Infinite series-parallel posets of 1-safe nets [1]

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Introduction

Languages of posets are a generalization of the usual languages of words and are intended to give a better description of concurrent systems. Pratt calls labelled posets pomsets and studies their theoretical aspects [Pra86]. Poset languages are closely related to the partial word languages studied by Grabowski [Gra81] and the trace languages introduced by Mazurkiewicz (on which there is a large body of work, see the book [DR95]).

In earlier work [LRR02], we generalized the standard theorems of automata theory (Kleene, Myhill, Nerode, Büchi) to languages of series-parallel (briefly sp-) posets. Our automaton model was 1-safe Petri nets [Pet62], a well known model which combines causality, concurrency and conflict. We showed that the sp-poset languages accepted by 1-safe nets are exactly those defined by the series-rational expressions [LW00], a syntax derived from Grabowski's paper [Gra81].

In this paper, we extend these ideas to languages of *infinite* series-parallel posets. These were first studied by Kuske [Kus00], who defined the ω-series-rational expressions. We show that the languages of infinite sp-posets accepted by 1-safe nets exactly match these expressions.

Since series-parallel posets can only represent a hierarchy of process structure and not the intricate intercommunication between processes, this work complements the approach of Mazurkiewicz trace theory.

In the next section, we define nets and runs on them. Section 2 introduces the different kinds of syntactic expressions and Section 3 defines the various net constructions we will need. In Section 4, we work in the reverse direction, analyzing net systems to produce syntactic expressions. Section 5 shows how our results can be made to work for two subclasses of nets: S-systems [Hack72] and CF-systems [Esp97].

1 Nets, processes and runs

A (labelled) net is a tuple (P, T, ℓ, F) where P is a set of places, T a (disjoint) set of transitions, $\ell : T \to A$ a function labelling the transitions and $F \subseteq (P \times T) \cup (T \times P)$ is the flow relation. For a place or transition x, its pre-set $F^{-1}(x)$ is conventionally denoted ${}^\bullet x$ and its post-set $F(x)$ is denoted $x^\bullet$. We will assume that F satisfies the condition that for each transition t, ${}^\bullet t$ and $t^\bullet$ are nonempty, and for each place p, either ${}^\bullet p$ or $p^\bullet$ is nonempty.

A (labelled) net system is a tuple $N = (P, T, \ell, F, M_0)$, where $M_0 : P \to \mathbf{N}$ is an initial marking. The run of a net system is described by a "token game" leading from the initial marking to other markings [Pet62]. Formally, a marking is a function $M : P \to \mathbf{N}$.

A net system is said to be 1-safe if for all markings M reachable from the initial marking and for all places p, we have $M(p) \leq 1$. Now markings can be interpreted as 0/1-vectors, or equivalently

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as subsets of places. We will therefore use $M_0 \subseteq P$ instead of $M_0 : P \to \mathbf{N}$ to describe the initial marking of 1-safe net systems.

To "accept" finite runs of net systems as in automata theory, we define an **accepting** 1-safe net system to be a tuple $\mathcal{N} = (N, \mathcal{F})$, where $\mathcal{F} \subseteq \wp P$ is a set of final markings.

Our definition of a "run" of a net system is what is known in net theory as a "process" [Sta81]. We will say that the net system $\mathcal{N}$ accepts an A-labelled poset x if x is the poset derived from a process of $\mathcal{N}$ from the initial marking M_0 to one of the final markings in $\mathcal{F}$. See, for example, [Rei85] for a definition of this notion, which works by associating with a net an unfolded version called an "occurrence" net. We call the set of all finite posets accepted $PL(\mathcal{N})$, the (finite) poset language of $\mathcal{N}$.

Similarly, to "accept" infinite runs of net systems, we define a **Büchi** 1-safe net system to be a tuple $\mathcal{B} = (N, \mathcal{G})$, where $\mathcal{G} \subseteq \wp P$ is a set of good markings. Infinite processes are defined by the same unfolding construction [Rei85] and the "good" ones are those in which one of the markings from $\mathcal{G}$ appears infinitely often in the occurrence net. As before, we can derive an infinite poset from such a process. The set of all infinite posets satisfying this acceptance condition is called $\omega PL(\mathcal{B})$.

An advantage of our definitions over Kuske's branching Büchi-automata is that they fit in better with standard ideas of a good run and do not require notions like the accepting *functions* used in his paper [Kus00] .

1.1 The sp-runs of a net system

We now restrict attention to the subclass of series-parallel (or N-free) posets [Gra81, Gis88], which are described inductively using a set of series-parallel terms.

We say $M_1 [a\rangle M_2$ holds if for some a-labelled transition t such that ${}^\bullet t \subseteq M_1$, $(M_1 \setminus {}^\bullet t) \cup t^\bullet = M_2$.

The term $u||v$ gives the parallel composition (disjoint union) of the posets u and v, leaving their elements unordered with respect to each other. $M_1\ [u||v\rangle\ M_2$ holds if each M_i can be split into disjoint components M_i^u and M_i^v such that $M_1^u\ [u\rangle\ M_2^u$ and $M_1^v\ [v\rangle\ M_2^v$.

