Constraining cosmological models

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Problem

- The problem is essentially finding out the range of parameters for models for which these are consistent with observations
- As models are defined by a large number of parameters, scanning the multi-dimensional space is computationally challanging
- If the number of parameters (np) is large, a brute force scanning on a grid with 100 samples in each direction require comparing observations with $10^{2\ np}$ models. In our studies, $np \geq 5$

Cosmology

- Observations show expansion of the universe is accelerating
- Cosmological constant Λ
- Many problems relating to origin and value of Λ
- Alternative – (a slowly varying component) with $\rho + 3p < 0$
- Toy models : Scalar fields with an appropriate potential

Equations

- Cosmological equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

- $w = \frac{P}{\rho}$; $\quad P = 0$ (non relativistic matter), $P = \frac{1}{3}\rho$ (radiation) and $P = -\rho$ (cosmological constant)

- Dark energy equation of state:

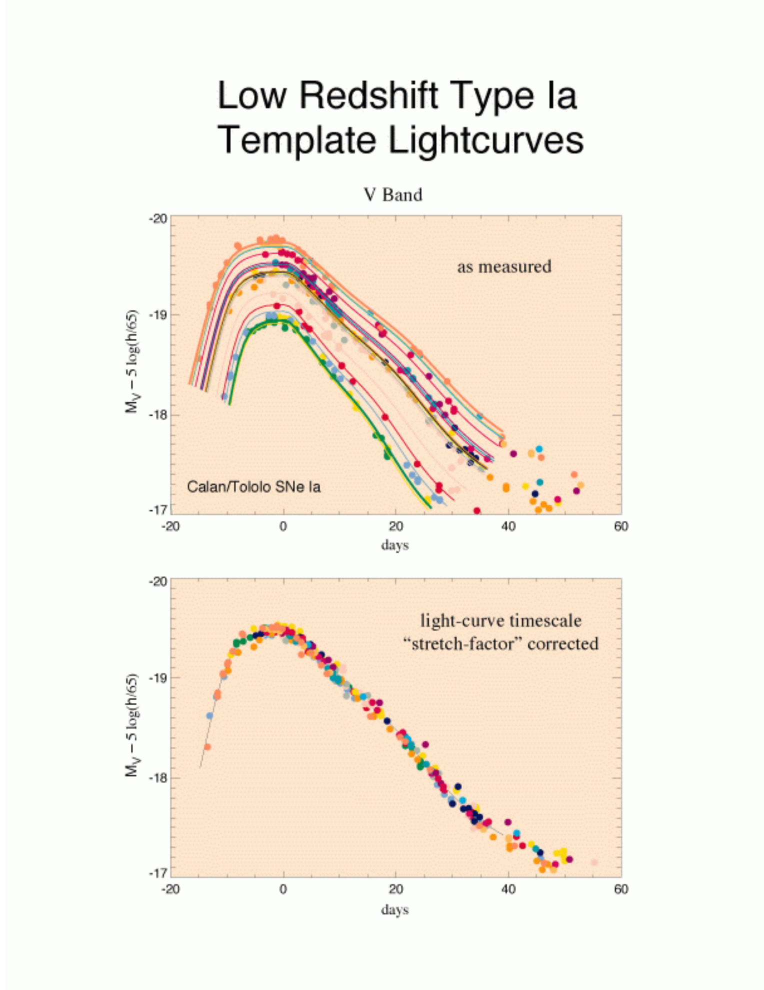
$$w(z) = w_0 + w_1 \frac{z}{(1+z)^p}; \qquad p = 1, 2$$

