3801$, leaving 325800. And finally at the third digit with quotient digit 6 ($8{\times}7 \bmod 10$), subtracting $6{\times}543 = 3258$ leaving 0. So the quotient is 678.

Notice however that the multiplies and subtractions don't need to extend past the low three digits of the dividend, since that's enough to determine the three quotient digits. For the last quotient digit no subtraction is needed at all. On a 2N×N division like this one, only about half the work of a normal basecase division is necessary.

For an N×M exact division producing Q=N−M quotient limbs, the saving over a normal basecase division is in two parts. Firstly, each of the Q quotient limbs needs only one multiply, not a 2×1 divide and multiply. Secondly, the crossproducts are reduced when $Q > M$ to $QM - M(M+1)/2$, or when $Q \le M$ to $Q(Q-1)/2$. Notice the savings are complementary. If Q is big then many divisions are saved, or if Q is small then the crossproducts reduce to a small number.

The modular inverse used is calculated efficiently by `modlimb_invert` in 'gmp-impl.h'. This does four multiplies for a 32-bit limb, or six for a 64-bit limb. 'tune/modlinv.c' has some alternate implementations that might suit processors better at bit twiddling than multiplying.

The sub-quadratic exact division described by Jebelean in "Exact Division with Karatsuba Complexity" is not currently implemented. It uses a rearrangement similar to the divide and conquer for normal division (see Section 16.2.3 [Divide and Conquer Division], page 93), but operating from low to high. A further possibility not currently implemented is "Bidirectional Exact Integer Division" by Krandick and Jebelean which forms quotient limbs from both the high and low ends of the dividend, and can halve once more the number of crossproducts needed in a 2N×N division.

A special case exact division by 3 exists in `mpn_divexact_by3`, supporting Toom-3 multiplication and `mpq` canonicalizations. It forms quotient digits with a multiply by the modular inverse of 3 (which is `0xAA..AAB`) and uses two comparisons to determine a borrow for the next limb. The multiplications don't need to be on the dependent chain, as long as the effect of the borrows is applied. Only a few optimized assembler implementations currently exist.

16.2.5 Exact Remainder

If the exact division algorithm is done with a full subtraction at each stage and the dividend isn't a multiple of the divisor, then low zero limbs are produced but with a remainder in the high limbs. For dividend a, divisor d, quotient q, and $b = 2^{\texttt{mp\_bits\_per\_limb}}$, then this remainder r is of the form

$$a = qd + rb^n$$

n represents the number of zero limbs produced by the subtractions, that being the number of limbs produced for q. r will be in the range $0 \le r < d$ and can be viewed as a remainder, but one shifted up by a factor of b^n.

Carrying out full subtractions at each stage means the same number of cross products must be done as a normal division, but there's still some single limb divisions saved. When d is a single limb some simplifications arise, providing good speedups on a number of processors.

`mpn_bdivmod`, `mpn_divexact_by3`, `mpn_modexact_1_odd` and the `redc` function in `mpz_powm` differ subtly in how they return r, leading to some negations in the above formula, but all are essentially the same.

Clearly r is zero when a is a multiple of d, and this leads to divisibility or congruence tests which are potentially more efficient than a normal division.

The factor of b^n on r can be ignored in a GCD when d is odd, hence the use of `mpn_bdivmod` in `mpn_gcd`, and the use of `mpn_modexact_1_odd` by `mpn_gcd_1` and `mpz_kronecker_ui` etc (see Section 16.3 [Greatest Common Divisor Algorithms], page 95).

Montgomery's REDC method for modular multiplications uses operands of the form of xb^{-n} and yb^{-n} and on calculating $(xb^{-n})(yb^{-n})$ uses the factor of b^n in the exact remainder to reach a product in the same form $(xy)b^{-n}$ (see Section 16.4.2 [Modular Powering Algorithm], page 97).

Notice that r generally gives no useful information about the ordinary remainder $a \bmod d$ since $b^n \bmod d$ could be anything. If however $b^n \equiv 1 \bmod d$, then r is the negative of the ordinary remainder. This occurs whenever d is a factor of $b^n - 1$, as for example with 3 in `mpn_divexact_by3`. Other such factors include 5, 17 and 257, but no particular use has been found for this.

16.2.6 Small Quotient Division

An N×M division where the number of quotient limbs Q=N−M is small can be optimized somewhat.

An ordinary basecase division normalizes the divisor by shifting it to make the high bit set, shifting the dividend accordingly, and shifting the remainder back down at the end of the calculation. This is wasteful if only a few quotient limbs are to be formed. Instead a division of just the top 2Q limbs of the dividend by the top Q limbs of the divisor can be used to form a trial quotient. This requires only those limbs normalized, not the whole of the divisor and dividend.

A multiply and subtract then applies the trial quotient to the M−Q unused limbs of the divisor and N−Q dividend limbs (which includes Q limbs remaining from the trial quotient division). The starting trial quotient can be 1 or 2 too big, but all cases of 2 too big and most cases of 1 too big are detected by first comparing the most significant limbs that will arise from the subtraction. An addback is done if the quotient still turns out to be 1 too big.

This whole procedure is essentially the same as one step of the basecase algorithm done in a Q limb base, though with the trial quotient test done only with the high limbs, not an entire Q limb "digit" product. The correctness of this weaker test can be established by following the argument of Knuth section 4.3.1 exercise 20 but with the $v_2\hat{q} > b\hat{r} + u_2$ condition appropriately relaxed.

16.3 Greatest Common Divisor

16.3.1 Binary GCD

At small sizes GMP uses an $O(N^2)$ binary style GCD. This is described in many textbooks, for example Knuth section 4.5.2 algorithm B. It simply consists of successively reducing operands a and b using $\gcd(a,b) = \gcd(\min(a,b), \operatorname{abs}(a-b))$, and also that if a and b are first made odd then $\operatorname{abs}(a-b)$ is even and factors of two can be discarded.

Variants like letting $a-b$ become negative and doing a different next step are of interest only as far as they suit particular CPUs, since on small operands it's machine dependent factors that determine performance.

The Euclidean GCD algorithm, as per Knuth algorithms E and A, reduces using $a \bmod b$ but this has so far been found to be slower everywhere. One reason the binary method does well is that the implied quotient at each step is usually small, so often only one or two subtractions are needed to get the same effect as a division. Quotients 1, 2 and 3 for example occur 67.7% of the time, see Knuth section 4.5.3 Theorem E.

When the implied quotient is large, meaning b is much smaller than a, then a division is worthwhile. This is the basis for the initial $a \bmod b$ reductions in `mpn_gcd` and `mpn_gcd_1` (the latter for both N×1 and 1×1 cases). But after that initial reduction, big quotients occur too rarely to make it worth checking for them.