The term uv stands for the poset formed by sequencing the posets u and v, that is, ordering all elements of u before those of v. We say that a marking M is connected if it cannot be decomposed into two markings M_1 and M_2 which have disjoint neighbourhoods. $M_1\ [uv\rangle\ M_2$ holds if there is an intermediate connected marking M such that $M_1\ [u\rangle\ M$ and $M\ [v\rangle\ M_2$.

Without the connectedness condition on intermediate markings, a run on $ab||cd$ could also be seen as a run on $(a||c)(b||d)$, which are distinct poset behaviours.

This completes our definition of the set of terms $SP(A)$, given by the syntax $u ::= a \mid u_1u_2 \mid u_1||u_2$, which we call **sp-terms**. Note that there is no way of describing the empty poset.

We have implicitly used the fact that sequencing and parallel composition are associative and that parallel composition is also commutative. With this notation for posets, it has been shown that sp-terms denote exactly the finite nonempty N-free posets [Gra81, Gis88], that is, those nonempty posets which satisfy

$$\forall a, b, c, d (a \le c,\ a \le d,\ b \le d,\ a \operatorname{co} b,\ c \operatorname{co} d \text{ implies } b \le c) \tag{1}$$

Here $a \operatorname{co} b$ says that a and b are unordered.

So far our terms only define finite N-free posets. To define infinite N-free posets, we use the syntax $w ::= \Pi_{i \ge 1} u_i \mid u_1 w_2 \mid w_1 || w_2$ [BÉ98], where the u's range over $SP(A)$ and the w's recursively define the ω-**sp-terms** $\omega SP(A)$.

We use the notation $M[w\rangle$ to mean a net has a run on the ω-sp-term w from the marking M. This is defined inductively, as well as the notion of when this run is a **good run** for a Büchi system.

We say $M_1[\Pi_{i\geq 1}u_i\rangle$ holds if there are connected markings M_i, $i > 1$ such that for all $i \geq 1$, $M_i\,[u_i\rangle\,M_{i+1}$. This definition uses that of runs on finite posets on the underlying net system. The infinite run is good if for some marking M_g in $\mathcal{G}$, for infinitely many i, $M_i = M_g$.

$M_1[u_1w_2\rangle$ holds if there is a connected marking M_2 such that $M_1\,[u_1\rangle\,M_2$ and $M_2[w_2\rangle$. The first one is a finite run and the second one an infinite run. If the second run is a good one, so is the run on u_1w_2.

Suppose the Büchi system $\mathcal{B} = (N, \mathcal{G})$ can be split into disjoint components $\mathcal{B}_i = (N_i, \mathcal{G}_i), i = 1, 2$ —that is, $P = P_1 \cup P_2$, $T = T_1 \cup T_2$, and F and M_0 split into their restrictions to the respective sets and $\mathcal{G} = \{M_g^1 \cup M_g^2 \mid M_i \subseteq P_i, M_g^i \in \mathcal{G}_i, i = 1, 2\}$. If we have for two markings M_1 and M_2 that for $i = 1, 2$, $M_i[w_i\rangle$ holds in $\mathcal{N}_i$, then we say that $(M_1 \cup M_2)[w_1||w_2\rangle$ holds in $\mathcal{N}$. This run is good if both of its "sub-runs" are good.

2 Language operations

Our basic syntax will be the rational expressions, which are exactly the same as for words, except that they are interpreted over posets.

$$r ::= a \mid r_1 \cup r_2 \mid r_1 r_2 \mid r_1^+$$

Adding parallel composition gives the series-rational expressions [LW00].

$$e ::= a \mid e_1 \cup e_2 \mid e_1 e_2 \mid e_1^+ \mid e_1||e_2$$

The **ω-series-rational expressions** [Kus00] can now be defined.

$$o ::= e^\omega \mid o_1 \cup o_2 \mid e_1 o_2 \mid o_1||o_2$$

With each (ω-)expression e (o) we associate an (ω-)sp-language $L(e)$ ($W(o)$ respectively). For example, $W(e^\omega) = L(e)^\omega$ (ω iteration), $W(e_1 o_2) = L(e_1)W(o_2)$ and $W(o_1||o_2) = W(o_1)||W(o_2)$. The operations on the right hand side are performed pointwise on terms.

Finally, an (ω-)sp-language over A is said to be (ω-)series-rational if it is of the form $L(e)$ ($W(o)$ respectively).

3 Net constructions

In this section, we will give constructions on nets which correspond to the expression operations. Many of these are known from earlier work [GM84, GV87, Tau89, DNM90, Old91]. In particular, we refer to [LRR02] for net constructions for series-rational expressions.

3.1 Infinite iteration

Assume a 1-safe net system $\mathcal{N} = (P, T, \ell, F, M_0, \mathcal{F})$ accepting the language $L \subseteq SP(A)$. Take *two* copies of the system, call them $\mathcal{N}_1$ and $\mathcal{N}_2$ and add a set of places $M_f^1 \times M_0^2$ for each $M_f^1 \in \mathcal{F}^1$, and a set of places $M_f^2 \times M_0^1$ for each $M_f^2 \in \mathcal{F}^2$. Link them as shown in the example Figure 1.