$w(0) = w_0$, $w'(0) = w_1$

- $p = 1:\ w(\infty) = w_0 + w_1$ and $p = 2:\ w(\infty) = w_0$

Supernova observations

- Supernova type Ia–standard candle

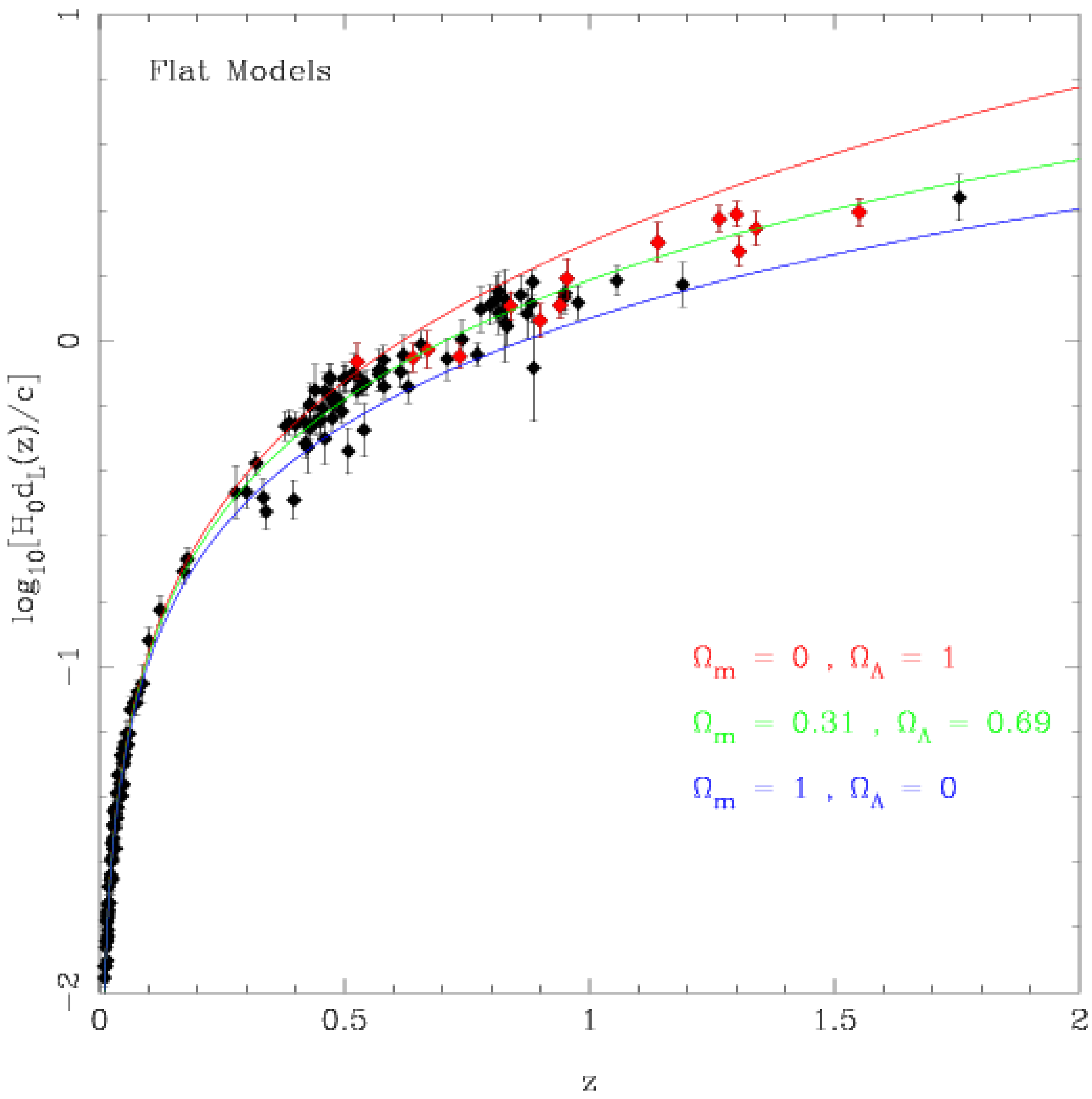


Supernova observations

- Distance modulus

$$m - M - K = 25 + 5 \log \left[3000(1+z)H_0 r(z)/c\right]$$

$\longrightarrow r(z)$ is the coordinate distance from the observer at $z = 0$ to the fundemental observer at redshift z



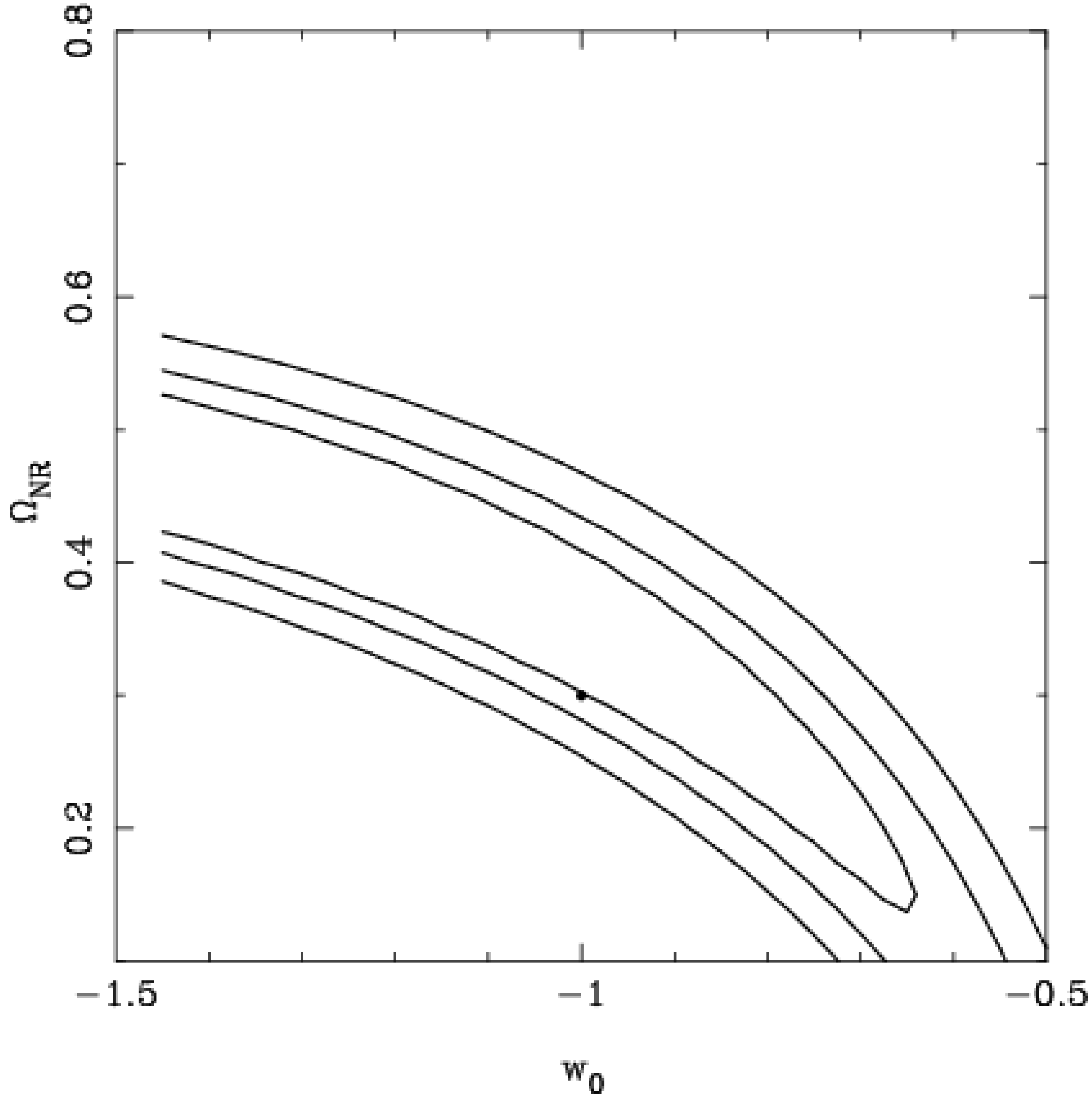
Flat Models
$\Omega_m = 0\ ,\ \Omega_\Lambda = 1$
$\Omega_m = 0.31\ ,\ \Omega_\Lambda = 0.69$
$\Omega_m = 1\ ,\ \Omega_\Lambda = 0$
$\log_{10}[H_0 d_L(z)/c]$
z
1
0
−1
−2
0
0.5
1
1.5
2

