Conformal Invariance in Inflation

Nilay Kundu

HRI, Allahabad

ISM, 2014

This talk is based on the following works

- arXiv:1401.1426 [hep-th]

  "Conformal Invariance and the Four Point Scalar Correlator in Slow-Roll Inflation"

  Archisman Ghosh, Suvrat Raju, Sandip Trivedi, NK

- arXiv:1410.2606 [hep-th]

  "Constraints from Conformal Symmetry on the Three Point Scalar Correlator in Inflation"

  Ashish Shukla, Sandip Trivedi, NK

Outline

- Introduction and Motivation
- Basic Set Up
- Conformal Invariance and the Scalar Four Point Correlator
- Ward Identities and the Scalar Three Point Correlator
- Conclusion

Introduction and Motivation

Introduction and Motivation

- Inflation explains the approximate homogeneity and isotropy of our Universe as well as provides a mechanism for producing tiny perturbations to break them.

- In cosmology we try to measure the correlation functions of these perturbations encoded in CMB.

Introduction and Motivation

The Universe during inflation was "approximately" de Sitter - with a symmetry group of $SO(4,1)$ - 3d CFT

One would like to ask $\Rightarrow$

How this symmetry group can be used to learn about the correlations of those perturbations?

Motivation $\Rightarrow$ robust and model independent.

- In situations where inflation occurs for $H \sim M_{string}$ - Higher derivative corrections might be important (Loop corrections are not important since $M_{string} \ll M_{Pl}$).
- Conclusions obtained from symmetry considerations alone still hold. Hence analysis based on symmetries are very powerful.

Introduction and Motivation

The general theme:

- This symmetry based analysis of cosmological correlators are relatively new
  - Maldacena's pioneering work (astro-ph/0210603)
  - Maldacena and Pimentel (1104.2846) $\Rightarrow \langle \gamma_{ij}\gamma_{kl}\gamma_{mn} \rangle$
  - Mata-Raju-Trivedi (1211.5482) $\Rightarrow \langle \zeta\zeta\gamma_{mn} \rangle$
- Convenient to consider Wave function of the Universe
- Invariance of the Wave function under spatial and time reparametrization gives Ward Identities under Conformal transformation.
- We systematically study others : Scalar 4 pt and 3 pt correlators.
- Also incorporate the effect of small breaking of conformal symmetry in the formalism.

Introduction and Motivation

The lessons in general :

The symmetry considerations

- Indeed fixes some correlators completely - full functional form + normalization.
  $\Rightarrow$ True for the three point correlator.

- Sometimes they uncover subtle and interesting issues : the correct question to ask regarding how the symmetries are implemented in the correlators $\Rightarrow$ For the scalar four point correlator

- Also, allow to derive relations between different correlators $\Rightarrow$ the three and four point correlator in a particular limit with important implications to future observations.

Basic Set Up

Basic Set Up

Isometries of dS_4 ⇔3d CFT⇔Euclid AdS_4

- The metric of dS space : $dS^2 = -dt^2 + e^{2Ht}(dx^i)^2$
- $SO(4,1)$ symmetry $\rightarrow$ 10 generators

Translations	3	$x^i \rightarrow x^i + v^i$
Rotations	3	$x^i \rightarrow R^i_j x^i$
Scaling	1	$t \rightarrow t + \frac{1}{H}\log\lambda$ and $x^i \rightarrow \frac{x^i}{\lambda}$
Special Conformal Transformations	3	$t \rightarrow t + 2b^j x_j$ and $x^i \rightarrow x^i - 2(b^j x^j)x^i + b^i((x^j)^2 - e^{-2Ht})$

- $SO(4,1)$ symmetry group of dS_4 $\equiv$ 3d Euclidean CFT $\equiv$ Euclidean AdS_4
  $\Rightarrow$ Analytic continuation takes $EAdS_4$ to dS_4 and vice-versa.
- We are not using any hologram of dS/inflation $\Rightarrow$ the symmetry considerations translates the questions regarding dS_4 to those in a 3d CFT and also use techniques from AdS/CFT.