Here is a detailed construction. Note that we use $+$ to indicate disjoint union.

Let $\mathcal{N}_i = (P_i, T_i, \ell_i, F_i, M_0^i, \mathcal{F}^i)$ be two copies of an accepting 1-safe net system for L. Then the net for its ω-iteration is $\mathcal{B} = (P, T, \ell, F, M_0, \mathcal{G})$ where

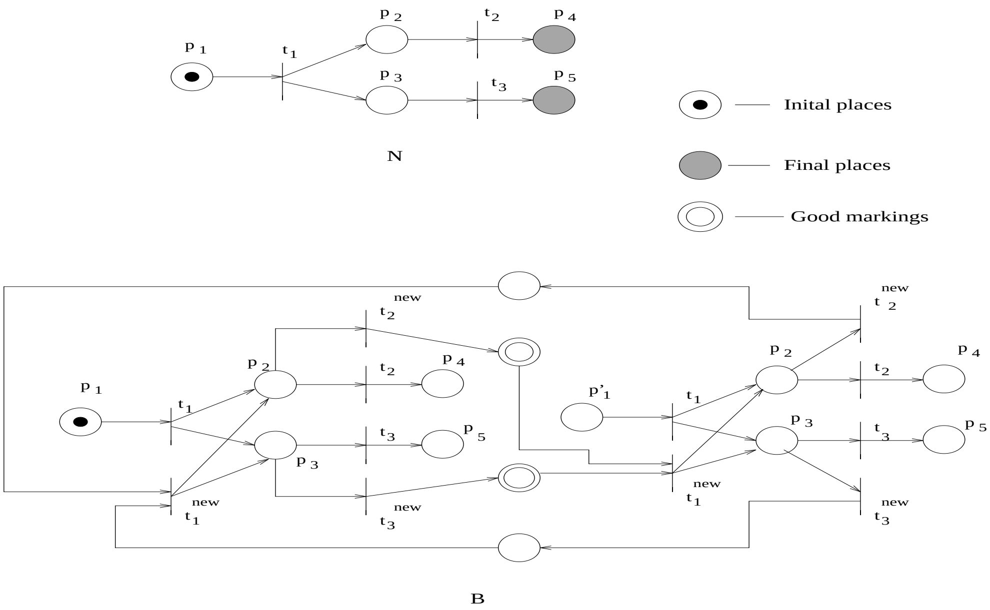

Figure 1: A net $\mathcal{B}$ for the ω-iteration of $\mathcal{N}$

- $P = P_1 + P_2 + P^{12}_{new} + P^{21}_{new};\;\; P^{12}_{new} = \bigcup\limits_{M^1_f \in \mathcal{F}^1} (M^1_f \times M^2_0);\;\; P^{21}_{new} = \bigcup\limits_{M^2_f \in \mathcal{F}^2} (M^2_f \times M^1_0)$

- $T = T_1 + T_2 + T^{12}_{new} + T^{21}_{new};\;\; T^{12}_{new} = \bigcup\limits_{M^1_f \in \mathcal{F}^1} ({}^\bullet(M^1_f) \cup (M^2_0)^\bullet);\;\; T^{21}_{new} = \bigcup\limits_{M^2_f \in \mathcal{F}^2} ({}^\bullet(M^2_f) \cup (M^1_0)^\bullet)$

- $\ell = \ell_1 \cup \ell_2 \cup \{(t \to \ell_i(t)) \mid t \in (T^{12}_{new} \cup T^{21}_{new}) \cap T_i, i = 1, 2\}$

- $F = F_1 \cup F_2 \cup F^{12}_{new} \cup F^{21}_{new};$
  $F^{12}_{new} = \{(p,t) \in P_1 \times T^{12}_{new} \mid (p,t) \in F_1\} \cup \{(t,p) \in T^{12}_{new} \times P_2 \mid (t,p) \in F_2\}$
  $\cup \{(t,(p_1,p_2)) \in T^{12}_{new} \times P^{12}_{new} \mid (t,p_1) \in F_1\} \cup \{((p_1,p_2),t) \in P^{12}_{new} \times T^{12}_{new} \mid (p_2,t) \in F_2\};$
  $F^{21}_{new} = \{(p,t) \in P_2 \times T^{21}_{new} \mid (p,t) \in F_2\} \cup \{(t,p) \in T^{21}_{new} \times P_1 \mid (t,p) \in F_1\}$
  $\cup \{(t,(p_1,p_2)) \in T^{21}_{new} \times P^{21}_{new} \mid (t,p_1) \in F_2\} \cup \{((p_1,p_2),t) \in P^{21}_{new} \times T^{21}_{new} \mid (p_2,t) \in F_1\}$

- $M_0 = M^1_0;\;\; \mathcal{G} = \bigcup\limits_{M^1_f \in \mathcal{F}^1} (M^1_f \times M^2_0)$

Given that $\mathcal{N}_1$ is 1-safe, when the intermediate marking P^{12}_{new} is reached, no tokens remain in the copy of $\mathcal{N}_1$, and now we use the same argument with $\mathcal{N}_2$ to argue that the new system is 1-safe.

