# Pure D-brane Systems and Black Hole Microstate Counting

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based on arXiv: 1405.0412

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# Plan of the talk

- Overview
- Our work
  - Our system
  - Warm up : an easier toy
  - Actual system
- Future plans

# Summary

- Long term goal:
  to produce exact microscopic counting in $N = 2$ theories using pure D brane systems.

- In this particular work, we test our ideas for an intersecting D brane system in type IIA string theory, compactified on $T^6$ and our computation yields the expected result.

# Overview

# Some Prehistory

- Black Hole Entropy $\sim$ Black Hole Area!
- Statistical underastanding? Black Hole microstates?
- Where would then misrostates come from?

# Here comes string theory

- Macroscopic story (small G, large GM )
  Low energy effective description of string theory → SUGRA
  Black Holes → (brane) solutions of SUGRA.
- Microscopic story (smaller G, small GM )
  p-branes → D-branes
  description involving stringy objects.
- Matching them :
  Witten index remains unchanged as one varies coupling.
  Calculate Witten index in microscopic description and see whether it matches the area of the corresponding Black Hole.

# Why D brane systems ?

- Only option for microscopic system in $N = 2$ theories (Calabi Yau compactifications).
- One to one correspondence with Black Hole microstates? (to be clear later . . . )

# Why D brane systems ?

- Only option for microscopic system in $N = 2$ theories (Calabi Yau compactifications).
- One to one correspondence with Black Hole microstates?
  (to be clear later . . . )
- Worth understanding state counting using pure D brane systems.

# Steps . . .

- simplest compactification ($T^6$), smallest charges.
  ( arXiv 1405.0412 )
- simplest compactification ($T^6$), arbitrary charges.
  ( work in progress )
- Calabi Yau compactification, arbitrary charges.

# Our Work

# Our System

# Our system

Table: Brane configuration

| brane | 123 | 45 | 67 | 89 |
|---|---|---|---|---|
| 1 D2 | | √ | | |
| 1 D2 | | | √ | |
| 1 D2 | | | | √ |
| 1 D6 | | √ | √ | √ |

# Some comments

- The index has been calculated in a "dual system" and for the particular case concerned, is known to be 12 .
  ( Shih,Strominger & Yin )
- A computation in the D brane system would be a non trivial check of U duality.
- The system corresponds to Black Holes only for large charges, which is NOT the case considered in our paper .

# What to do ?

- Calculate Witten Index for the given brane system.
- Only minimum energy modes are relevant
  $\rightarrow$ concentrate on 0 modes.
- Witten Index in the SUSY QM ( that lives on the intersection of the branes ).
- Q: But how to get that SUSY QM ?

# What to do ?

- Calculate open string spectrum in this brane background.
- Count the d.o.f and arrange in SUSY multiplets.
- SUSY dictates their interactions.
- Witten Index = Euler characteristic of the vacuum manifold.
- Write down the potential, calculate the Euler number of the vacuum manifold.

# Warm up: 2 intersecting branes

# 2 Intersecting D-branes

Table: Brane configuration

| brane | 123 | 45 | 67 | 89 |
|---|---|---|---|---|
| 1 D2 | | √ | | |
| 1 D2 | | | √ | |

# SUSY multiplets

Preserved number of supercharges $= 16/2 = 8$

$\Rightarrow$ Arrange fields in $\mathcal{N} = 2$ multiplets .

Table: $\mathcal{N} = 2$ multiplets

| Fields | $\mathcal{N} = 2$ multiplet |
|---|---|
| $V^{(i)}, \Phi_3^{(i)}$ | $\mathcal{N} = 2$ vector multiplets |
| $\Phi_1^{(i)}, \Phi_2^{(i)}$ | $\mathcal{N} = 2$ hypermultiplet |
| $Z^{(12)}, Z^{(21)}$ | $\mathcal{N} = 2$ hypermultiplet |

# Physical interpretation of bosonic fields

Table: Interpretation of on brane fields

| Fields | Physical Interpretation |
|---|---|
| $V^{(1)}$ | $1, 2, 3$ coordinates of 1-st brane. |
| $\Phi_1^{(1)}$ | Wilson lines of the 1-st brane along $4, 5$ . |
| $\Phi_2^{(1)}$ | $6, 7$ coordinates of 1-st brane. |
| $\Phi_3^{(1)}$ | $8, 9$ coordinates of 1-st brane. |

# Interactions of the multiplets

Table: Interactions

| Fields | Interactions |
|---|---|
| $V, \Phi_1, \Phi_2, \Phi_3$ | $\mathcal{N} = 4$ SYM (free for U(1) ) |
| $V^{(1)} - V^{(2)}, \Phi_3^{(1)} - \Phi_3^{(2)}, Z^{(12)}, Z^{(21)}$ | $\mathcal{N} = 2$ vector + $\mathcal{N} = 2$ hyper |

# Superpotentials

- $\mathcal{N} = 4$ :
  No superpotential for Ableian case.