Basic Set Up

Assumptions in our analysis-I

- The $SO(4,1)$ symmetries of dS_4 are slightly broken during inflation - due to a scalar field (inflaton) and its slow rolling.
- Our analysis will apply to situations where the full inflationary dynamics including the scalar sector approximately preserve the full $SO(4,1)$ symmetry.
  - Geometry is fixed to be dS_4 but the scalar sector might not respect the full conformal group.
- There might also be higher derivative corrections respecting Conformal symmetry.

Basic Set Up

Canonical v/s General slow-roll model of inflation

- The action for the Canonical slow-roll model :

$$S = \int d^4x \frac{\sqrt{-g}}{8\pi G_N}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]$$

- In General slow-roll model :

$$S = \int d^4x \frac{\sqrt{-g}}{8\pi G_N}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) + \frac{c_1}{\Lambda^2}R^2 + \frac{c_2}{\Lambda^4}R^3 + \ldots\right]$$

  - ellipses $\rightarrow$ Additional higher derivative terms on metric or inflaton.
  - c_1 and c_2 are dimensionless constants.
  - $\Lambda \rightarrow$ Some high energy cut-off scale, e.g. String scale M_{st}
  - Additional terms get important when Hubble scale $H \sim M_{st}$

Basic Set Up

The perturbations : $\delta\phi, \zeta, \gamma_{ij}$

- We work in ADM formalism :

$$ds^2 = -N^2 dt^2 + h_{ij} \left(dx^i + N^i dt\right) \left(dx^j + N^j dt\right)$$

$$h_{ij} = e^{2Ht}\left(\delta_{ij}(1+2\zeta) + \gamma_{ij}\right) \quad \text{and} \quad \phi = \bar{\phi} + \delta\phi$$

- Scalar Perturbations : $\delta\phi, \zeta$ and Tensor Perturbation : γ_{ij}

- Action for small perturbations to leading order is

$$S_{\delta\phi} = \int d^4x \frac{\sqrt{-g}}{16\pi G}\left[-\frac{1}{2}(\nabla\delta\phi)^2\right]$$

- This action cares about the background through geometry only and the symmetries of the geometry is shared by the perturbations as well.

Basic Set Up

Slow-roll parameters

- Slow-roll parameters: Quantifies the breaking of Conformal Symmetry :

$$1.\ \epsilon = -\frac{\dot{H}}{H^2} \ll 1 \qquad 2.\ \delta = \frac{\ddot{H}}{2H\dot{H}} \ll 1 \qquad \text{and}\ \ 3.\ \frac{\dot{\bar{\phi}}}{H} \ll 1$$

- In Canonical slow-roll model : $H^2 = \frac{V}{3}$ and $\dot{\bar{\phi}} \sim -\frac{V'}{3H}$

$$\epsilon = \left(\frac{V'}{2V}\right)^2 \ll 1; \qquad \delta = \epsilon - \frac{V''}{V} \ll 1 \ \Rightarrow \ \frac{\dot{\bar{\phi}}}{H} = \sqrt{2\epsilon} \ \gg \epsilon$$

- For General slow-roll cases too, it can be argued that

$$\frac{\dot{\bar{\phi}}}{H} \sim \sqrt{\epsilon} \ \gg \epsilon$$

Basic Set Up

Summary of Assumptions

- Gravity + Scalar (Inflaton) sector respects approximate Conformal Symmetry.
- Single Scalar Field
- Bunch Davies Vacuum in far past
- Slow-roll conditions hold in general.
- One important point is that we will work in momentum space $\Rightarrow$ Often calculations in CFT are done in coordinate space $\Rightarrow$ Fourier transforming position space answers is not straightforward. (Issues of contact terms).