Proposition 1 $\omega PL(\mathcal{B}) = PL(\mathcal{N})^\omega$.

Proof. First we prove the forward inclusion. Let $x \in \omega PL(\mathcal{B})$. Then there is a good run of $\mathcal{B}$ on x. It has to go through one of the product markings $M^1_f \times M^2_0$ in P^{12}_{new}. The sub-run from M_0 to the product marking can be simulated by an accepting run $M_0\,[x_1\rangle\, M^1_f$ of $\mathcal{N}$ where u_1 is a finite prefix of x. Because of the second set of product markings P^{21}_{new}, we also have that the remaining part of x can be broken up into $u_i, u_{i+1}, i \geq 2$, such that the even u_i go from P^{12}_{new} to P^{21}_{new} and the odd u_i go the other way. Each of these sub-runs can be simulated by accepting runs of $\mathcal{N}$. Hence each u_i is in L, which yields the inclusion.

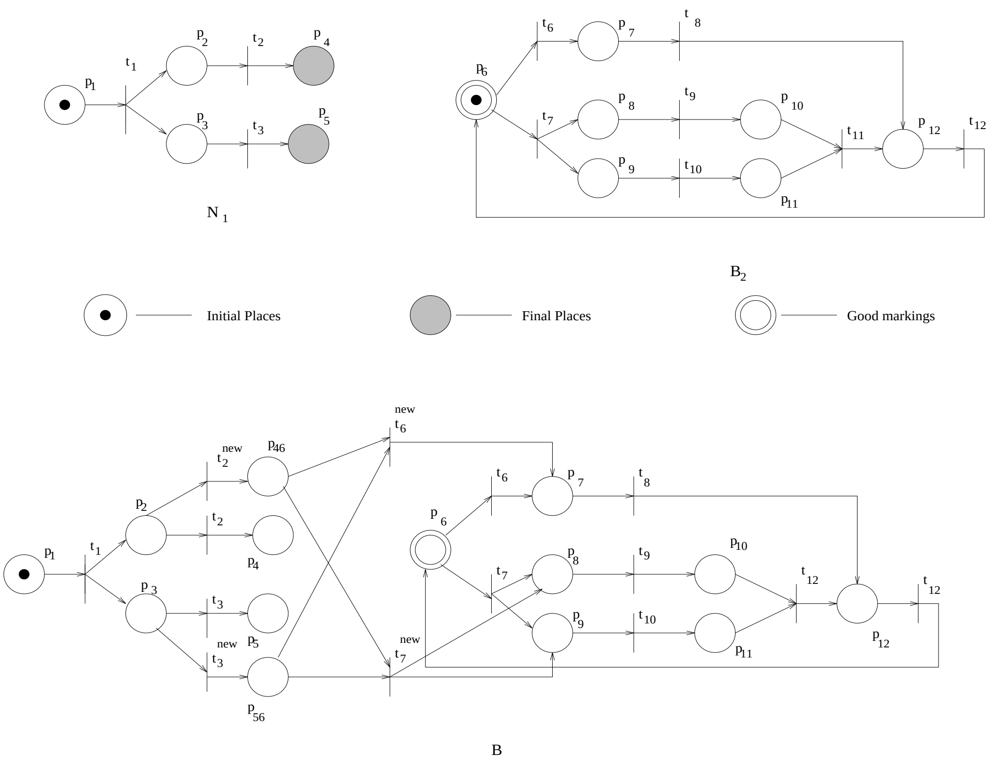

Figure 2: A net $\mathcal{B}$ for sequencing $\mathcal{N}_1$ before $\mathcal{B}_2$

For the reverse inclusion, let $u_i \in PL(\mathcal{N}), i \geq 1$. Hence there is a sub-run on u_1 from M_0 to one of the product markings P^{12}_{new} in $\mathcal{B}$ and a sub-run on $u_i u_{i+1}, i \geq 2$ from P^{12}_{new} back to P^{12}_{new} via P^{21}_{new}. Hence there is a run on $u_1 u_2 u_3 \ldots$ from M_0 in $\mathcal{B}$. It is a good run since one of the markings in P^{12}_{new} must occur infinitely on it. This yields the inclusion. $\Box$

3.2 Sequencing an infinite run after a finite one

Let $N_i = (P_i, T_i, \ell_i, F_i, M_0^i)$ be 1-safe net systems, $\mathcal{N}_1 = (N_1, \mathcal{F}^1)$ be an accepting system for L_1 and $\mathcal{B}_2 = (N_2, \mathcal{G}^2)$ be a Büchi system for W_2. Then the sequential composition of nets N_1, N_2 is $\mathcal{B} = (P, T, \ell, F, M_0, \mathcal{G})$ where