- $\mathcal{N} = 2$ :

$$\mathcal{W} \sim Z^{(12)}(\Phi_3^{(1)} - \Phi_3^{(2)})Z^{(21)}$$

Mixed strings sense separation of branes.

# Goldstones

Table: Goldstones

| Goldstone | Physical interpretation |
|---|---|
| $A_\mu^{(1)} + A_\mu^{(2)}$ | c.o.m along flat directions |
| $\phi_1^{(1)}$ | Wilson line |
| $\phi_2^{(2)}$ | Wilson line |
| $\phi_2^{(1)}$ | 1st brane moving along 2nd brane |
| $\phi_1^{(2)}$ | 2nd brane moving along 1st brane |
| $\phi_3^{(1)} + \phi_3^{(2)}$ | c.o.m along $x^8, x^9$ |

7 Goldstones $\rightarrow$ 6 Goldstinos $\rightarrow$ $4 \times 6 = 24$ broken SUSY

$\therefore 32 - 24 = 8$ remaining SUSY.

# The actual problem

# The actual problem

- The brane configuration :

Table: Brane configuration

| brane | 123 | 45 | 67 | 89 |
|---|---|---|---|---|
| 1 D2 | | √ | | |
| 1 D2 | | | √ | |
| 1 D2 | | | | √ |
| 1 D6 | | √ | √ | √ |

- preserved SUSY : $\mathcal{N} = 1$
- The Lagrangian :

$$L = \sum_{i=1}^{4} (N = 4\, SYM)_i + \sum_{(ij);i,j=1}^{4} (N = 2)_{(ij)} + W$$

# Various pieces of W

- 3 string interaction :

$$\mathcal{W}_2 = \sqrt{2}C \sum_{(ij);\ i,\ j=1}^{4} Z^{ij} Z^{jk} Z^{ki}$$

- turn on metric and B field fluctuations :
  Effects :
  - $$\mathcal{W}_3 = c^{(12)}(\Phi_3^1 - \Phi_3^2) + \dots$$

    Prohibits mixed strings from vanishing.
  - Introduces F.I parameters.
    Can support mixed strings in the vacuum.

# The vacuum manifold

- $V = V_D + V_F$
- $V_D$ + gauge redundancy
  $\rightarrow$ a toric variety for mixed strings of complex dimension 9.
- $V_F$ $\rightarrow$ intersection of hypersurfaces in the toric variety.

# The equations (in homogeneous coordinates)

- $\Phi$ eqns :

$$z_{ij}z_{ji} = -c_{ij} \; ; i,j = 1,2,3,4$$

- $z$ eqns :
  - $\Phi$-s are fixed in terms of $Z$-s
  - consistency conditions:

$$z_{23}z_{31}z_{12} + z_{23}z_{34}z_{42} = z_{32}z_{21}z_{13} + z_{32}z_{24}z_{43}$$

$$z_{24}z_{41}z_{12} + z_{24}z_{43}z_{32} = z_{42}z_{21}z_{14} + z_{42}z_{23}z_{34}$$

$$z_{34}z_{41}z_{13} + z_{34}z_{42}z_{23} = z_{43}z_{31}z_{14} + z_{43}z_{32}z_{24}$$

- 9 equations on 9 variables
  $\rightarrow$ vacuum manifold is 0 dimensional

# Affine coordinates

$$
\begin{aligned}
u_1 &\equiv z_{12}z_{21} \\
u_2 &\equiv z_{23}z_{32} \\
u_3 &\equiv z_{31}z_{13} \\
u_4 &\equiv z_{14}z_{41} \\
u_5 &\equiv z_{24}z_{42} \\
u_6 &\equiv z_{34}z_{43} \\
u_7 &\equiv z_{12}z_{24}z_{41} \\
u_8 &\equiv z_{13}z_{34}z_{41} \\
u_9 &\equiv z_{23}z_{34}z_{42}
\end{aligned}
$$

# The final result

**Number of solutions $=12$**

exactly the expected result !