Basic Set Up

Gauge fixing

- By coordinate choice the shift and lapse functions can be set to :

$$N = 1 \qquad \text{and} \qquad N^i = 0$$

$$ds^2 = -dt^2 + e^{2Ht}(\delta_{ij}(1+2\zeta) + \gamma_{ij})dx^i dx^j$$

- Remaining Gauge invariance :
  - Spatial Reparametrization : $x^i \to x^i + v^i(x)$

    Fix by imposing transversality of γ_{ij} : $\partial_i \gamma_{ij} = 0$
  - Time Reparametrization (at late times) : $t \to t + \epsilon(x)$

    This leads to two choices for the scalar degree of freedom.

Basic Set Up

Gauge choice : For Scalars : I

- Gauge 1 : $\delta\phi = 0$ and $\zeta \neq 0$

  Gauge 2 : $\delta\phi \neq 0$ and $\zeta = 0$

- To go between the two gauges : time reparametrization

$$t \to t - \frac{\zeta}{H} \quad \text{to get} \quad \delta\phi = -\frac{\dot{\phi}}{H}\,\zeta$$

- Gauge 1 is useful to work with, once the modes cross horizon $\rightarrow$ since ζ becomes time independent then.

Basic Set Up

Gauge choice : Exact dS Limit

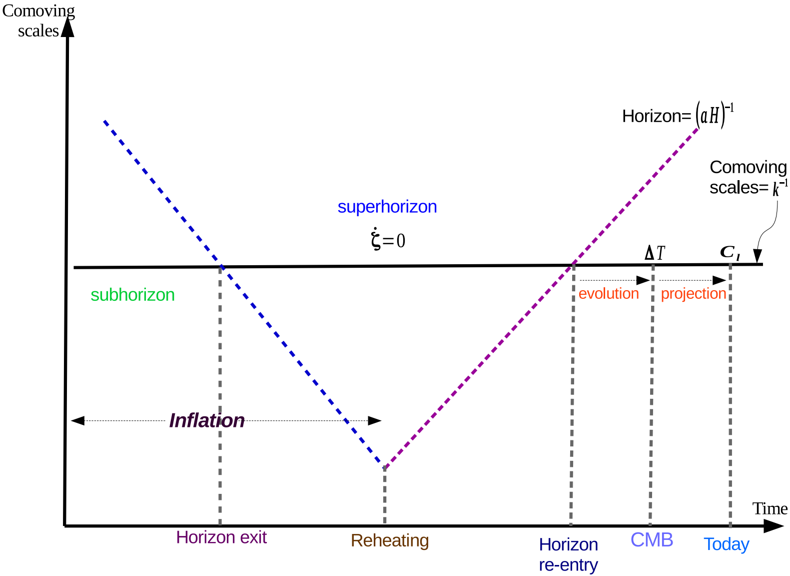
- It is convenient to work in Gauge 2 during inflation before the modes cross the horizon.

$$\zeta = -\frac{1}{\sqrt{2\epsilon}}\delta\phi$$

- The slow-roll limit, $\epsilon \to 0$, the discussion in terms of ζ might be confusing.

- Therefore Gauge 2 is useful to think about the conformally invariant limit in a straightforward manner
  $\Rightarrow$ In this gauge we work in exact dS limit and incorporate the effect of breaking of conformal invariance by going over to Gauge 1 at the end.

Perturbations During and After Inflation:



Basic Set Up

Wave Function of the Universe

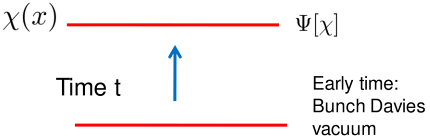
- Symmetry considerations are useful in terms of Wave function. Invariance under symmetry translates to the invariance of the wave function.
- In particle physics S-matrix is more natural. But in cosmology we are interested in the state of the Universe at a time instant.
- We will compute the wave function at late times when the modes have crossed the horizon and have stopped time evolving.