- $P = P_1 + P_2 + P^{12}_{new};\ \ P^{12}_{new} = \bigcup\limits_{M_f^1 \in \mathcal{F}^1} (M_f^1 \times M_0^2)$
- $T = T_1 + T_2 + T^{12}_{new};\ \ T^{12}_{new} = \bigcup\limits_{M_f^1 \in \mathcal{F}^1} ({}^\bullet(M_f^1) \cup (M_0^2)^\bullet)$
- $\ell = \ell_1 \cup \ell_2 \cup \{(t \to \ell_i(t)) \mid t \in T^{12}_{new} \cap T_i, i = 1, 2\}$
- $F = F_1 \cup F_2 \cup F^{12}_{new};$
  $F^{12}_{new} = \{(p,t) \in P_1 \times T_{new} \mid (p,t) \in F_1\} \cup \{(t,p) \in T_{new} \times P_2 \mid (t,p) \in F_2\}$
  $\cup \{(t,(p_1,p_2)) \in T_{new} \times P_{new} \mid (t,p_1) \in F_1\} \cup \{((p_1,p_2),t) \in P_{new} \times T_{new} \mid (p_2,t) \in F_2\}$
- $M_0 = M_0^1;\ \ \mathcal{G} = \mathcal{G}^2.$

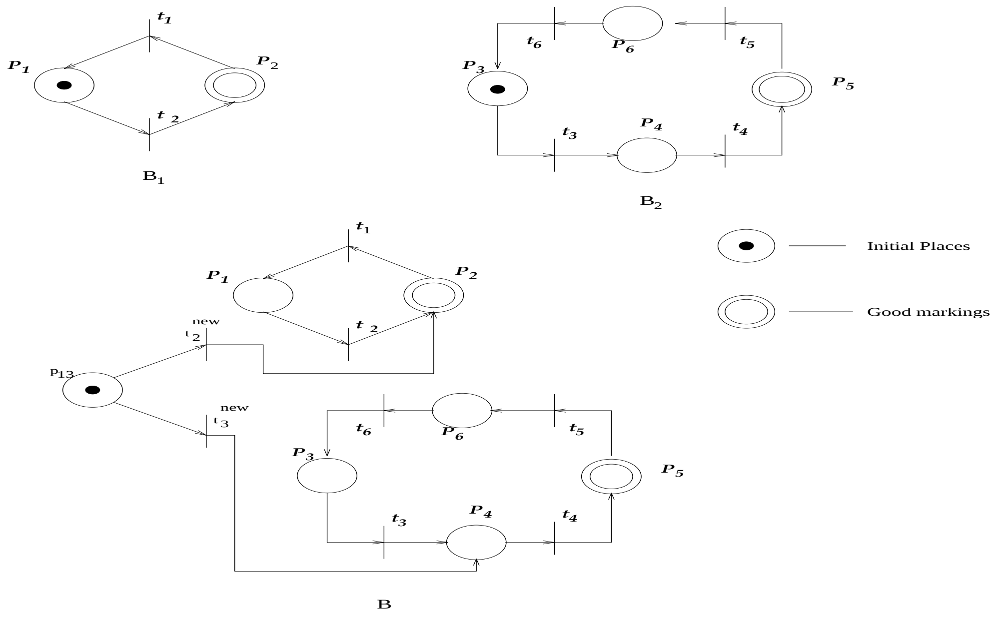

Figure 3: A net $\mathcal{B}$ for the union of $\mathcal{B}_1$ and $\mathcal{B}_2$

The new system uses a product of $M_f^1 \times M_0^2$ for each $M_f^1 \in \mathcal{F}^1$, making copies of transitions coming in to M_f^1 as well as leaving M_0^2. M_0^1 is the initial marking, and $\mathcal{G}^2$ the good markings. This is illustrated in Figure 2.

Proposition 2 $\omega PL(\mathcal{B}) = PL(\mathcal{N}_1)\ \omega PL(\mathcal{B}_2)$.

Proof. First we prove the forward inclusion. Let $x \in \omega PL(\mathcal{B})$. Then there is a good run of $\mathcal{B}$ on x. It has to go through one of the product markings $M_f^1 \times M_0^2$. The sub-run from M_0 to the product marking can be simulated by an accepting run $M_0\, [u\rangle\, M_f^1$ of $\mathcal{N}_1$ and the remaining part of the run by a good run $M_0^2[w\rangle$ of $\mathcal{B}_2$. This implies that $x = uw$ is in the right hand side.

For the reverse inclusion, let $u \in PL(\mathcal{N}_1)$ and $w \in \omega PL(\mathcal{B}_2)$. Hence there is a sub-run on u from M_0 to one of the product markings in $\mathcal{B}$ and a sub-run on w from this product marking. Hence there is a run on uw from M_0 in $\mathcal{B}$. It is good since the run on w was a good run of $\mathcal{B}_2$. □

3.3 Union

Let $\mathcal{B}_i = (P_i, T_i, \ell_i, F_i, M_0^i, \mathcal{G}^i$ be 1-safe net systems for W_i (i=1,2). Then the net for the union of B_1, B_2 is $\mathcal{B} = (P, T, \ell, F, M_0, \mathcal{G})$ where

- $P = P_1 + P_2 + P_{new}$; $P_{new} = (M_0^1 \times M_0^2)$
- $T = T_1 + T_2 + T_{new}$; $T_{new} = (M_0^1)^\bullet \cup (M_0^2)^\bullet$
- $\ell = \ell_1 \cup \ell_2 \cup \{(t \to \ell_i(t)) \mid t \in T_{new} \cap T_i, i = 1, 2\}$
- $F = F_1 \cup F_2 \cup \{((p_1, p_2), t) \in P_{new} \times T_{new} \mid (p_1, t) \in F_1$ or $(p_2, t) \in F_2\}$
$\cup \{(t, p) \in T_{new} \times (P_1 \cup P_2) \mid (t, p) \in F_1$ or $(t, p) \in F_2\}$
- $M_0 = P_{new}$; $\mathcal{G} = \mathcal{G}_1 \cup \mathcal{G}_2$.

