# Pigeon Hole Principle:Class Notes

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## 1 Pigeonhole Principle(PHP1)

**Theorem 1.** *If $n+1$ objects are put into $n$ boxes, then at least one box contains two or more objects.*

**Example 1.** *Among 8 people there are at least two persons who have the same birthday.*

**Example 2.** *Among 13 people there are at least two persons whose month of birth is same.*

**Example 3.** *Suppose you are blindfolded. You are given a tray of $n$ pairs of socks all mixed up. How many socks would you pick so that there is certainly a pair of socks?*

**Solution:** Consider $n$ boxes each box denoting a pair. Each socks denotes a pigeon. Choose $n+1$ pigeons so that there will be a box which contains 2 socks. Thus if we choose $n+1$ socks we get a pair. □

**Example 4.** *In a party there are $n$ people. Prove that there are at least two persons who know exactly the same number of people.*

**Solution:** Let the pigeon holes denote how many people a person can know. A person can know at most $n-1$ people. So the number of pigeonholes is $n-1$. Let each person be the pigeon. Now if a person knows $i$ people then the person is put in the $i^{th}$ box. There are $n$ people. So there must be one box which contains two persons. □

**Example 5.** *Pick three natural numbers. Prove that there exist a pair whose sum or difference is divisible by* 3.

**Solution:** Observe that a number when divided by 3 has remainder $0, 1, 2$. Identify these remainders as the pigeonholes. So there are 3 pigeonholes. Given a number it goes to the $i^{th}$ pigeonhole if the reminder after dividing it by 3 is $i$ where $i = 0, 1$ or 2. Since there are 3 numbers there can be two cases. Case(i) When each number goes to distinct box. Take the numbers in the box 1 and 2 and add them. Case(ii) When a box contains at least two numbers. Then take the differnce of those two numbers. □

## 2 Pigeonhole Principle(PHP2)

**Theorem 2.** *If $n$ or more pigeons are distributed among $k > 0$ pigeonholes, then at least one pigeonhole contains at least $\lceil \frac{n}{k} \rceil$ pigeons.*

**Example 6.** *Prove that among $k$ numbers there exist a number which is at least as large as the average of the given numbers.*

**Solution:** Consider $k$ pigeonholes. Let $\sum_{i=1}^{i=k} a_i = n$. (where $a_i$'s are the given numbers) Let there be $n$ pigeons. Put $a_i$ pigeons in the $i^{th}$ box. By the above theorem we have $a_i \geq \lceil \frac{n}{k} \rceil$. But the average of the given numbers is $\frac{n}{k}$. Hence we have a number which is at least the average. $\square$

**Example 7.** *A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruit that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?*

**Solution:** $8 + 6 + 9 - 3 + 1 = 21$. $\square$

**References:**


1. **www.math.ust.hk/ mabfchen/Math391I/Pigeonhole.pdf.**
2. **www.math.uvic.ca/faculty/gmacgill/guide/pigeonhole.pdf**