Basic Set Up

Wave Function: Formal definition and conformally invariant

The wave function may be defined as a functional of the late time values of the perturbations $\delta\phi, \gamma_{ij}$ through the path integral

$$\psi[\chi(x)] = \int^{\chi(x)} \mathcal{D}\tilde{\chi}\ e^{i\ S[\tilde{\chi}]},$$



Conformal invariance of $\psi[\chi(x)]$ follows from this definition.

Basic Set Up

Wave Function: Gaussian and Non-Gaussian Terms

- We know the perturbations are nearly Gaussian and hence write the wave function as

$$\psi[\delta\phi,\gamma_{ij}] = \exp\Big[\Big(-\frac{1}{2}\int d^3x\, d^3y\ \delta\phi(x)\ \delta\phi(y)\ \langle O(x)O(y)\rangle$$
$$-\frac{1}{2}\int d^3x\, d^3y\ \gamma_{ij}(x)\ \gamma_{kl}(y)\ \langle T^{ij}(x)T^{kl}(y)\rangle$$
$$-\frac{1}{4}\int d^3x\, d^3y\, d^3z\ \delta\phi(x)\ \delta\phi(y)\ \gamma_{ij}(z)\ \langle O(x)O(y)T^{ij}(z)\rangle$$
$$+\frac{1}{4!}\int d^3x\, d^3y\, d^3z\, d^3w\ \delta\phi(x)\ \delta\phi(y)\ \delta\phi(z)\ \delta\phi(w)\ \langle O(x)O(y)O(z)O(w)\rangle + \cdots\Big)\Big].$$

- $\langle O(x)O(y)\rangle$, $\langle O(x)O(y)T^{ij}(z)\rangle$ for now are just arbitrary coefficient functions determining the Wave function.

Basic Set Up

Coefficient Functions $\rightarrow$ Correlators in CFT

- The boundary values of the fields in *dS* transforms under the symmetry, leading to the statement of the invariance of wave function

$$\delta\phi' = \delta\phi + \delta(\delta\phi) \quad \Rightarrow \quad \psi[\delta\phi] = \psi[\delta\phi'] = \psi[\delta\phi + \delta(\delta\phi)]$$

- Invariance of the wave function fix the transformations of the coefficient functions as correlation functions in a CFT.

$$O'(x) = O(x) + \delta O(x) \Rightarrow O(x) : \text{dimension 3 operators in CFT}$$

- One obtains Ward Identities for those correlators

$$\langle \delta O(x_1) O(x_2) O(x_3) \rangle + \langle O(x_1) \delta O(x_2) O(x_3) \rangle + \langle O(x_1) O(x_2) \delta O(x_3) \rangle = 0$$

- This way we recast our study of symmetries for cosmological perturbations to that of constraints put on correlation functions in a CFT $\Rightarrow$ This is the central idea of our analysis.

Basic Set Up

Ward Identities of conformal symmetry : Completely fix $\langle T_{ij} OO \rangle$

- The Ward identities $\Rightarrow$ differential equations for $\langle T_{ij} OO \rangle$

$$\langle \delta O(x_1) O(x_2) T_{ij}(x_3)\rangle + \langle O(x_1) \delta O(x_2) T_{ij}(x_3)\rangle + \langle O(x_1) O(x_2) \delta T_{ij}(x_3)\rangle = 0$$

$$\partial_i \langle T_{ij}(x) O(y_1) O(y_2)\rangle = \delta^3(x-y_1)\langle \partial_{y_1^j} O(y_1) O(y_2)\rangle + \delta^3(x-y_2)\langle O(y_1)\partial_{y_2^j} O(y_2)\rangle$$

$$\langle T_{ii}(x) O(y_1) O(y_2)\rangle = -3\delta^3(x-y_1)\langle O(y_1) O(y_2)\rangle - 3\delta^3(x-y_2)\langle O(y_1) O(y_2)\rangle$$

- In MRT they completely fixed this correlator.