Supernova observations

- Method used for comparison: goodness of fit and likelihood methods
- Compute χ^2 for each set of parameters and look for minimum
- Mark out the confidence levels in the parameter space

Parameters

- $\star$ $\Omega_M \equiv \rho_M/\rho_c$ Matter density parameter $(\rho_c = 8\pi G/3H_0^2)$
- $\star$ $\Omega_\Lambda = 1 - \Omega_M$: Flat models
- $\star$ w_0 Present value of dark energy equation of state parameter
- $\star$ w_1 Present value of variation of w
- Marginalize over Hubble constant


0.8
0.6
0.4
0.2
Ω_{NR}
−1.5
−1
−0.5
w_0

Cosmic Microwave Background Radiation

• First clear indication of "Big Bang"; discovered in 1965 (Penzias and Wilson)

• Planckian spectrum –temperature $T = 2.725$

• Origin of CMB

⋆ Very early universe–baryonic matter in highly ionised form–radiation strongly coupled to baryons

⋆ As universe expanded, matter cooled and atoms formed $\simeq 3000k$

⋆ Photon mean free path increased to greater than the present Hubble radius–these photons detected in CMB

⋆ Photons appear to come from a sphere centered at the observer–last scattering surface

CMB anisotropies

- Prediction–CMB should show angular variations in temperature
- Fluctuations in the early universe $\longrightarrow$ inhomogeneities on the last scattering surface
- CMB photons also influenced by scattering and gravitational effects ion their passage from last scattering surface to the observer
- COBE ruled out baryon dominated universe and hot dark matter universe–consistent with cold dark matter models
- Wilkinson Microwave Anisotropy Probe (WMAP) – latest data on CMB anisotropies on large scales

CMB anisotropies – Mathematical framework

- Temperature anisotropies in the CMB

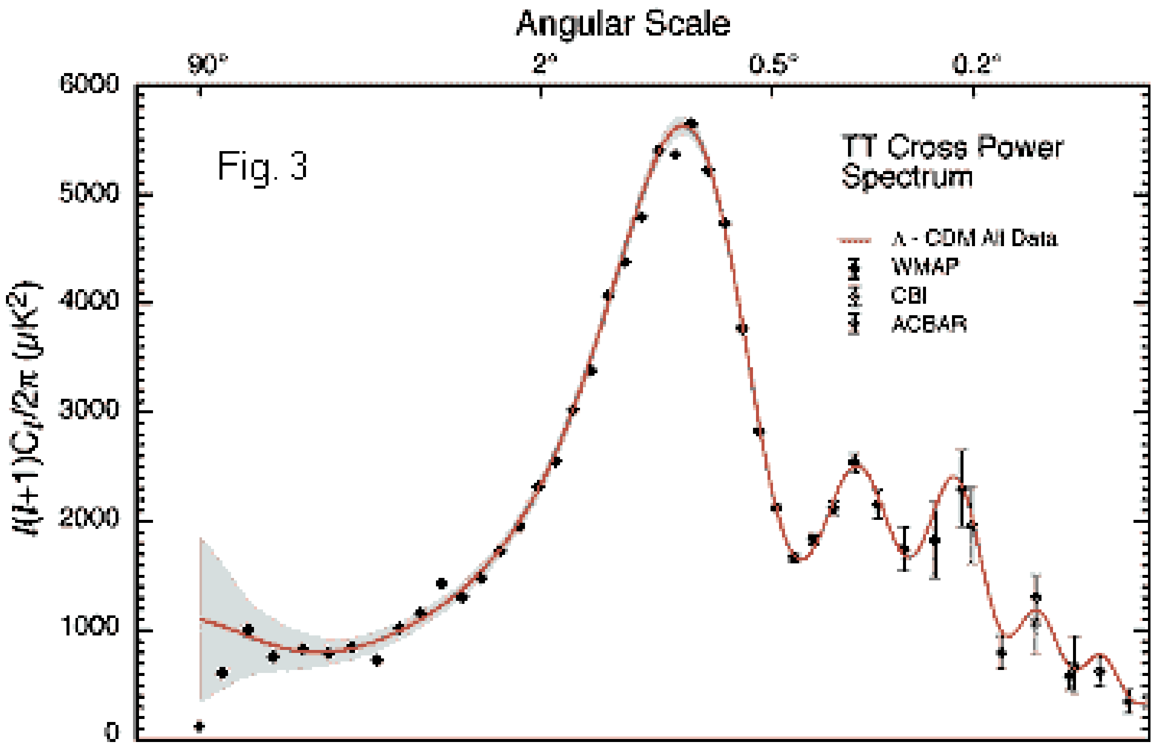
$$\frac{\Delta T}{T}(\hat{n}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}$$

- Anisotropies small: $(\Delta T/T)_{RMS} = 10^{-5}$
- Correlation function of the temperature field