16.3.2 Accelerated GCD

For sizes above `GCD_ACCEL_THRESHOLD`, GMP uses the Accelerated GCD algorithm described independently by Weber and Jebelean (the latter as the "Generalized Binary" algorithm), see Appendix B [References], page 113. This algorithm is still $O(N^2)$, but is much faster than the binary algorithm since it does fewer multi-precision operations. It consists of alternating the k-ary reduction by Sorenson, and a "dmod" exact remainder reduction.

For operands u and v the k-ary reduction replaces u with $nv - du$ where n and d are single limb values chosen to give two trailing zero limbs on that value, which can be stripped. n and d are calculated using an algorithm similar to half of a two limb GCD (see `find_a` in '`mpn/generic/gcd.c`').

When u and v differ in size by more than a certain number of bits, a dmod is performed to zero out bits at the low end of the larger. It consists of an exact remainder style division applied to an appropriate number of bits (see Section 16.2.4 [Exact Division], page 93, and see Section 16.2.5 [Exact Remainder], page 94). This is faster than a k-ary reduction but useful only when the operands differ in size. There's a dmod after each k-ary reduction, and if the dmod leaves the operands still differing in size then it's repeated.

The k-ary reduction step can introduce spurious factors into the GCD calculated, and these are eliminated at the end by taking GCDs with the original inputs $\gcd(u, \gcd(v, g))$ using the binary algorithm. Since g is almost always small this takes very little time.

At small sizes the algorithm needs a good implementation of `find_a`. At larger sizes it's dominated by `mpn_addmul_1` applying n and d.

16.3.3 Extended GCD

The extended GCD calculates $\gcd(a, b)$ and also cofactors x and y satisfying $ax+by = \gcd(a,b)$. Lehmer's multi-step improvement of the extended Euclidean algorithm is used. See Knuth section 4.5.2 algorithm L, and '`mpn/generic/gcdext.c`'. This is an $O(N^2)$ algorithm.

The multipliers at each step are found using single limb calculations for sizes up to `GCDEXT_THRESHOLD`, or double limb calculations above that. The single limb code is faster but doesn't produce full-limb multipliers, hence not making full use of the `mpn_addmul_1` calls.

When a CPU has a data-dependent multiplier, meaning one which is faster on operands with fewer bits, the extra work in the double-limb calculation might only save some looping overheads, leading to a large `GCDEXT_THRESHOLD`.

Currently the single limb calculation doesn't optimize for the small quotients that often occur, and this can lead to unusually low values of `GCDEXT_THRESHOLD`, depending on the CPU.

An analysis of double-limb calculations can be found in "A Double-Digit Lehmer-Euclid Algorithm" by Jebelean (see Appendix B [References], page 113). The code in GMP was developed independently.

It should be noted that when a double limb calculation is used, it's used for the whole of that GCD, it doesn't fall back to single limb part way through. This is because as the algorithm proceeds, the inputs a and b are reduced, but the cofactors x and y grow, so the multipliers at each step are applied to a roughly constant total number of limbs.

16.3.4 Jacobi Symbol

`mpz_jacobi` and `mpz_kronecker` are currently implemented with a simple binary algorithm similar to that described for the GCDs (see Section 16.3.1 [Binary GCD], page 95). They're not very fast when both inputs are large. Lehmer's multi-step improvement or a binary based multi-step algorithm is likely to be better.

When one operand fits a single limb, and that includes `mpz_kronecker_ui` and friends, an initial reduction is done with either `mpn_mod_1` or `mpn_modexact_1_odd`, followed by the binary algorithm on a single limb. The binary algorithm is well suited to a single limb, and the whole calculation in this case is quite efficient.

In all the routines sign changes for the result are accumulated using some bit twiddling, avoiding table lookups or conditional jumps.

16.4 Powering Algorithms

16.4.1 Normal Powering

Normal `mpz` or `mpf` powering uses a simple binary algorithm, successively squaring and then multiplying by the base when a 1 bit is seen in the exponent, as per Knuth section 4.6.3. The "left to right" variant described there is used rather than algorithm A, since it's just as easy and can be done with somewhat less temporary memory.

16.4.2 Modular Powering

Modular powering is implemented using a 2^k-ary sliding window algorithm, as per "Handbook of Applied Cryptography" algorithm 14.85 (see Appendix B [References], page 113). k is chosen according to the size of the exponent. Larger exponents use larger values of k, the choice being made to minimize the average number of multiplications that must supplement the squaring.

The modular multiplies and squares use either a simple division or the REDC method by Montgomery (see Appendix B [References], page 113). REDC is a little faster, essentially saving N single limb divisions in a fashion similar to an exact remainder (see Section 16.2.5 [Exact Remainder], page 94). The current REDC has some limitations. It's only $O(N^2)$ so above `POWM_THRESHOLD` division becomes faster and is used. It doesn't attempt to detect small bases, but rather always uses a REDC form, which is usually a full size operand. And lastly it's only applied to odd moduli.

16.5 Root Extraction Algorithms

16.5.1 Square Root

Square roots are taken using the "Karatsuba Square Root" algorithm by Paul Zimmermann (see Appendix B [References], page 113). This is expressed in a divide and conquer form, but as noted in the paper it can also be viewed as a discrete variant of Newton's method.

In the Karatsuba multiplication range this is an $O(\frac{3}{2}M(N/2))$ algorithm, where $M(n)$ is the time to multiply two numbers of n limbs. In the FFT multiplication range this grows to a bound of $O(6M(N/2))$. In practice a factor of about 1.5 to 1.8 is found in the Karatsuba and Toom-3 ranges, growing to 2 or 3 in the FFT range.

The algorithm does all its calculations in integers and the resulting `mpn_sqrtrem` is used for both `mpz_sqrt` and `mpf_sqrt`. The extended precision given by `mpf_sqrt_ui` is obtained by padding with zero limbs.

16.5.2 Nth Root

Integer Nth roots are taken using Newton's method with the following iteration, where A is the input and n is the root to be taken.

$$a_{i+1} = \frac{1}{n}\left(\frac{A}{a_i^{n-1}} + (n-1)a_i\right)$$

The initial approximation a_1 is generated bitwise by successively powering a trial root with or without new 1 bits, aiming to be just above the true root. The iteration converges quadratically when started from a good approximation. When n is large more initial bits are needed to get good convergence. The current implementation is not particularly well optimized.

16.5.3 Perfect Square

`mpz_perfect_square_p` is able to quickly exclude most non-squares by checking whether the input is a quadratic residue modulo some small integers.