$$\langle O(k_1) O(k_2) T_{ij}(k_3)\rangle e^{s,ij} = -2(2\pi)^3 \delta(\sum \boldsymbol{k}_J) e^{s,ij} k_{1i} k_{2j} S(k_1, k_2, k_3).$$

with $S(k_1, k_2, k_3) = (k_1 + k_2 + k_3) - \dfrac{\sum_{i>j} k_i k_j}{(k_1 + k_2 + k_3)} - \dfrac{k_1 k_2 k_3}{(k_1 + k_2 + k_3)^2}$

Conformal Invariance and the Four Point Scalar Correlator

The Four Point Scalar Correlator

- The scalar four point correlator is not fixed. In canonical slow-roll model of inflation Serry-Sloth-Vernizzi (0811.3934) already computed it using “in-in” formalism $\Rightarrow$ not invariant under the symmetries.

- This puzzling feature motivated us to embark on calculating it ourselves using technique borrowed from AdS/CFT correspondence in the exact dS limit

The Four Point Scalar Correlator

Analytic continuation of dS ⇔ Euclidean AdS

- Starting with Euclidean AdS_4 $\Rightarrow$

$$ds^2 = R^2_{\text{AdS}} \left(\frac{dz^2 + \sum_{i=1}^{3} (dx^i)^2}{z^2} \right) \text{ with } z \in [0, \infty]$$

- Upon the analytic continuation

$$\Rightarrow z = -i\,\eta, \quad R_{\text{AdS}} = \frac{i}{H}$$

- We get back the dS_4 metric in conformal coordinates,

$$ds^2 = \frac{1}{H^2 \eta^2} \left(-d\eta^2 + \sum_{i=1}^{3} (dx^i)^2 \right); \text{ with } \eta \in [-\infty, 0]$$

The Four Point Scalar Correlator

dS Wave function $\Leftrightarrow$ AdS Partition function

- In leading semi-classical approximation the partition function in AdS space is defined as a functional of the boundary values of the fields as $z \to 0$, i.e.

$$Z[\chi(x)] = e^{-S^{\text{AdS}}_{\text{on-shell}}[\chi(x)]}$$

- Similarly, the wave function in dS space can also be obtained in saddle point approximation,

$$\psi[\chi(x)] = e^{i\, S^{dS}_{\text{on-shell}}[\chi(x)]}$$

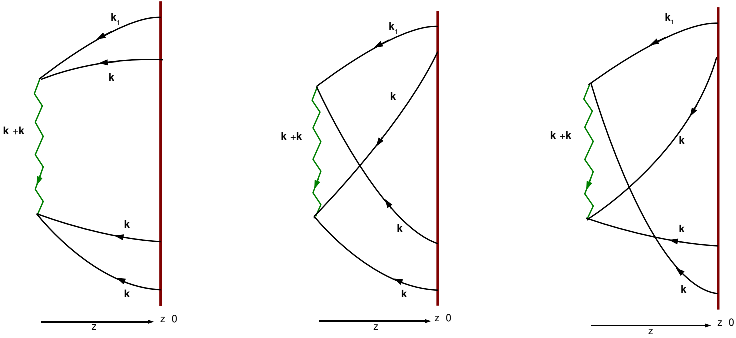
- The analytic continuation gives,

$$S^{ds}_{\text{on-shell}}[\phi(\boldsymbol{k}), H] = -i\, S^{\text{AdS}}_{\text{on-shell}}\left[\phi(\boldsymbol{k}), R_{\text{AdS}} = \frac{i}{H}\right].$$

$\Rightarrow$ Upon analytic continuation we obtain the wave function.

The Four Point Scalar Correlator

Feynman-Witten diagram



(a) S-Channel (a) T-Channel (a) U-Channel

Figure : Three different contribution corresponding to S,T and U-channel to the scalar four point correlator are shown in the three figures. The brown solid vertical line represents the 3-dimensional boundary of AdS_4 at $z = 0$, the black solid lines are boundary to bulk scalar propagators whereas the green wavy lines are graviton propagators in the bulk.

