Pigeon Hole Principle:Assignments

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Q 1. *Suppose there are* 12 *children in a family. There are* 4 *bedroom. Prove that there exist a bedroom which accomodates at least* 3 *children.*

Q 2. *Show that among any* $n+1$ *numbers one can find* 2 *numbers so that their difference is divisible by* n.

(Hint: Given any $n+1$ numbers how many distinct remainders are obtained if those numbers are divided by n.)

Q 3. *Prove that if seven distinct numbers are selected from* $1, 2, ..., 11$, *then there exist two numbers among these selected numbers which sum to* 12.

(Hint: Consider all the pairs whose sum is 12.)

Q 4. *Given* 101 *integers from* $1, 2, ..., 200$, *there are at least two integers such that one of them is divisible by the other.*

(Hint: What happens if you write the numbers as $2^k.a$ where a is an odd number.)

Q 5. *Prove that if five points are selected from the interior of a* 1×1 *square, then there are two points whose distance is less than* $\sqrt{2}/2$.

(Hint: Relate the pigeons with the points chosen. The length of the diagonal of a 1×1 square is $\sqrt{2}$.)