The first test is modulo 256 which means simply examining the least significant byte. Only 44 different values occur as the low byte of a square, so 82.8% of non-squares can be immediately excluded. Similar tests modulo primes from 3 to 29 exclude 99.5% of those remaining, or if a limb is 64 bits then primes up to 53 are used, excluding 99.99%. A single N×1 remainder using `PP` from '`gmp-impl.h`' quickly gives all these remainders.

A square root must still be taken for any value that passes the residue tests, to verify it's really a square and not one of the 0.086% (or 0.000156% for 64 bits) non-squares that get through. See Section 16.5.1 [Square Root Algorithm], page 97.

16.5.4 Perfect Power

Detecting perfect powers is required by some factorization algorithms. Currently `mpz_perfect_power_p` is implemented using repeated Nth root extractions, though naturally only prime roots need to be considered. (See Section 16.5.2 [Nth Root Algorithm], page 98.)

If a prime divisor p with multiplicity e can be found, then only roots which are divisors of e need to be considered, much reducing the work necessary. To this end divisibility by a set of small primes is checked.

16.6 Radix Conversion

Radix conversions are less important than other algorithms. A program dominated by conversions should probably use a different data representation.

16.6.1 Binary to Radix

Conversions from binary to a power-of-2 radix use a simple and fast $O(N)$ bit extraction algorithm.

Conversions from binary to other radices use one of two algorithms. Sizes below `GET_STR_PRECOMPUTE_THRESHOLD` use a basic $O(N^2)$ method. Repeated divisions by b^n are made, where b is the radix and n is the biggest power that fits in a limb. But instead of simply using the remainder r from such divisions, an extra divide step is done to give a fractional limb representing

r/b^n. The digits of r can then be extracted using multiplications by b rather than divisions. Special case code is provided for decimal, allowing multiplications by 10 to optimize to shifts and adds.

Above GET_STR_PRECOMPUTE_THRESHOLD a sub-quadratic algorithm is used. For an input t, powers b^{n2^i} of the radix are calculated, until a power between t and $\sqrt{t}$ is reached. t is then divided by that largest power, giving a quotient which is the digits above that power, and a remainder which is those below. These two parts are in turn divided by the second highest power, and so on recursively. When a piece has been divided down to less than GET_STR_DC_THRESHOLD limbs, the basecase algorithm described above is used.

The advantage of this algorithm is that big divisions can make use of the sub-quadratic divide and conquer division (see Section 16.2.3 [Divide and Conquer Division], page 93), and big divisions tend to have less overheads than lots of separate single limb divisions anyway. But in any case the cost of calculating the powers b^{n2^i} must first be overcome.

GET_STR_PRECOMPUTE_THRESHOLD and GET_STR_DC_THRESHOLD represent the same basic thing, the point where it becomes worth doing a big division to cut the input in half. GET_STR_PRECOMPUTE_THRESHOLD includes the cost of calculating the radix power required, whereas GET_STR_DC_THRESHOLD assumes that's already available, which is the case when recursing.

Since the base case produces digits from least to most significant but they want to be stored from most to least, it's necessary to calculate in advance how many digits there will be, or at least be sure not to underestimate that. For GMP the number of input bits is multiplied by chars_per_bit_exactly from mp_bases, rounding up. The result is either correct or one too big.

Examining some of the high bits of the input could increase the chance of getting the exact number of digits, but an exact result every time would not be practical, since in general the difference between numbers 100. . . and 99. . . is only in the last few bits and the work to identify 99. . . might well be almost as much as a full conversion.

mpf_get_str doesn't currently use the algorithm described here, it multiplies or divides by a power of b to move the radix point to the just above the highest non-zero digit (or at worst one above that location), then multiplies by b^n to bring out digits. This is $O(N^2)$ and is certainly not optimal.

The r/b^n scheme described above for using multiplications to bring out digits might be useful for more than a single limb. Some brief experiments with it on the base case when recursing didn't give a noticable improvement, but perhaps that was only due to the implementation. Something similar would work for the sub-quadratic divisions too, though there would be the cost of calculating a bigger radix power.

Another possible improvement for the sub-quadratic part would be to arrange for radix powers that balanced the sizes of quotient and remainder produced, ie. the highest power would be an b^{nk} approximately equal to $\sqrt{t}$, not restricted to a 2^i factor. That ought to smooth out a graph of times against sizes, but may or may not be a net speedup.

16.6.2 Radix to Binary

Conversions from a power-of-2 radix into binary use a simple and fast $O(N)$ bitwise concatenation algorithm.

Conversions from other radices use one of two algorithms. Sizes below SET_STR_THRESHOLD use a basic $O(N^2)$ method. Groups of n digits are converted to limbs, where n is the biggest power of the base b which will fit in a limb, then those groups are accumulated into the result by multiplying by b^n and adding. This saves multi-precision operations, as per Knuth section 4.4

part E (see Appendix B [References], page 113). Some special case code is provided for decimal, giving the compiler a chance to optimize multiplications by 10.

Above `SET_STR_THRESHOLD` a sub-quadratic algorithm is used. First groups of n digits are converted into limbs. Then adjacent limbs are combined into limb pairs with $xb^n + y$, where x and y are the limbs. Adjacent limb pairs are combined into quads similarly with $xb^{2n} + y$. This continues until a single block remains, that being the result.

The advantage of this method is that the multiplications for each x are big blocks, allowing Karatsuba and higher algorithms to be used. But the cost of calculating the powers b^{n2^i} must be overcome. `SET_STR_THRESHOLD` usually ends up quite big, around 5000 digits, and on some processors much bigger still.

`SET_STR_THRESHOLD` is based on the input digits (and tuned for decimal), though it might be better based on a limb count, so as to be independent of the base. But that sort of count isn't used by the base case and so would need some sort of initial calculation or estimate.

The main reason `SET_STR_THRESHOLD` is so much bigger than the corresponding `GET_STR_PRECOMPUTE_THRESHOLD` is that `mpn_mul_1` is much faster than `mpn_divrem_1` (often by a factor of 10, or more).

16.7 Other Algorithms

16.7.1 Factorial

Factorials $n!$ are calculated by a simple product from 1 to n, but arranged into certain sub-products.

First as many factors as fit in a limb are accumulated, then two of those multiplied to give a 2-limb product. When two 2-limb products are ready they're multiplied to a 4-limb product, and when two 4-limbs are ready they're multiplied to an 8-limb product, etc. A stack of outstanding products is built up, with two of the same size multiplied together when ready.