The Four Point Scalar Correlator

Wave Function to 4 point Correlator

- The wave function schematically looks like

$$\psi[\delta\phi, \gamma^s] = \exp\Big[-\frac{1}{2}\int \delta\phi\delta\phi\ \langle OO\rangle - \frac{1}{2}\int \gamma^s\gamma^{s'}\ \langle T^s T^{s'}\rangle$$
$$-\frac{1}{4}\int \delta\phi\delta\phi\gamma^s\ \langle OOT^s\rangle + \frac{1}{4!}\int \delta\phi\delta\phi\delta\phi\delta\phi\ \langle OOOO\rangle\Big].$$

- Knowing the wave function as a functional of the late time values of the perturbations we can compute the scalar correlator.

$$\langle\delta\phi(\boldsymbol{x}_1)\delta\phi(\boldsymbol{x}_2)\delta\phi(\boldsymbol{x}_3)\delta\phi(\boldsymbol{x}_4)\rangle$$
$$= \mathcal{N}\int \mathcal{D}[\delta\phi]\mathcal{D}[\gamma_{ij}]\ \delta\phi(\boldsymbol{x}_1)\delta\phi(\boldsymbol{x}_2)\delta\phi(\boldsymbol{x}_3)\delta\phi(\boldsymbol{x}_4)\ |\psi[\delta\phi, \gamma_{ij}]|^2$$

- Using the relation $\zeta = -\frac{1}{\sqrt{2\epsilon}}\delta\phi$, one obtains the desired correlator $\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4)\rangle$.

- It agrees with existing result !!.

The Four Point Scalar Correlator

Conformal Invariance : Wave Function vs Correlator

- To check conformal invariance for the wave function it just remains to check that for $\langle OOOO \rangle$

$$\psi[\delta\phi, \gamma^s] = \exp\Big[-\frac{1}{2}\int \delta\phi\delta\phi \, \langle OO \rangle - \frac{1}{2}\int \gamma^s \gamma^{s'} \, \langle T^s T^{s'} \rangle - \frac{1}{4}\int \delta\phi\delta\phi\gamma^s \, \langle OOT^s \rangle + \frac{1}{4!}\int \delta\phi\delta\phi\delta\phi\delta\phi \, \langle OOOO \rangle \Big].$$

- We will subject the correlator $\langle OOOO \rangle$ to conformal ward identity and it satisfies.

$$\langle \delta O(k_1) O(k_2) O(k_3) O(k_4) \rangle + \langle O(k_1) \delta O(k_2) O(k_3) O(k_4) \rangle + \langle O(k_1) O(k_2) \delta O(k_3) O(k_4) \rangle + \langle O(k_1) O(k_2) O(k_3) \delta O(k_4) \rangle = 0$$

$\delta O(k_1) \rightarrow$ Change in $O(k_1)$ under SCT

- The wave function is conformally invariant but the correlator $\langle \delta\phi\delta\phi\delta\phi\delta\phi \rangle$ is not.

The Four Point Scalar Correlator

Conformal Invariance : subtlety for Correlator

- This subtlety is due to the fact that one needs do a path integral over the graviton

$$\langle \delta\phi\delta\phi\delta\phi\delta\phi\rangle = \int \mathcal{D}[\delta\phi]\mathcal{D}[\gamma_{ij}] \prod_{i=1}^{4} \delta\phi(\boldsymbol{x}_i)\, |\psi[\delta\phi, \gamma_{ij}]|^2$$

- Before doing the path integral one needs to fix the gauge for the graviton $\Rightarrow \partial_i\gamma_{ij} = \gamma_{ii} = 0$, A conformal transformation does not preserve this choice of gauge.

$$\delta\gamma_{ij}(x) = 2M_j^m\gamma_{im} + 2M_i^m\gamma_{mj} - (x^2 b^m - 2x^m(\boldsymbol{x}\cdot\boldsymbol{b}))\frac{\partial\gamma_{ij}(\boldsymbol{x})}{\partial x^m}$$

$$\partial_i\delta\gamma_{ij} = -6b^k\gamma_{kj} \neq 0$$

- One needs to do a compensating coordinate reparametrization to restore the gauge. Under the combined transformation the invariance works and we learn to ask the correct question.