$$C(\theta) = \frac{1}{4\pi} \sum_{l} (2l+1) C_l P_l(\cos\theta)$$

$$C_l = \sum_{m=-l}^{l} a_{lm}^{*} a_{lm}$$

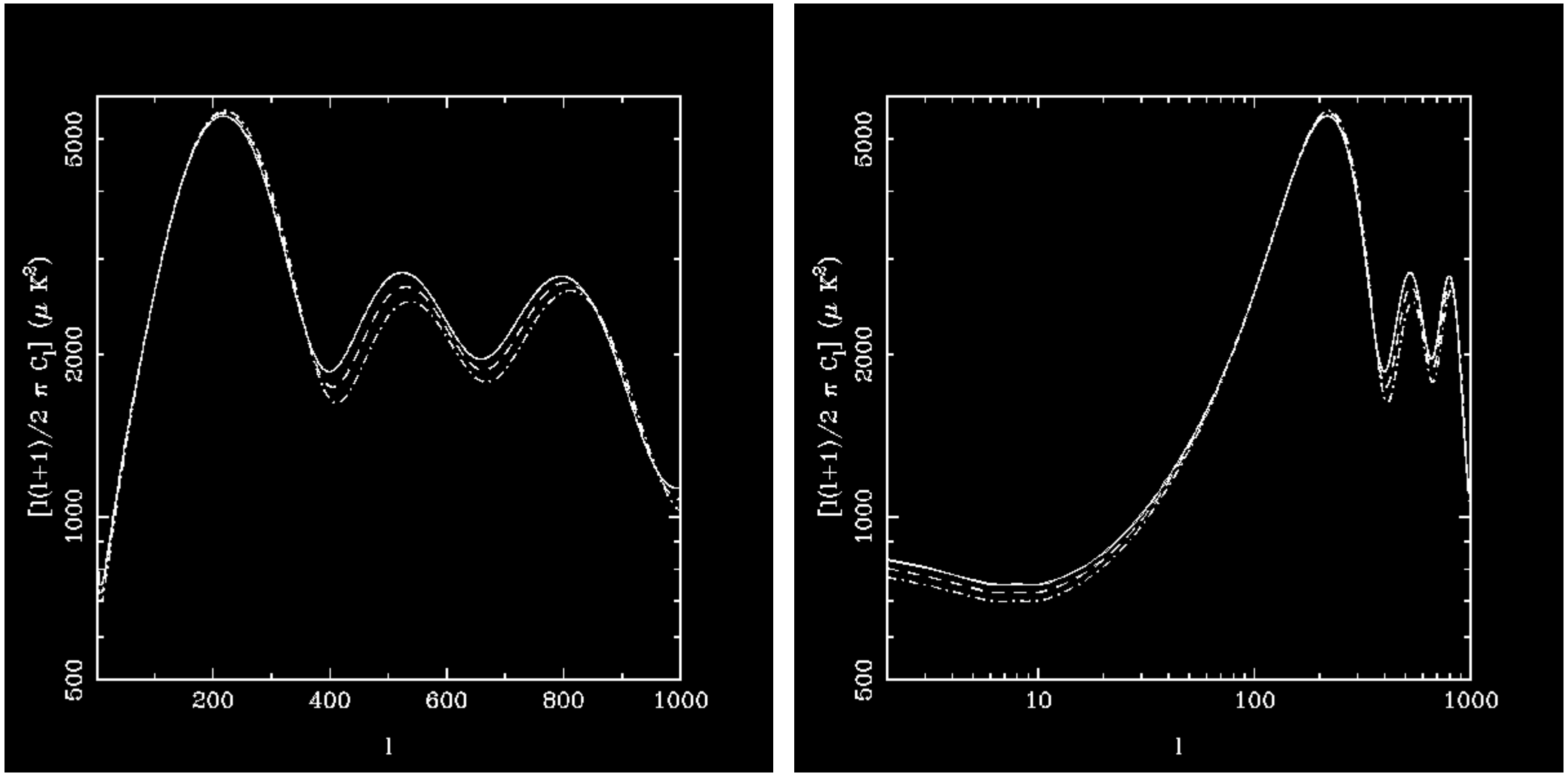
CMB anisotropies – WMAP data



Parameters

- Present value of Hubble constant–H_0
- Baryonic density parameter–Ω_B
- Cold dark matter density parameter–Ω_{CDM}
- Spectral index–n
- Optical depth–τ
- Present value of dark energy parameter– w_0
- Present value of derivative of dark energy parameter–w_1

- Ω_B – Density parameter for baryons ($\Omega_M = \Omega_B + \Omega_{CDM}$)