Arranging for multiplications to have operands the same (or nearly the same) size means the Karatsuba and higher multiplication algorithms can be used. And even on sizes below the Karatsuba threshold an N×N multiply will give a basecase multiply more to work on.

An obvious improvement not currently implemented would be to strip factors of 2 from the products and apply them at the end with a bit shift. Another possibility would be to determine the prime factorization of the result (which can be done easily), and use a powering method, at each stage squaring then multiplying in those primes with a 1 in their exponent at that point. The advantage would be some multiplies turned into squares.

16.7.2 Binomial Coefficients

Binomial coefficients $\binom{n}{k}$ are calculated by first arranging $k \le n/2$ using $\binom{n}{k} = \binom{n}{n-k}$ if necessary, and then evaluating the following product simply from $i = 2$ to $i = k$.

$$\binom{n}{k} = (n-k+1)\prod_{i=2}^{k}\frac{n-k+i}{i}$$

It's easy to show that each denominator i will divide the product so far, so the exact division algorithm is used (see Section 16.2.4 [Exact Division], page 93).

The numerators $n-k+i$ and denominators i are first accumulated into as many fit a limb, to save multi-precision operations, though for `mpz_bin_ui` this applies only to the divisors, since n is an `mpz_t` and $n-k+i$ in general won't fit in a limb at all.

An obvious improvement would be to strip factors of 2 from each multiplier and divisor and count them separately, to be applied with a bit shift at the end. Factors of 3 and perhaps 5 could even be handled similarly. Another possibility, if n is not too big, would be to determine the prime factorization of the result based on the factorials involved, and power up those primes appropriately. This would help most when k is near $n/2$.

16.7.3 Fibonacci Numbers

The Fibonacci functions `mpz_fib_ui` and `mpz_fib2_ui` are designed for calculating isolated F_n or F_n,F_{n-1} values efficiently.

For small n, a table of single limb values in `__gmp_fib_table` is used. On a 32-bit limb this goes up to F_{47}, or on a 64-bit limb up to F_{93}. For convenience the table starts at F_{-1}.

Beyond the table, values are generated with a binary powering algorithm, calculating a pair F_n and F_{n-1} working from high to low across the bits of n. The formulas used are

$$\begin{aligned} F_{2k+1} &= 4F_k^2 - F_{k-1}^2 + 2(-1)^k \\ F_{2k-1} &= F_k^2 + F_{k-1}^2 \\ F_{2k} &= F_{2k+1} - F_{2k-1} \end{aligned}$$

At each step, k is the high b bits of n. If the next bit of n is 0 then F_{2k},F_{2k-1} is used, or if it's a 1 then F_{2k+1},F_{2k} is used, and the process repeated until all bits of n are incorporated. Notice these formulas require just two squares per bit of n.

It'd be possible to handle the first few n above the single limb table with simple additions, using the defining Fibonacci recurrence $F_{k+1} = F_k + F_{k-1}$, but this is not done since it usually turns out to be faster for only about 10 or 20 values of n, and including a block of code for just those doesn't seem worthwhile. If they really mattered it'd be better to extend the data table.

Using a table avoids lots of calculations on small numbers, and makes small n go fast. A bigger table would make more small n go fast, it's just a question of balancing size against desired speed. For GMP the code is kept compact, with the emphasis primarily on a good powering algorithm.

`mpz_fib2_ui` returns both F_n and F_{n-1}, but `mpz_fib_ui` is only interested in F_n. In this case the last step of the algorithm can become one multiply instead of two squares. One of the following two formulas is used, according as n is odd or even.

$$\begin{aligned} F_{2k} &= F_k(F_k + 2F_{k-1}) \\ F_{2k+1} &= (2F_k + F_{k-1})(2F_k - F_{k-1}) + 2(-1)^k \end{aligned}$$

F_{2k+1} here is the same as above, just rearranged to be a multiply. For interest, the $2(-1)^k$ term both here and above can be applied just to the low limb of the calculation, without a carry or borrow into further limbs, which saves some code size. See comments with `mpz_fib_ui` and the internal `mpn_fib2_ui` for how this is done.

16.7.4 Lucas Numbers

`mpz_lucnum2_ui` derives a pair of Lucas numbers from a pair of Fibonacci numbers with the following simple formulas.

$$\begin{aligned} L_k &= F_k + 2F_{k-1} \\ L_{k-1} &= 2F_k - F_{k-1} \end{aligned}$$

`mpz_lucnum_ui` is only interested in L_n, and some work can be saved. Trailing zero bits on n can be handled with a single square each.

$$L_{2k} = L_k^2 - 2(-1)^k$$

And the lowest 1 bit can be handled with one multiply of a pair of Fibonacci numbers, similar to what mpz_fib_ui does.

$$L_{2k+1} = 5F_{k-1}(2F_k + F_{k-1}) - 4(-1)^k$$

16.8 Assembler Coding

The assembler subroutines in GMP are the most significant source of speed at small to moderate sizes. At larger sizes algorithm selection becomes more important, but of course speedups in low level routines will still speed up everything proportionally.

Carry handling and widening multiplies that are important for GMP can't be easily expressed in C. GCC asm blocks help a lot and are provided in 'longlong.h', but hand coding low level routines invariably offers a speedup over generic C by a factor of anything from 2 to 10.

16.8.1 Code Organisation

The various 'mpn' subdirectories contain machine-dependent code, written in C or assembler. The 'mpn/generic' subdirectory contains default code, used when there's no machine-specific version of a particular file.

Each 'mpn' subdirectory is for an ISA family. Generally 32-bit and 64-bit variants in a family cannot share code and will have separate directories. Within a family further subdirectories may exist for CPU variants.

16.8.2 Assembler Basics

mpn_addmul_1 and mpn_submul_1 are the most important routines for overall GMP performance. All multiplications and divisions come down to repeated calls to these. mpn_add_n, mpn_sub_n, mpn_lshift and mpn_rshift are next most important.

On some CPUs assembler versions of the internal functions mpn_mul_basecase and mpn_sqr_basecase give significant speedups, mainly through avoiding function call overheads. They can also potentially make better use of a wide superscalar processor.

The restrictions on overlaps between sources and destinations (see Chapter 8 [Low-level Functions], page 53) are designed to facilitate a variety of implementations. For example, knowing mpn_add_n won't have partly overlapping sources and destination means reading can be done far ahead of writing on superscalar processors, and loops can be vectorized on a vector processor, depending on the carry handling.

16.8.3 Carry Propagation

The problem that presents most challenges in GMP is propagating carries from one limb to the next. In functions like mpn_addmul_1 and mpn_add_n, carries are the only dependencies between limb operations.