The Four Point Scalar Correlator

No analogue of this subtlety in AdS space

- This subtlety of conformal invariance for dS space correlators doesn't exist in AdS space : One needs to proceed differently in calculating a correlation function from a wave function as opposed to the partition function (in the presence of sources) in AdS.

- In AdS space one computes the partition function, and the boundary value of the metric is a non-dynamical source. Taking derivative with respect to the source produces correlation function.

- In dS, the metric at late times is a genuine degree of freedom and hence to calculate correlation functions from the wave function of the Universe at late times, one must fix gauge completely.

Ward Identities and the Three Point Scalar Correlator

The Three Point Scalar Correlator

- The scalar three point correlator : $\langle\zeta\zeta\zeta\rangle \Rightarrow$ observationally most important for non-Gaussianity.
- It is suppressed in canonical slow roll inflation $\Rightarrow$ vanishes up to leading order in approximate conformal invariance during inflation.

The Three Point Scalar Correlator

The vanishing at leading order

- The corresponding term in the wave function to consider is

$$\psi = exp\left[- \int \delta\phi\delta\phi\delta\phi \, \langle OOO \rangle \right]$$

- In a 3 dim CFT: $\langle OOO \rangle \Rightarrow$ 3 point function of an exactly marginal operator $\Rightarrow \langle OOO \rangle$ must vanish when all the slow roll parameters go to zero.

- From the gravity calculation $\Rightarrow \delta\phi$ becomes a massless scalar with no potential $\Rightarrow$ 3 point function $\langle \delta\phi\delta\phi\delta\phi \rangle$ vanishes.

- We need to go beyond the exact de-Sitter limit $\Rightarrow$to incorporate the breaking of the conformal symmetry in our formalism. and compute the leading non-vanishing contribution.

The Three Point Scalar Correlator

The orders of parameters at play

- In attempts to consider the departure from conformally invariant dS limit $\Rightarrow$ The background might itself change because H is no longer a constant

  $\Rightarrow$ Corrections to dS $\sim \mathcal{O}(\epsilon) \sim (\frac{\dot{\phi}^2}{H^2})$

  $\Rightarrow$ the stress tensor gets corrections $\sim \mathcal{O}(\frac{\dot{\phi}^2}{H^2})$ or higher.

- For studying $\langle OOO \rangle \Rightarrow$ we can neglect departures from dS and work up to limit $\sim \mathcal{O}(\frac{\dot{\phi}}{H})$

- In canonical slow-roll model $\Rightarrow \frac{\dot{\phi}}{H} \sim \sqrt{\epsilon} \quad \gg \epsilon \ll 1 \Rightarrow$ This is true even in general slow-roll models.

- Up to $\mathcal{O}(\frac{\dot{\phi}}{H}) : \delta\phi \to \delta\phi + \delta(\delta\phi) + \tilde{\delta}(\delta\phi)$ with $\tilde{\delta}(\delta\phi) \sim \frac{\dot{\phi}}{H}$

The Three Point Scalar Correlator

Ward identities for “approximate” conformal symmetry

- The wave function schematically looks like

$$\psi[\delta\phi, \gamma^s] = \exp\Bigg[\bigg(-\frac{1}{2}\int \delta\phi\delta\phi \ \langle OO\rangle - \frac{1}{4}\int \delta\phi\delta\phi\delta\phi \ \langle OOO\rangle + \frac{1}{4!}\int \delta\phi\delta\phi\delta\phi\delta\phi \ \langle OOOO\rangle\bigg)\Bigg].$$