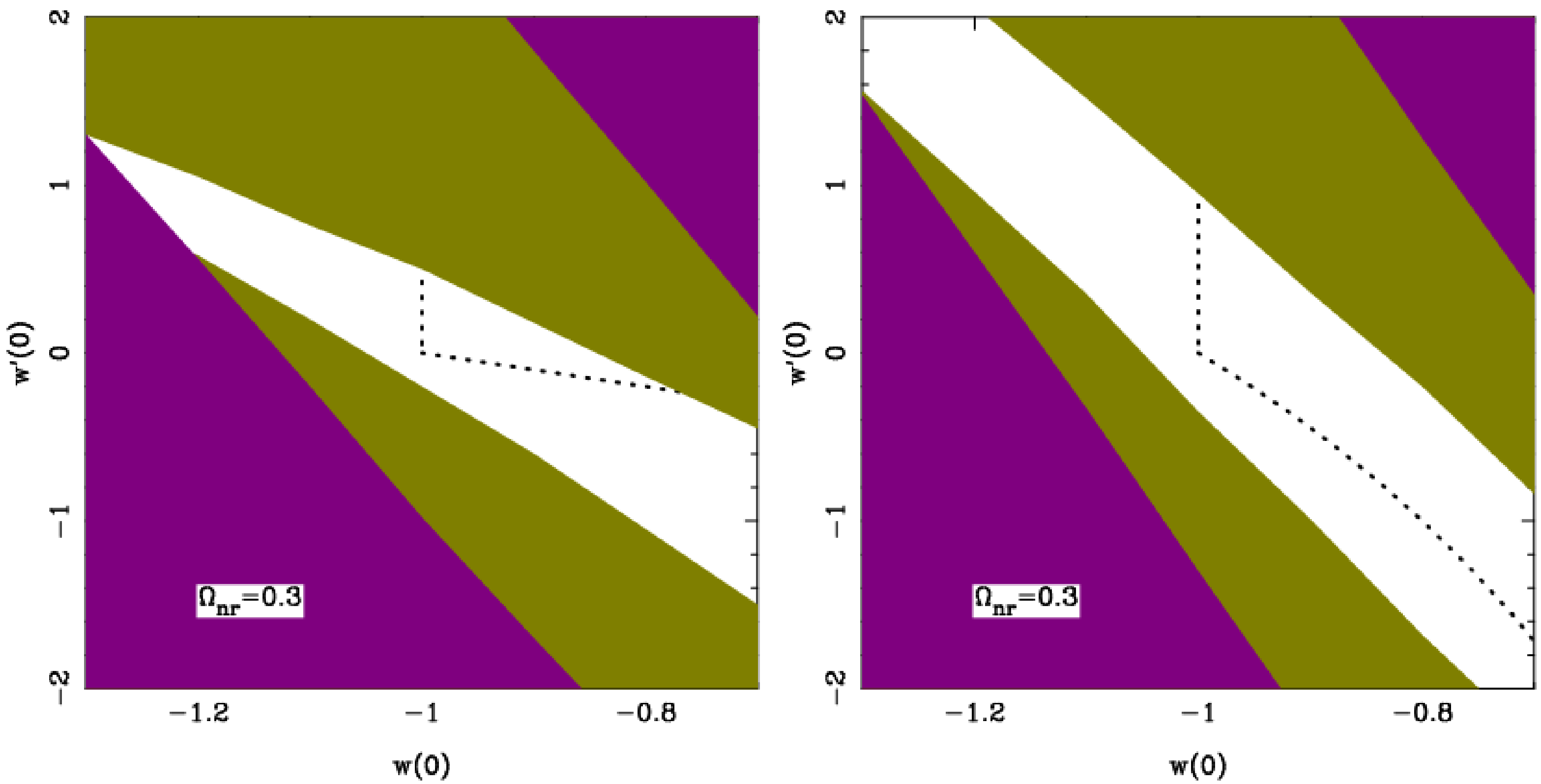
Priors

- $H_0 = 70.0 km Mpc^{-1} s^{-1}$
- $n = 1$
- $\Omega_B = 0.05$
- $\tau = 0$
- $-1.5 \leq w_0 \leq -0.7$
- $-2 \leq w_1 \leq 2$

$\star$ CMBFAST (version 4.5.1) (http://www.cmbfast.org)
$\longrightarrow$ allows varying dark energy

$\star$ Likelihood program provided by the WMAP team (http://map.gsfc.nasa.gov)

Allowed range of dark energy parameters



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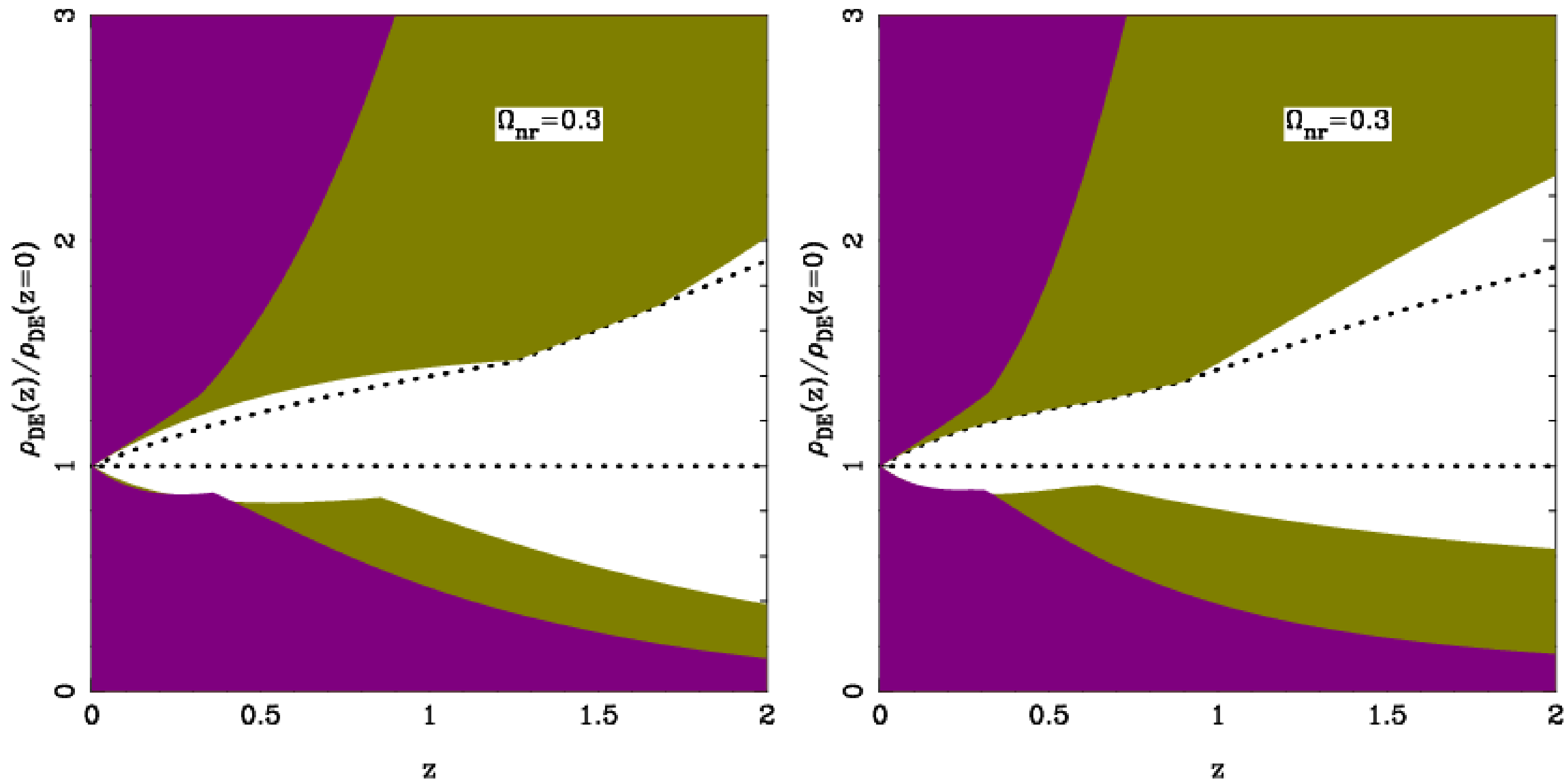
Allowed range of dark energy density



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Priors

- $60.0 \leq H_0 \leq 80.0 \quad kmMpc^{-1}s^{-1}$
- $0.86 \leq n \leq 1.2$
- $0.03 \leq \Omega_B \leq 0.06$
- $0 \leq \tau \leq 0.4$
- $-2.0 \leq w_0 \leq -0.4$
- $-5 \leq w_1 \leq 5$