On processors with carry flags, a straightforward CISC style adc is generally best. AMD K6 mpn_addmul_1 however is an example of an unusual set of circumstances where a branch works out better.

On RISC processors generally an add and compare for overflow is used. This sort of thing can be seen in 'mpn/generic/aors_n.c'. Some carry propagation schemes require 4 instructions, meaning at least 4 cycles per limb, but other schemes may use just 1 or 2. On wide superscalar processors performance may be completely determined by the number of dependent instructions between carry-in and carry-out for each limb.

On vector processors good use can be made of the fact that a carry bit only very rarely propagates more than one limb. When adding a single bit to a limb, there's only a carry out if that limb was

0xFF...FF which on random data will be only 1 in $2^{\texttt{mp\_bits\_per\_limb}}$. 'mpn/cray/add_n.c' is an example of this, it adds all limbs in parallel, adds one set of carry bits in parallel and then only rarely needs to fall through to a loop propagating further carries.

On the x86s, GCC (as of version 2.95.2) doesn't generate particularly good code for the RISC style idioms that are necessary to handle carry bits in C. Often conditional jumps are generated where adc or sbb forms would be better. And so unfortunately almost any loop involving carry bits needs to be coded in assembler for best results.

16.8.4 Cache Handling

GMP aims to perform well both on operands that fit entirely in L1 cache and those which don't.

Basic routines like mpn_add_n or mpn_lshift are often used on large operands, so L2 and main memory performance is important for them. mpn_mul_1 and mpn_addmul_1 are mostly used for multiply and square basecases, so L1 performance matters most for them, unless assembler versions of mpn_mul_basecase and mpn_sqr_basecase exist, in which case the remaining uses are mostly for larger operands.

For L2 or main memory operands, memory access times will almost certainly be more than the calculation time. The aim therefore is to maximize memory throughput, by starting a load of the next cache line which processing the contents of the previous one. Clearly this is only possible if the chip has a lock-up free cache or some sort of prefetch instruction. Most current chips have both these features.

Prefetching sources combines well with loop unrolling, since a prefetch can be initiated once per unrolled loop (or more than once if the loop covers more than one cache line).

On CPUs without write-allocate caches, prefetching destinations will ensure individual stores don't go further down the cache hierarchy, limiting bandwidth. Of course for calculations which are slow anyway, like mpn_divrem_1, write-throughs might be fine.

The distance ahead to prefetch will be determined by memory latency versus throughput. The aim of course is to have data arriving continuously, at peak throughput. Some CPUs have limits on the number of fetches or prefetches in progress.

If a special prefetch instruction doesn't exist then a plain load can be used, but in that case care must be taken not to attempt to read past the end of an operand, since that might produce a segmentation violation.

Some CPUs or systems have hardware that detects sequential memory accesses and initiates suitable cache movements automatically, making life easy.

16.8.5 Floating Point

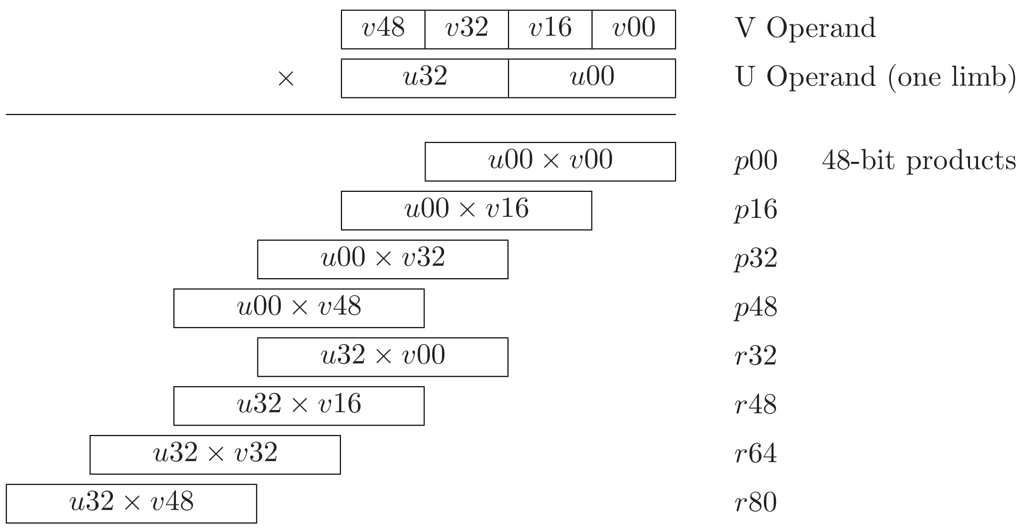
Floating point arithmetic is used in GMP for multiplications on CPUs with poor integer multipliers. It's mostly useful for mpn_mul_1, mpn_addmul_1 and mpn_submul_1 on 64-bit machines, and mpn_mul_basecase on both 32-bit and 64-bit machines.

With IEEE 53-bit double precision floats, integer multiplications producing up to 53 bits will give exact results. Breaking a 64×64 multiplication into eight $16\times32 \rightarrow 48$ bit pieces is convenient. With some care though six $21\times32 \rightarrow 53$ bit products can be used, if one of the lower two 21-bit pieces also uses the sign bit.

For the mpn_mul_1 family of functions on a 64-bit machine, the invariant single limb is split at the start, into 3 or 4 pieces. Inside the loop, the bignum operand is split into 32-bit pieces. Fast conversion of these unsigned 32-bit pieces to floating point is highly machine-dependent. In some cases, reading the data into the integer unit, zero-extending to 64-bits, then transferring to the floating point unit back via memory is the only option.

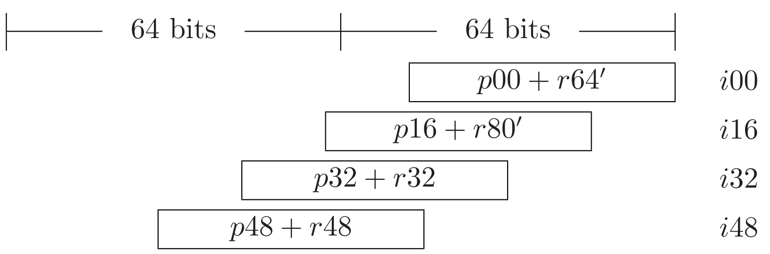
Converting partial products back to 64-bit limbs is usually best done as a signed conversion. Since all values are smaller than 2^{53}, signed and unsigned are the same, but most processors lack unsigned conversions.