- Finally, for SCT

$$\mathcal{L}^b_{k_i}\langle O(k_1)O(k_2)O(k_3)\rangle' = 2\,\frac{\dot{\phi}}{H}\left[b\cdot\frac{\partial}{\partial k_4}\right]\left\{\langle O(k_1)O(k_2)O(k_3)O(k_4)\rangle'\Big|_{k_4\to 0}\right\},$$

- and for scaling

$$\left(k_a\cdot\frac{\partial}{\partial k_a}\right)\langle O(k_1)O(k_2)O(k_3)\rangle = \frac{\dot{\phi}}{H}\langle O(k_1)O(k_2)O(k_3)O(k_4)\rangle\Big|_{k_4\to 0}.$$

The Three Point Scalar Correlator

Implications of the ward identities

$$\mathcal{L}^b_{k_i}\langle O(k_1)O(k_2)O(k_3)\rangle' = 2\,\frac{\dot{\bar{\phi}}}{H}\left[b\cdot\frac{\partial}{\partial k_4}\right]\left\{\langle O(k_1)O(k_2)O(k_3)O(k_4)\rangle'\Big|_{k_4\to 0}\right\},$$

- The Ward identities hold for general slow-roll models but as an important check they are indeed satisfied by the results obtained in the canonical slow-roll model.

- The uniqueness : Given a four point function how to solve the Ward Identities to determine the three point function. We found that it nearly fixes $\langle OOO\rangle$ in terms of $\langle OOOO\rangle$ up to an additional term, the homogeneous solution of the Ward identities. Under generic assumptions even that is argued to be negligible.

- When the generic assumptions fail, the known functional form will subject it to observational tests.

The Three Point Scalar Correlator

Implications of the ward identities

- As long as the dynamics is approximately conformally invariant, and the slow roll approximation is valid, the magnitude of $\langle\zeta\zeta\zeta\rangle$ should be suppressed, of the same order as in the canonical slow roll model of inflation.

- If the non-Gaussianity in the near future is measured to be much bigger for $\langle\zeta\zeta\zeta\rangle$. Our analysis confirms, that would not only rule out the canonical slow roll model of inflation, but in fact any model approximately conformally invariant with slow roll approximation.

- These Ward identities also implies that, if $\langle\zeta\zeta\zeta\rangle$ is observed and found to deviate from its functional form in the canonical slow roll model, $\langle\zeta\zeta\zeta\zeta\rangle$ must also be different, suggesting perhaps that higher spin fields are involved during inflation.

The Three Point Scalar Correlator

Implications of the ward identities

- Our procedure is different from the standard conformal perturbation theory. It is a perturbative expansion in the coupling constant that breaks conformal symmetry whereas we try to solve the ward identities once the symmetry breaking is taken into account. In other words, we try to solve a Callan-Symanzik eqn with small beta function and hence is more efficient than the first.

- Finally, it will be interesting to extend our analysis to $\mathcal{O}(\frac{\dot{\phi}^2}{H^2}) \sim \mathcal{O}(\epsilon)$, where the background dS will get corrected. It will be interesting to understand that in our set-up and explain effects due breaking of conformal symmetry, such as tilt of scalar and tensor power spectrum, $n_s, n_t \sim \epsilon$.

Summary of the talk

Summary

- Conformal Invariance is indeed a powerful constraint.
- The way conformal invariance is implemented might turn out to be subtle, gauge fixing issues due to gravitons.
- The correlators discussed here can serve as a good model independent and robust probe of approximate conformal invariance during inflation.
- The smallness of the non-Gaussianity when conformal symmetry is a good approximation is a damper but consistent with Planck.
- Perhaps future observations like LSS etc can measure them.
- There is a possibility that the non-Gaussianity will be measured to be large and approximate conformal invariance will be ruled out.

Thank you