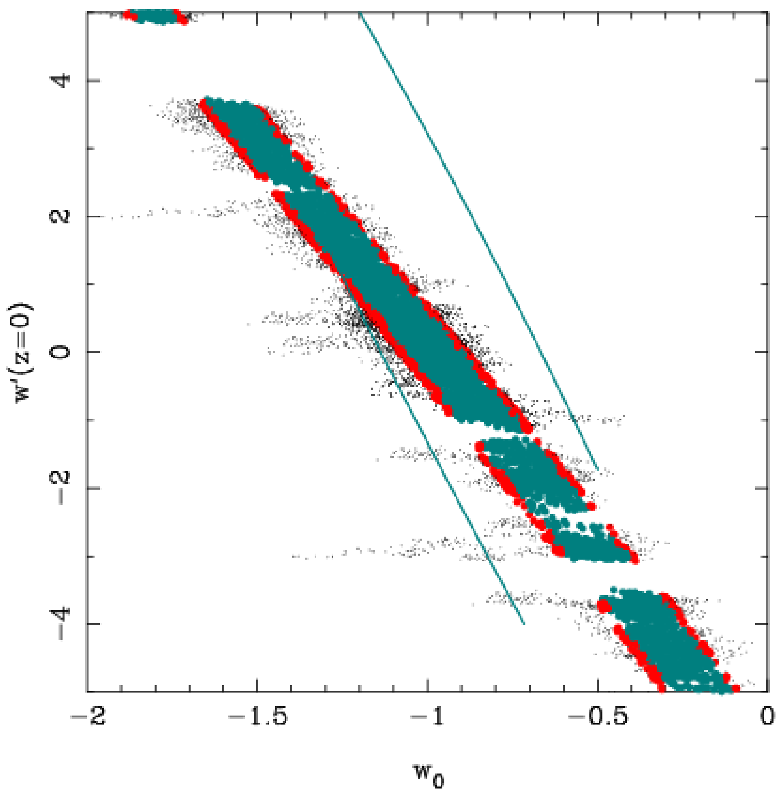
Full parameter analysis

- Grid Based methods
- Random sampling in parameter space
- These methods are computationally expensive

Markov chain monte carlo method (MCMC)

- We combined CMBFAST and WMAP likelihood program and looked for minimum χ^2 using MCMC
- MCMC–Based on metropolis algorithm
- Start from a random point $\mathbf{r}_i$ in parameter space and compute C_l and $\chi^2(\mathbf{r}_i)$
- Consider a small random displacement $\mathbf{r}_{i+1} = \mathbf{r}_i + d\mathbf{r}$ and compute $\chi^2(\mathbf{r}_{i+1})$
- If $\chi^2(\mathbf{r}_{i+1}) \leq \chi^2(\mathbf{r}_i)$ then $i \rightarrow i+1$. Go to the first step
- Else: Compute $\Delta\chi^2 = \chi^2(\mathbf{r}_{i+1}) - \chi^2(\mathbf{r}_i)$ and $\exp[-\alpha\ \Delta\chi^2]$
- Compare this with a random number $0 \leq \beta \leq 1$
- If $\beta \leq \exp[-\alpha\ \Delta\chi^2]$ then $i \rightarrow i+1$. Go to the first step