For this expression, the new system adds, to a copy of both $\mathcal{N}_i$, a fresh set of places for the marking $M_0^1 \times M_0^2$ (which is the new initial marking, and a final marking if either M_0^i was a final marking) and a fresh copy of transitions enabled by each M_0^i. Figure 3 shows how the net for the union is obtained.

Proposition 3 $\omega PL(\mathcal{B}) = \omega PL(\mathcal{B}_1) \cup \omega PL(\mathcal{B}_2)$.

Proof. If $\mathcal{B}_i$ began by firing (p_i, t) and (t, q), the new system begins by firing $((p_1, p_2), t_{new})$ and (t_{new}, q). After the initial conflict is resolved, tokens circulate in one of the two copies. Thus the posets accepted are those of either of the systems. □

3.4 Parallel composition

The parallel composition of two 1-safe net systems $\mathcal{B}_i = (P_i, T_i, \ell_i, F_i, M_0^i, \mathcal{G}^i)$ for W_i (i=1,2) is the simple disjoint union $\mathcal{B} = (P, T, \ell, F, M_0, \mathcal{G})$ where

- $P = P_1 + P_2$; $T = T_1 + T_2$; $\ell = \ell_1 \cup \ell_2$; $F = F_1 \cup F_2$
- $M_0 = M_0^1 \cup M_0^2$; $\mathcal{G} = \{M_g^1 \cup M_g^2 \mid M_g^1 \in \mathcal{G}^1,\ M_g^2 \in \mathcal{G}^2\}$

Proposition 4 $\omega PL(\mathcal{B}) = \omega PL(\mathcal{B}_1) || \omega PL(\mathcal{B}_2)$.

Proof. The splitting of the good markings has been engineered to make both inclusions go through.

First we prove the forward direction. Let there be a good run on x in $\mathcal{B}$. Since $\mathcal{B}$ is a disjoint union of $\mathcal{B}_1$ and $\mathcal{B}_2$, the run can be split into two sub-runs on x_1 corresponding to $\mathcal{B}_1$ and x_2 corresponding to $\mathcal{B}_2$. Since the run was good, there is a good marking M_g which appears infinitely often in the run. Let M_g^i be its restriction to $\mathcal{B}_i, i = 1, 2$. Then each $\mathcal{B}_i$ has a good run on x_i and $x = x_1 || x_2$ is in the right hand side.

For the reverse inclusion, let $\mathcal{B}_i, i = 1, 2$ have a good run on x_i visiting the good markings M_g^i respectively. Then $\mathcal{B}$ has a run on $x_1 || x_2$. It is good because $M_g^1 \cup M_g^2$ is a good marking of $\mathcal{B}$. □

4 Expessions for nets

Lemma 5 *Given a 1-safe Büchi system $\mathcal{B} = (N, \mathcal{G})$, there is an ω-series-rational expression for $\omega PL(\mathcal{B}) \cap \omega SP(A)$.*

Proof. If the system $\mathcal{B}$ has more than one good marking, the desired expression is the union of the ones found for each good marking separately. So we assume that there is only one good marking M_g. Let $M_g = M_g^1 \cup \ldots \cup M_g^k$, where each $M_g^i, i = 1, \ldots, k$ is connected and each $M_g^i \cup M_g^j, i \neq j$ is not connected.

Now consider the 1-safe *accepting* system $\mathcal{N}$ formed by adding to N new places $P_{new} = \{p_1, \ldots, p_k\}$ and new transitions $t_1, \ldots, t_k$, labelled $a_1, \ldots, a_k$ respectively where these letters are *outside* the alphabet A, and setting ${}^\bullet p_i = \{t_i\}$ and ${}^\bullet t_i = M_g^i$. Let the only final marking of $\mathcal{N}$ be P_{new}. (Its initial marking continues to be the initial marking of $\mathcal{B}$.)

By [LRR02], there is a series-rational expression e for $PL(\mathcal{N})$.

Now let $\mathcal{N}_i$ be the *accepting* systems with the same underlying net as $\mathcal{B}$, except that the initial marking as well as the only final marking are both M_g^i.

Again, by [LRR02], there are series-rational expressions e_i for $PL(\mathcal{N}_i)$.

In the series-rational expression e, replace each letter a_i by the expression e_i^ω to get an ω-series-rational expression o.

