Logic Tutorial

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Propositional Logic Let $\mathcal{P}$ be a non-empty set of propositional atoms. Then the set of propositional well formed formula $\mathcal{F}$ is :

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\begin{aligned}
p \in \mathcal{P} &\text{ then } p \in \mathcal{F} \\
f \in \mathcal{F} &\text{ then } \sim f \in \mathcal{F} \\
\odot \in \{\vee, \wedge, \implies, \iff\}, f, g \in \mathcal{F} &\text{ then } (f \odot g) \in \mathcal{F}
\end{aligned}
$$

- Give examples and non examples of well formed formula.
- Write out the formula parse tree for the following formulae : $(p \wedge q) \vee (\sim (\sim p \vee (q \wedge r)) \wedge q)$
- Give an algorithm to test if a propositional formula is a tautology or not. Analyse its running time.
- Every f $\in \mathcal{F}$ has matching paranthesis. Prove this via induction.
- Each f $\in \mathcal{F}$ has a unique parsing tree. Prove this using induction.
- Give a propositional logic encoding of the problem of coloring a graph using only k colors.
- Give a propositional logic encoding of the pigeon hole principle for n pigeons in n-1 holes.

First Order Logic

- Given the language $\{R^2\}$ where R is a binary relational symbol. Write out first order formulas which would be satisfied by structures in the language that are equivalence relations.
- Given the language $\{+\}$ where $+$ is a binary function symbol. Write out first order formulas that would be satisfied only when the structure interpreted using the language is a group.

- Given the language $\{<^2\}$ where $<$ is a binary relational symbol. Write out the formulas that would characterise a dense linear order with no end point. Give some models that satisy the property.
- Let $\{E\}$ be the language where E is a binary relational symbol. Give some models that satisfy these FO formula :
  - $\exists x, \sim (\exists y E(x,y))$
  - $\forall x \forall y E(x,y)$
  - $\forall x, \exists y, E(x,y) \wedge \forall z (E(x,z) \implies z = y)$
- Given $M = (\mathbb{N}, 0, s, +)$ where s is the successor function, define a first order formula $\psi(a,b)$ which will be true iff $a < b$

Second Order Logic Given the language of graphs, ie, E where E is a binary relational symbol.

- Write a second order formula that will be satisfiable by graphs that are three colourable.
- Write a second formula that will be satisfied by graphs that are not connected.
- Write a formula $\psi(a,b)$ which will be satisfied iff $(a,b) \in TC(E)$ where TC is the transitive closure.
- Write a formula that will be satisified by only planar graphs.