Preliminary result

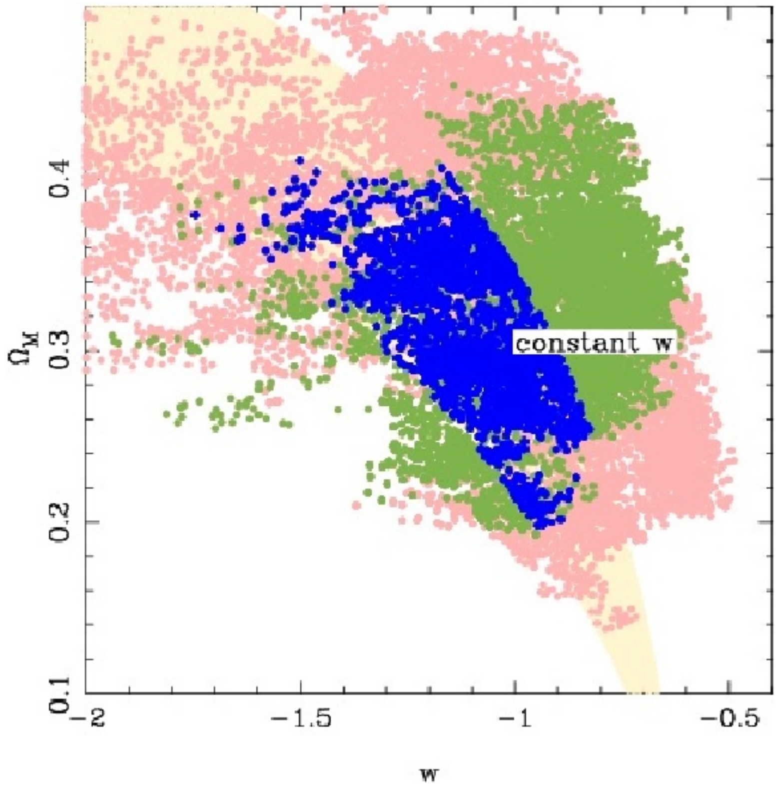


- Result completely consistent with the earlier grid based calculation

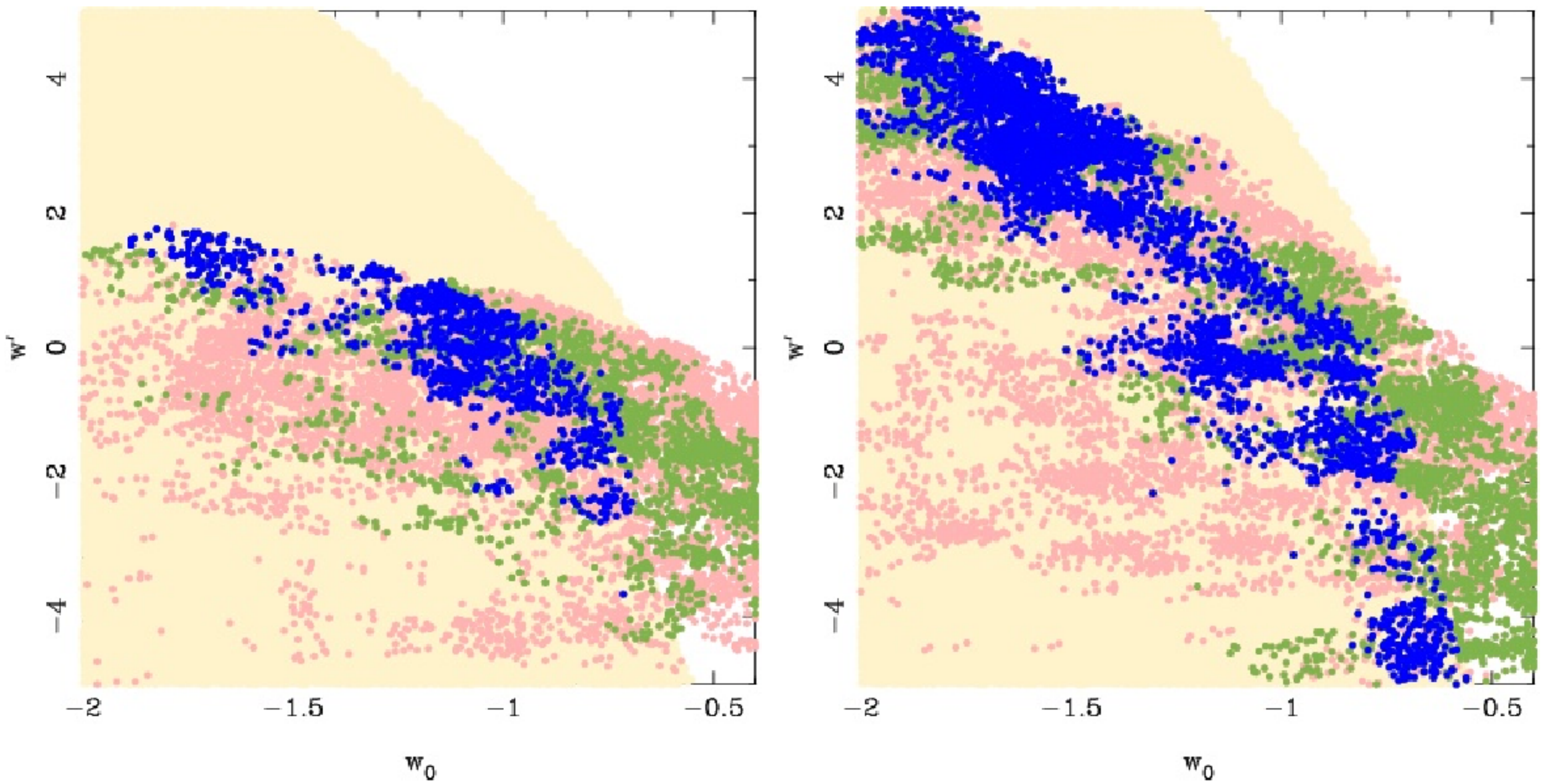
Structure Formation

- Constraints from structure formation restrict the allowed variation of dark energy in a significant manner
- The mass of a typical rich cluster corresponds to the Lagrangian scale of $8\mathrm{h}^{-1}\mathrm{Mpc}$
- Cluster abundance obervations therefore constrain σ_8, the rms fluctuations in density contrast at $8\mathrm{h}^{-1}\mathrm{Mpc}$
- The number density of clusters depends strongly on σ_8 and Ω_{nr}
- ROSAT deep cluster survey and are given by $\sigma_8 = (0.58 \pm 0.06) \times \Omega_{nr}^{-0.47+0.16\Omega_{nr}}$ at 90% confidence level
- The cosmological model should predict σ_8 in the allowed range in order to be consistent with observations