Here is a diagram showing 16×32 bit products for an `mpn_mul_1` or `mpn_addmul_1` with a 64-bit limb. The single limb operand V is split into four 16-bit parts. The multi-limb operand U is split in the loop into two 32-bit parts.



$p32$ and $r32$ can be summed using floating-point addition, and likewise $p48$ and $r48$. $p00$ and $p16$ can be summed with $r64$ and $r80$ from the previous iteration.

For each loop then, four 49-bit quantities are transfered to the integer unit, aligned as follows,



The challenge then is to sum these efficiently and add in a carry limb, generating a low 64-bit result limb and a high 33-bit carry limb ($i48$ extends 33 bits into the high half).

16.8.6 SIMD Instructions

The single-instruction multiple-data support in current microprocessors is aimed at signal processing algorithms where each data point can be treated more or less independently. There's generally not much support for propagating the sort of carries that arise in GMP.

SIMD multiplications of say four 16×16 bit multiplies only do as much work as one 32×32 from GMP's point of view, and need some shifts and adds besides. But of course if say the SIMD form is fully pipelined and uses less instruction decoding then it may still be worthwhile.

On the 80x86 chips, MMX has so far found a use in `mpn_rshift` and `mpn_lshift` since it allows 64-bit operations, and is used in a special case for 16-bit multipliers in the P55 `mpn_mul_1`. 3DNow and SSE haven't found a use so far.

16.8.7 Software Pipelining

Software pipelining consists of scheduling instructions around the branch point in a loop. For example a loop taking a checksum of an array of limbs might have a load and an add, but the load wouldn't be for that add, rather for the one next time around the loop. Each load then is effectively scheduled back in the previous iteration, allowing latency to be hidden.

Naturally this is wanted only when doing things like loads or multiplies that take a few cycles to complete, and only where a CPU has multiple functional units so that other work can be done while waiting.

A pipeline with several stages will have a data value in progress at each stage and each loop iteration moves them along one stage. This is like juggling.

Within the loop some moves between registers may be necessary to have the right values in the right places for each iteration. Loop unrolling can help this, with each unrolled block able to use different registers for different values, even if some shuffling is still needed just before going back to the top of the loop.

16.8.8 Loop Unrolling

Loop unrolling consists of replicating code so that several limbs are processed in each loop. At a minimum this reduces loop overheads by a corresponding factor, but it can also allow better register usage, for example alternately using one register combination and then another. Judicious use of `m4` macros can help avoid lots of duplication in the source code.

Unrolling is commonly done to a power of 2 multiple so the number of unrolled loops and the number of remaining limbs can be calculated with a shift and mask. But other multiples can be used too, just by subtracting each n limbs processed from a counter and waiting for less than n remaining (or offsetting the counter by n so it goes negative when there's less than n remaining).

The limbs not a multiple of the unrolling can be handled in various ways, for example

- A simple loop at the end (or the start) to process the excess. Care will be wanted that it isn't too much slower than the unrolled part.
- A set of binary tests, for example after an 8-limb unrolling, test for 4 more limbs to process, then a further 2 more or not, and finally 1 more or not. This will probably take more code space than a simple loop.
- A `switch` statement, providing separate code for each possible excess, for example an 8-limb unrolling would have separate code for 0 remaining, 1 remaining, etc, up to 7 remaining. This might take a lot of code, but may be the best way to optimize all cases in combination with a deep pipelined loop.
- A computed jump into the middle of the loop, thus making the first iteration handle the excess. This should make times smoothly increase with size, which is attractive, but setups for the jump and adjustments for pointers can be tricky and could become quite difficult in combination with deep pipelining.

One way to write the setups and finishups for a pipelined unrolled loop is simply to duplicate the loop at the start and the end, then delete instructions at the start which have no valid antecedents, and delete instructions at the end whose results are unwanted. Sizes not a multiple of the unrolling can then be handled as desired.

17 Internals

This chapter is provided only for informational purposes and the various internals described here may change in future GMP releases. Applications expecting to be compatible with future releases should use only the documented interfaces described in previous chapters.

17.1 Integer Internals

`mpz_t` variables represent integers using sign and magnitude, in space dynamically allocated and reallocated. The fields are as follows.

`_mp_size` The number of limbs, or the negative of that when representing a negative integer. Zero is represented by `_mp_size` set to zero, in which case the `_mp_d` data is unused.

`_mp_d` A pointer to an array of limbs which is the magnitude. These are stored "little endian" as per the `mpn` functions, so `_mp_d[0]` is the least significant limb and `_mp_d[ABS(_mp_size)-1]` is the most significant. Whenever `_mp_size` is non-zero, the most significant limb is non-zero.

Currently there's always at least one limb allocated, so for instance `mpz_set_ui` never needs to reallocate, and `mpz_get_ui` can fetch `_mp_d[0]` unconditionally (though its value is then only wanted if `_mp_size` is non-zero).

`_mp_alloc`

`_mp_alloc` is the number of limbs currently allocated at `_mp_d`, and naturally `_mp_alloc >= ABS(_mp_size)`. When an `mpz` routine is about to (or might be about to) increase `_mp_size`, it checks `_mp_alloc` to see whether there's enough space, and reallocates if not. `MPZ_REALLOC` is generally used for this.

The various bitwise logical functions like `mpz_and` behave as if negative values were twos complement. But sign and magnitude is always used internally, and necessary adjustments are made during the calculations. Sometimes this isn't pretty, but sign and magnitude are best for other routines.

Some internal temporary variables are setup with `MPZ_TMP_INIT` and these have `_mp_d` space obtained from `TMP_ALLOC` rather than the memory allocation functions. Care is taken to ensure that these are big enough that no reallocation is necessary (since it would have unpredictable consequences).

17.2 Rational Internals

`mpq_t` variables represent rationals using an `mpz_t` numerator and denominator (see Section 17.1 [Integer Internals], page 106).

The canonical form adopted is denominator positive (and non-zero), no common factors between numerator and denominator, and zero uniquely represented as 0/1.

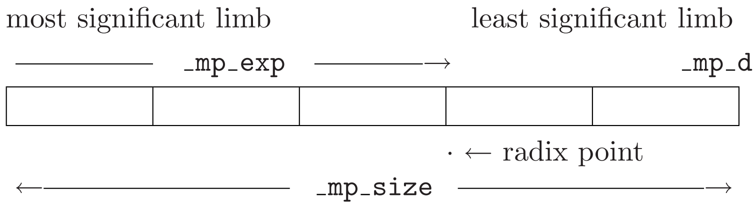
It's believed that casting out common factors at each stage of a calculation is best in general. A GCD is an $O(N^2)$ operation so it's better to do a few small ones immediately than to delay and have to do a big one later. Knowing the numerator and denominator have no common factors can be used for example in `mpq_mul` to make only two cross GCDs necessary, not four.