An analysis of our construction shows that o is the required expression for $\omega PL(\mathcal{B})$. □

We now tie our results up, together with the results of Kuske. For a precise definition of some of the terms used in the latter work, we refer to the paper [Kus00]. Intuitively, width provides an upper bound on the number of processors required for a maximally parallel implementation of the poset corresponding to an sp-term. For instance, the term $a(b||c(a||b))$ has width three. The width of an sp-language is the supremum of the widths of its terms.

Theorem 6 *The following classes of ω-sp-languages are equivalent:*

(1) The ω-sp-languages accepted by 1-safe systems.

(2) The ω-series-rational languages.

(3) The ω-sp-languages of bounded width accepted by branching Büchi-automata.

(4) The recognizable ω-sp-languages of bounded width.

(5) The MSO-definable ω-sp-languages of bounded width.

Proof. Let X be the language of infinite posets accepted by a Büchi 1-safe system. Hence $W = X \cap \omega SP(A)$ is an ω-sp-language. By the lemma above, there is an ω-series-rational expression for it. This proves that (1) implies (2).

The net constructions of the previous Section 3 inductively preserve the property of 1-safeness and N-freeness, so we get a Büchi 1-safe system accepting W. This proves that (2) implies (1).

The equivalence of the ω-series-rational languages to the languages in (3), (4) and (5), which we cite here for completeness, come from the paper [Kus00]. □

5 Net subclasses

We now show that our results can be made to work for two subclasses of net systems.

Our classification of net systems is based on the inputs and outputs of transitions. A transition t is said to be **forking** if $|t^\bullet| > 1$ and **joining** if $|{}^\bullet t| > 1$. A transition can be both forking and joining.

An **S-system** [Hack72] is a net system where neither forking nor joining transitions are allowed. In other words, for each transition, its pre-set and post-set are both singletons. A **communication-free (CF) system** [Esp97] is one where joining transitions are not allowed. Accepting/Büchi S- and CF-systems are defined by adding final/good markings as before.

Observe that an S-system (even an unsafe one) is equivalent to another one, which is a disjoint union of 1-safe S-systems with an initial marking of size one. (That is, they accept the same sp-language.) Call these 1-safe components of the original system. By the S-system property, exactly one token will circulate throughout a 1-safe S-component, so they are behaviourally the same as state machines [Hack72].

We will also be interested in two subclasses of $SP(A)$ and $\omega SP(A)$.

The outermost parallel (ω-)sp-terms $(\omega)OSP(A)$ are those where no parallel composition is nested inside a sequential product. Let $s_i, i \geq 1$, be sequential terms (that is, words). Then the outermost parallel ω-sp-terms are defined by $w ::= \Pi_{i\geq 1} s_i \mid w_1||w_2$.

$(\omega)OSP(A)$ characterizes (in)finite posets which are disjoint unions of chains. The posets in the language accepted by an S-system satisfy the property that they are disjoint unions of chains, that is, they are outermost parallel.

The right parallel sp-terms $RSP(A)$ are those where a subterm of the form $(u_1||u_2)u_3$ is disallowed, that is, only the right multiplicand of a sequential product is allowed to be a parallel composition. The right parallel ω-sp-terms $\omega RSP(A)$ are now given by $w ::= \Pi_{i\geq 1}s_i \mid s_1w_2 \mid w_1||w_2$.

$(\omega)RSP(A)$ characterizes (in)finite join-free posets, that is, those posets where there do not exist distinct elements x, y, z such that $x \leq z$, $y \leq z$ and x, y are unordered. The posets accepted by a CF-system are right parallel.

Clearly $A^+ \subset OSP(A) \subset RSP(A) \subset SP(A)$ and $A^\omega \subset \omega OSP(A) \subset \omega RSP(A) \subset \omega SP(A)$. If an ω-sp-language is a subset of $\omega RSP(A)$, we call it an ω-rsp-language; ω-osp-languages are similarly defined.

We now turn to series-rational expressions.

An expression with parallel composition only at the outermost level is said to be **outermost (ω-)series-rational**.

An expression where parallel composition is only allowed on the right hand side of a sequential product is said to be right series-rational. More precisely, the **right (ω-)series-rational expressions** are defined by the clauses for e (o) below.

$$r ::= a \mid r_1 \cup r_2 \mid r_1 r_2 \mid r_1^+, \quad e ::= r \mid r_1 e_2 \mid e_1||e_2, \quad o ::= r^\omega \mid r_1 o_2 \mid o_1||o_2$$

An (ω-)sp-language over A is said to be right/outermost (ω-)series-rational if it is of the form $L(e)$ ($W(o)$ resp.) for some right/outermost (ω-)series-rational expression $e(o)$ over A. From the definitions, it follows that the outermost (ω-)series-rational languages are subsets of $(\omega)OSP(A)$ and the right (ω-)series-rational languages are subsets of $(\omega)RSP(A)$. Conversely, if an (ω-)rsp/osp-language is (ω-)series-rational, the expression for it must be right/outermost (ω-)series-rational.