Preliminary result

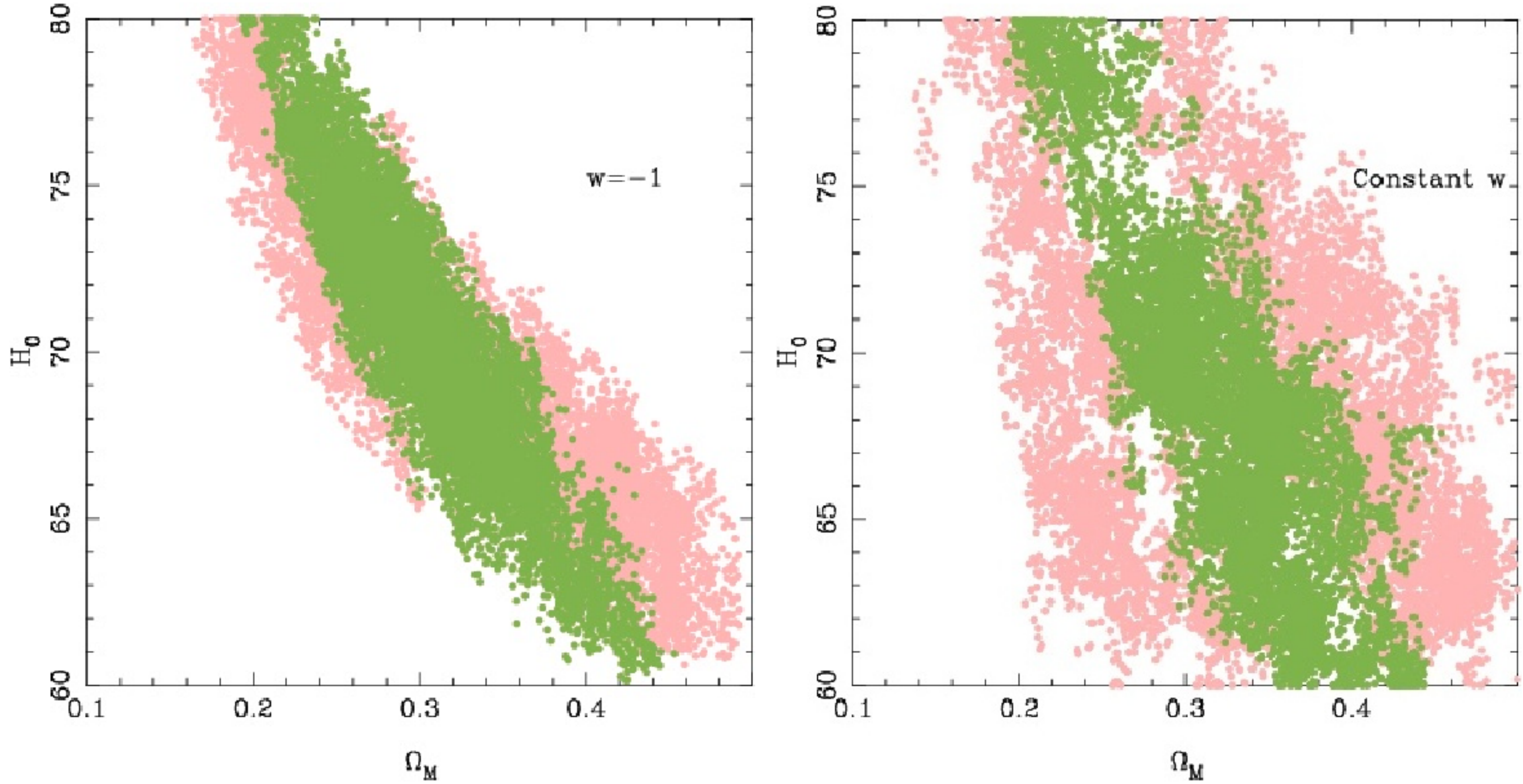


Preliminary result

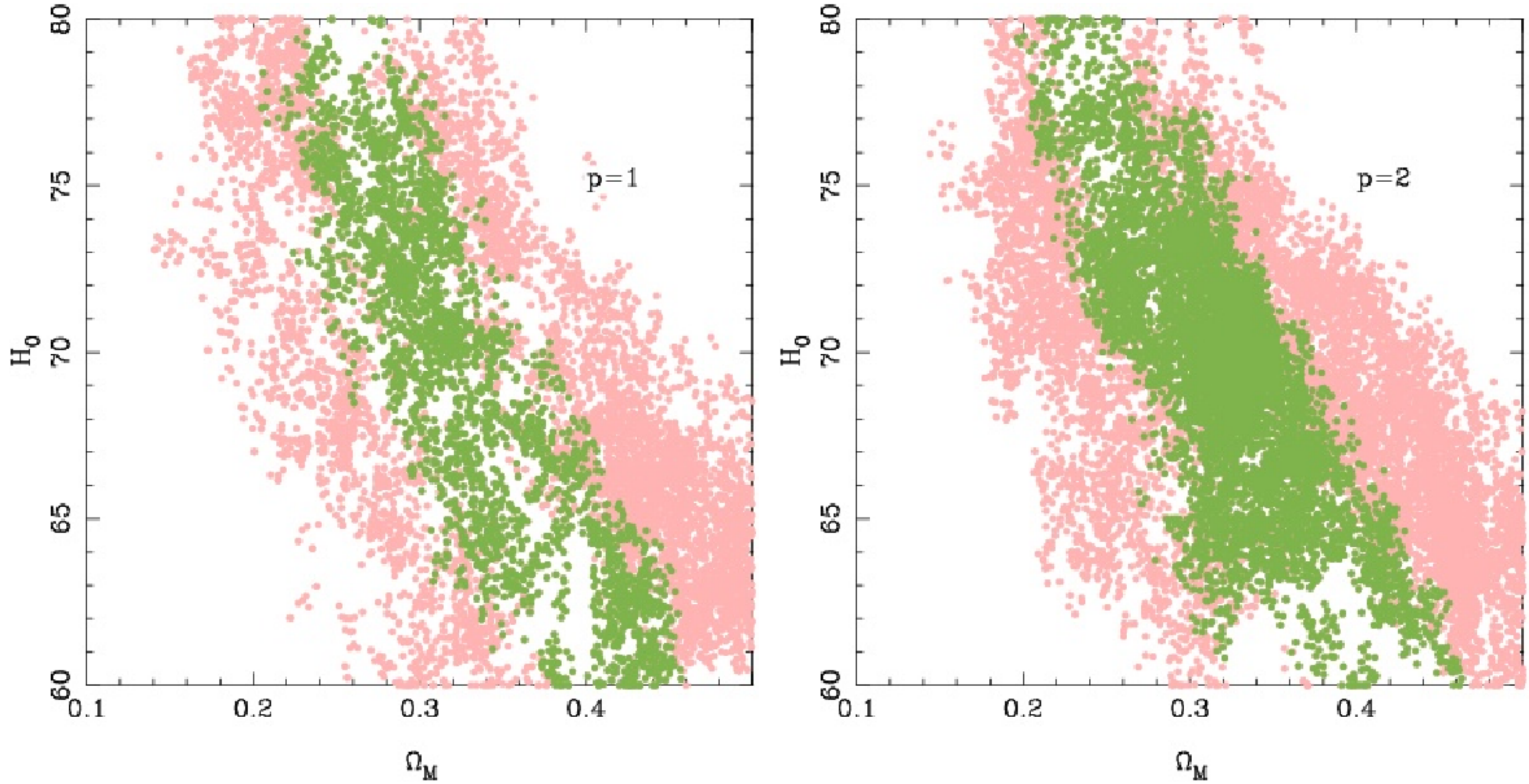


- All other parameters projected on this plane

Preliminary result



Preliminary result



Summary

- Combination of supernova observations and CMB observations puts stronger constraints on low redshift evolution of dark energy as compared to individual observations
- Results are completely consistent with the cosmological constant as a source of dark energy
- Allowing perturbations in dark energy further strengthens our conclusions
- These observations if further combined with requirements of structure formation fix the cosmological parameters better