This general approach to common factors is badly sub-optimal in the presence of simple factorizations or little prospect for cancellation, but GMP has no way to know when this will occur. As per Section 3.11 [Efficiency], page 20, that's left to applications. The `mpq_t` framework might still suit, with `mpq_numref` and `mpq_denref` for direct access to the numerator and denominator, or of course `mpz_t` variables can be used directly.

17.3 Float Internals

Efficient calculation is the primary aim of GMP floats and the use of whole limbs and simple rounding facilitates this.

`mpf_t` floats have a variable precision mantissa and a single machine word signed exponent. The mantissa is represented using sign and magnitude.



The fields are as follows.

`_mp_size` The number of limbs currently in use, or the negative of that when representing a negative value. Zero is represented by `_mp_size` and `_mp_exp` both set to zero, and in that case the `_mp_d` data is unused. (In the future `_mp_exp` might be undefined when representing zero.)

`_mp_prec` The precision of the mantissa, in limbs. In any calculation the aim is to produce `_mp_prec` limbs of result (the most significant being non-zero).

`_mp_d` A pointer to the array of limbs which is the absolute value of the mantissa. These are stored "little endian" as per the `mpn` functions, so `_mp_d[0]` is the least significant limb and `_mp_d[ABS(_mp_size)-1]` the most significant.

The most significant limb is always non-zero, but there are no other restrictions on its value, in particular the highest 1 bit can be anywhere within the limb.

`_mp_prec+1` limbs are allocated to `_mp_d`, the extra limb being for convenience (see below). There are no reallocations during a calculation, only in a change of precision with `mpf_set_prec`.

`_mp_exp` The exponent, in limbs, determining the location of the implied radix point. Zero means the radix point is just above the most significant limb. Positive values mean a radix point offset towards the lower limbs and hence a value ≥ 1, as for example in the diagram above. Negative exponents mean a radix point further above the highest limb.

Naturally the exponent can be any value, it doesn't have to fall within the limbs as the diagram shows, it can be a long way above or a long way below. Limbs other than those included in the `{_mp_d,_mp_size}` data are treated as zero.

The following various points should be noted.

Low Zeros The least significant limbs `_mp_d[0]` etc can be zero, though such low zeros can always be ignored. Routines likely to produce low zeros check and avoid them to save time in subsequent calculations, but for most routines they're quite unlikely and aren't checked.

Mantissa Size Range

The `_mp_size` count of limbs in use can be less than `_mp_prec` if the value can be represented in less. This means low precision values or small integers stored in a high precision `mpf_t` can still be operated on efficiently.

`_mp_size` can also be greater than `_mp_prec`. Firstly a value is allowed to use all of the `_mp_prec+1` limbs available at `_mp_d`, and secondly when `mpf_set_prec_raw` lowers _mp_prec it leaves _mp_size unchanged and so the size can be arbitrarily bigger than _mp_prec.

Rounding
All rounding is done on limb boundaries. Calculating _mp_prec limbs with the high non-zero will ensure the application requested minimum precision is obtained.

The use of simple "trunc" rounding towards zero is efficient, since there's no need to examine extra limbs and increment or decrement.

Bit Shifts
Since the exponent is in limbs, there are no bit shifts in basic operations like mpf_add and mpf_mul. When differing exponents are encountered all that's needed is to adjust pointers to line up the relevant limbs.

Of course mpf_mul_2exp and mpf_div_2exp will require bit shifts, but the choice is between an exponent in limbs which requires shifts there, or one in bits which requires them almost everywhere else.

Use of _mp_prec+1 Limbs
The extra limb on _mp_d (_mp_prec+1 rather than just _mp_prec) helps when an mpf routine might get a carry from its operation. mpf_add for instance will do an mpn_add of _mp_prec limbs. If there's no carry then that's the result, but if there is a carry then it's stored in the extra limb of space and _mp_size becomes _mp_prec+1.

Whenever _mp_prec+1 limbs are held in a variable, the low limb is not needed for the intended precision, only the _mp_prec high limbs. But zeroing it out or moving the rest down is unnecessary. Subsequent routines reading the value will simply take the high limbs they need, and this will be _mp_prec if their target has that same precision. This is no more than a pointer adjustment, and must be checked anyway since the destination precision can be different from the sources.

Copy functions like mpf_set will retain a full _mp_prec+1 limbs if available. This ensures that a variable which has _mp_size equal to _mp_prec+1 will get its full exact value copied. Strictly speaking this is unnecessary since only _mp_prec limbs are needed for the application's requested precision, but it's considered that an mpf_set from one variable into another of the same precision ought to produce an exact copy.

Application Precisions
__GMPF_BITS_TO_PREC converts an application requested precision to an _mp_prec. The value in bits is rounded up to a whole limb then an extra limb is added since the most significant limb of _mp_d is only non-zero and therefore might contain only one bit.

__GMPF_PREC_TO_BITS does the reverse conversion, and removes the extra limb from _mp_prec before converting to bits. The net effect of reading back with mpf_get_prec is simply the precision rounded up to a multiple of mp_bits_per_limb.

Note that the extra limb added here for the high only being non-zero is in addition to the extra limb allocated to _mp_d. For example with a 32-bit limb, an application request for 250 bits will be rounded up to 8 limbs, then an extra added for the high being only non-zero, giving an _mp_prec of 9. _mp_d then gets 10 limbs allocated. Reading back with mpf_get_prec will take _mp_prec subtract 1 limb and multiply by 32, giving 256 bits.

Strictly speaking, the fact the high limb has at least one bit means that a float with, say, 3 limbs of 32-bits each will be holding at least 65 bits, but for the purposes of mpf_t it's considered simply to be 64 bits, a nice multiple of the limb size.

17.4 Raw Output Internals

mpz_out_raw uses the following format.

size	data bytes

The size is 4 bytes written most significant byte first, being the number of subsequent data bytes, or the twos complement negative of that when a negative integer is represented. The data bytes are the absolute value of the integer, written most significant byte first.

The most significant data byte is always non-zero, so the output is the same on all systems, irrespective of limb size.

In GMP 1, leading zero bytes were written to pad the data bytes to a multiple of the limb size. `mpz_inp_raw` will still accept this, for compatibility.

The use of "big endian" for both the size and data fields is deliberate, it makes the data easy to read in a hex dump of a file. Unfortunately it also means that the limb data must be reversed when reading or writing, so neither a big endian nor little endian system can just read and write `_mp_d`.