5.1 rsp-languages

Lemma 7 *Given a 1-safe Büchi CF-system $\mathcal{B}$, $\omega PL(\mathcal{B}) \cap \omega RSP(A)$ can be described by a right ω-series-rational expression.*

Proof. We follow the proof of Lemma 5.First notice that since we are restricting ourselves to $\omega RSP(A)$, we can ignore all runs through join transitions, since these will generate a poset which is not in $\omega RSP(A)$.

Hence, for the accepting system $\mathcal{N}$ defined in the proof of Lemma 5 (which is a CF-system by our assumption here), by [LRR02], we get a *right series-rational* expression e for $PL(\mathcal{N})$.

Now we observe a property of 1-safe nets. Suppose a fork transition f is in a cycle of non-joining transitions. It must have an output place p which is not in the cycle. Once f fires, a token is placed in p. A different token circulates through the cycle. Since there are no joining transitions, no waiting is involved and the token comes back to f, which fires again, placing a second token in p, contradicting 1-safeness.

Hence, for the accepting 1-safe systems $\mathcal{N}_i$ of Lemma 5 which describe a cycle (of non-joining transitions by assumption of CF-system) from a good marking to itself, there cannot be fork transitions in the cycle. That is, $PL(\mathcal{N}_i)$ is described by *rational* expressions r_i.

In the right series-rational expression e, replace each letter a_i by the expression r_i^ω to get a right ω-series-rational expression o. □

Theorem 8 *The following classes of ω-rsp-languages are equivalent:*

(1) The ω-sp-languages accepted by 1-safe Büchi CF-systems.

(2) The ω-rsp-languages accepted by 1-safe Büchi systems.

(3) The right ω-series-rational languages.

(4) The recognizable ω-rsp-languages of bounded width.

(5) The MSO-definable ω-rsp-languages of bounded width.

Proof. $(1 \Rightarrow 2)$: Obvious.

$(2 \Rightarrow 3)$: Let W be the language of infinite posets accepted by a 1-safe Büchi system. Then the ω-rsp-language $W \cap \omega RSP(A)$ is right ω-series-rational by Lemma 7.

$(3 \Rightarrow 1)$: Observe now that the sequencing construction of Section 3.2 satisfies the following property: if $\mathcal{N}_1$ is a 1-safe component of an accepting S-system and $\mathcal{B}_2$ is a 1-safe Büchi CF-system, then the constructed net $\mathcal{B}$ is a 1-safe Büchi CF-system. Hence, from the net constructions of Section 3, a right ω-series-rational language is accepted by a 1-safe Büchi CF-system.

$(3 \Longleftrightarrow 4 \Longleftrightarrow 5)$: Consequences of [Kus00, LRR02]. □

5.2 osp-langauges

The following results can be derived by simple modifications of the preceding arguments.

Theorem 9 *The following classes of ω-osp-languages are equivalent:*

(1) The ω-sp-languages accepted by 1-safe S-systems.

(2) The ω-osp-languages accepted by 1-safe systems.

(3) The outermost ω-series-rational languages.

(4) The recognizable ω-osp-languages of bounded width.

(5) The MSO-definable ω-osp-languages of bounded width.

Figure 4 summarizes the results of this paper.

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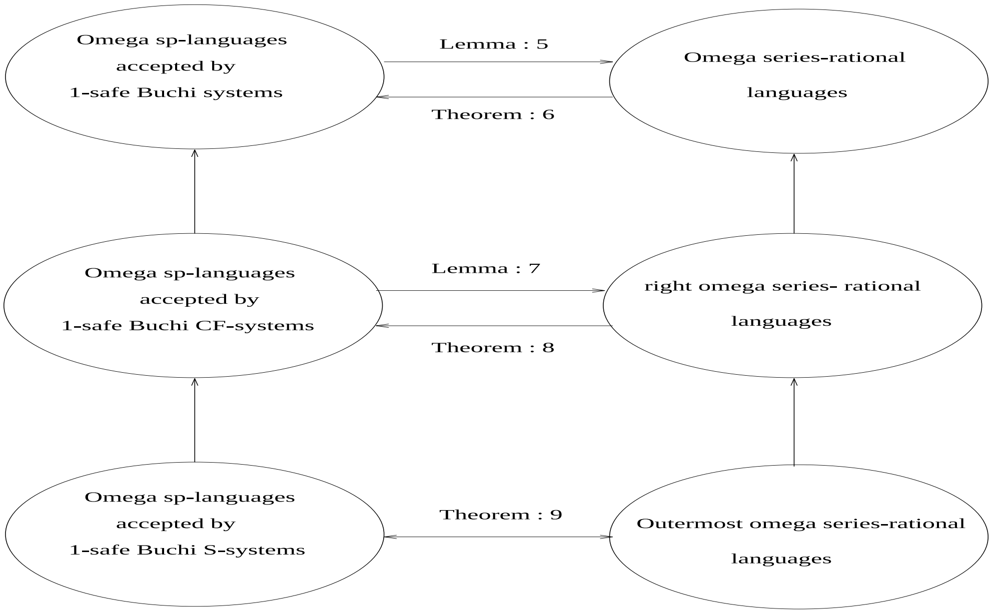

Figure 4: Relationships

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