# Future plans

# The task ahead

- $(1, 1, 1, 2)$ case
- $(1, 1, 1, N_4)$ case .
- $(N_1, N_2, N_3, N_4)$
- trek to Calabi Yau!

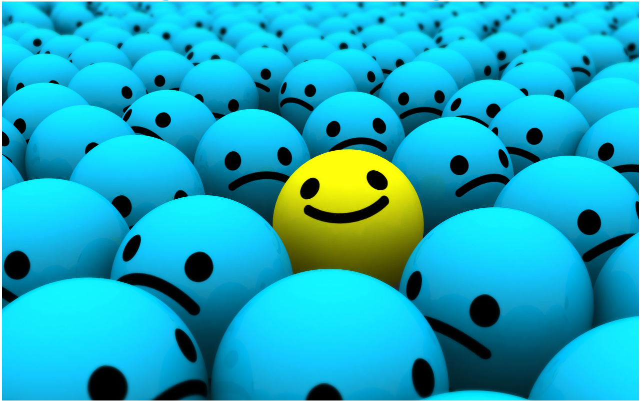
# Some developments : (1,1,1,2) case

- Approach 1: Gauge invariant combinations of equations in terms of gauge invariant observables.
  Too many equations and too many variables (along with compensating syzygies.)

# Some developments : (1,1,1,2) case

- ~~Approach 1~~
- Approach 2: Gauge fix.
  - elimination $\rightarrow$ 5 variables, 5 equations of degree 14,12,10,11,8.
  - Question: number of roots of this polynomial system ?
    Bernshtein's formula : number of roots on $\mathcal{C}^{*n}$ = certain linear combination of volumes of Minkowski sum of Newton Polytopes.
  - tried in SAGE.
    does not seem to work :(

# LETS HAVE LUNCH!



# The equations (in affine coordinates)

$$
\begin{aligned}
m_{13}\,u_7^2\,u_9^2 - m_{23}\,m_{34}\,m_{24}^2\,u_7\,u_8 + m_{24}\,u_7\,u_8\,u_9^2 - m_{24}\,m_{23}\,m_{12}\,u_8^2 &= 0 \\
u_7^2\,u_9 - u_7\,u_9^2 + m_{23}\,m_{24}\,m_{34}\,u_7 - m_{12}\,m_{14}\,m_{24}\,u_9 &= 0 \\
u_8^2\,u_9 + u_8\,u_9^2 - m_{23}\,m_{24}\,m_{34}\,u_8 - m_{13}\,m_{14}\,m_{34}\,u_9 &= 0\,.
\end{aligned}
$$

with $m_{ij} = -c_{ij}$

# The system concerned

| Original System<br>IIB on $T^6$ , D1-D5 system<br>( some results are known here ) | D Dual<br>IIA on $T^6$ , only R-R charges<br>( computations ⇒ check of U duality ) |
|---|---|
| KK along 4<br>momentum along 5<br>D1-brane along 5<br>D5-brane along 56789<br>momentum along 4 | D2-branes along 45<br>D2-branes along 67<br>D2-branes along 89<br>D6-branes along 456789<br>D4-branes along 4589 |

# Dualities relating two systems

1. T duality along 4-5
2. T duality along 6-7
3. S duality
4. T duality along 5-8-9

# Thumb Rules: S Duality

| Initial configuration | Final configuration |
|---|---|
| momentum | momentum |
| F1 | D1 |
| D1 | F1 |
| KK monopole | KK monopole |
| NS5 brane | D5 brane |
| D3 brane | D3 brane |

Table: S Duality

# Thumb Rules: T Duality

| **Initial configuration** | **Final configuration** |
|---|---|
| momentum (4) | F1 (4) |
| F1 (4) | momentum (4) |
| momentum ($a$), $a \neq 4$ | momentum ($a$) |
| F1 ($a$), $a \neq 4$ | F1 ($a$) |
| KK monopole (4) | NS5 (56789) |
| NS5 (5-6-7-8-9) | KK monopole (4) |
| KK monopole ($a$), $a \neq 4$ | KK monopole ($a$), $a \neq 4$ |
| NS 5 (4) $\times T^4$ | NS5 (4) $\times T^4$ |

Table: T Duality (along $X^4$)