17.5 C++ Interface Internals

A system of expression templates is used to ensure something like `a=b+c` turns into a simple call to `mpz_add` etc. For `mpf_class` and `mpfr_class` the scheme also ensures the precision of the final destination is used for any temporaries within a statement like `f=w*x+y*z`. These are important features which a naive implementation cannot provide.

A simplified description of the scheme follows. The true scheme is complicated by the fact that expressions have different return types. For detailed information, refer to the source code.

To perform an operation, say, addition, we first define a "function object" evaluating it,

```
struct __gmp_binary_plus
{
  static void eval(mpf_t f, mpf_t g, mpf_t h) { mpf_add(f, g, h); }
};
```

And an "additive expression" object,

```
__gmp_expr<__gmp_binary_expr<mpf_class, mpf_class, __gmp_binary_plus> >
operator+(const mpf_class &f, const mpf_class &g)
{
  return __gmp_expr
    <__gmp_binary_expr<mpf_class, mpf_class, __gmp_binary_plus> >(f, g);
}
```

The seemingly redundant `__gmp_expr<__gmp_binary_expr<...>>` is used to encapsulate any possible kind of expression into a single template type. In fact even `mpf_class` etc are `typedef` specializations of `__gmp_expr`.

Next we define assignment of `__gmp_expr` to `mpf_class`.

```
template <class T>
mpf_class & mpf_class::operator=(const __gmp_expr<T> &expr)
{
  expr.eval(this->get_mpf_t(), this->precision());
  return *this;
}

template <class Op>
```

```
void __gmp_expr<__gmp_binary_expr<mpf_class, mpf_class, Op> >::eval
(mpf_t f, unsigned long int precision)
{
  Op::eval(f, expr.val1.get_mpf_t(), expr.val2.get_mpf_t());
}
```

where `expr.val1` and `expr.val2` are references to the expression's operands (here `expr` is the `__gmp_binary_expr` stored within the `__gmp_expr`).

This way, the expression is actually evaluated only at the time of assignment, when the required precision (that of `f`) is known. Furthermore the target `mpf_t` is now available, thus we can call `mpf_add` directly with `f` as the output argument.

Compound expressions are handled by defining operators taking subexpressions as their arguments, like this:

```
template <class T, class U>
__gmp_expr
<__gmp_binary_expr<__gmp_expr<T>, __gmp_expr<U>, __gmp_binary_plus> >
operator+(const __gmp_expr<T> &expr1, const __gmp_expr<U> &expr2)
{
  return __gmp_expr
    <__gmp_binary_expr<__gmp_expr<T>, __gmp_expr<U>, __gmp_binary_plus> >
    (expr1, expr2);
}
```

And the corresponding specializations of `__gmp_expr::eval`:

```
template <class T, class U, class Op>
void __gmp_expr
<__gmp_binary_expr<__gmp_expr<T>, __gmp_expr<U>, Op> >::eval
(mpf_t f, unsigned long int precision)
{
  // declare two temporaries
  mpf_class temp1(expr.val1, precision), temp2(expr.val2, precision);
  Op::eval(f, temp1.get_mpf_t(), temp2.get_mpf_t());
}
```

The expression is thus recursively evaluated to any level of complexity and all subexpressions are evaluated to the precision of `f`.

Appendix A Contributors

Torbjorn Granlund wrote the original GMP library and is still developing and maintaining it. Several other individuals and organizations have contributed to GMP in various ways. Here is a list in chronological order:

Gunnar Sjoedin and Hans Riesel helped with mathematical problems in early versions of the library.

Richard Stallman contributed to the interface design and revised the first version of this manual.

Brian Beuning and Doug Lea helped with testing of early versions of the library and made creative suggestions.

John Amanatides of York University in Canada contributed the function `mpz_probab_prime_p`.

Paul Zimmermann of Inria sparked the development of GMP 2, with his comparisons between bignum packages.

Ken Weber (Kent State University, Universidade Federal do Rio Grande do Sul) contributed `mpz_gcd`, `mpz_divexact`, `mpn_gcd`, and `mpn_bdivmod`, partially supported by CNPq (Brazil) grant 301314194-2.

Per Bothner of Cygnus Support helped to set up GMP to use Cygnus' configure. He has also made valuable suggestions and tested numerous intermediary releases.

Joachim Hollman was involved in the design of the `mpf` interface, and in the `mpz` design revisions for version 2.

Bennet Yee contributed the initial versions of `mpz_jacobi` and `mpz_legendre`.

Andreas Schwab contributed the files '`mpn/m68k/lshift.S`' and '`mpn/m68k/rshift.S`' (now in '`.asm`' form).

The development of floating point functions of GNU MP 2, were supported in part by the ESPRIT-BRA (Basic Research Activities) 6846 project POSSO (POlynomial System SOlving).

GNU MP 2 was finished and released by SWOX AB, SWEDEN, in cooperation with the IDA Center for Computing Sciences, USA.

Robert Harley of Inria, France and David Seal of ARM, England, suggested clever improvements for population count.

Robert Harley also wrote highly optimized Karatsuba and 3-way Toom multiplication functions for GMP 3. He also contributed the ARM assembly code.

Torsten Ekedahl of the Mathematical department of Stockholm University provided significant inspiration during several phases of the GMP development. His mathematical expertise helped improve several algorithms.

Paul Zimmermann wrote the Divide and Conquer division code, the REDC code, the REDC-based mpz_powm code, the FFT multiply code, and the Karatsuba square root. The ECMNET project Paul is organizing was a driving force behind many of the optimizations in GMP 3.

Linus Nordberg wrote the new configure system based on autoconf and implemented the new random functions.

Kent Boortz made the Macintosh port.

Kevin Ryde worked on a number of things: optimized x86 code, m4 asm macros, parameter tuning, speed measuring, the configure system, function inlining, divisibility tests, bit scanning, Jacobi symbols, Fibonacci and Lucas number functions, printf and scanf functions, perl interface, demo expression parser, the algorithms chapter in the manual, '`gmpasm-mode.el`', and various miscellaneous improvements elsewhere.

Steve Root helped write the optimized alpha 21264 assembly code.

Gerardo Ballabio wrote the '`gmpxx.h`' C++ class interface and the C++ `istream` input routines.

GNU MP 4.0 was finished and released by Torbjorn Granlund and Kevin Ryde. Torbjorn's work was partially funded by the IDA Center for Computing Sciences, USA.

(This list is chronological, not ordered after significance. If you have contributed to GMP but are not listed above, please tell `tege@swox.com` about the omission!)

Thanks goes to Hans Thorsen for donating an SGI system for the GMP test system environment.

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