Gauge Invariant 1PI Effective Superstring Field Theory

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Puri, December 2014

PLAN

1. Motivation

2. Off-shell amplitudes

3. 1PI effective string field theory

A.S., arXiv:1408.0571, 1411.7478, to appear

Earlier work

Roji Pius, Arnab Rudra, A.S., 1311.1257, 1401.7014, 1404.6254
B. Zwiebach, hep-th/9206084; Hata, Zwiebach, hep-th9301097;
Saroja, A.S., hep-th/9202087

Motivation

In the conventional approach to perturbative string theory we are supposed to directly compute on-shell S-matrix elements

– integrals of certain CFT correlation functions over the moduli spaces of Riemann surfaces with punctures.

However this approach is insufficient for addressing many issues even within the perturbation theory.

1. Mass renormalization

2. Vacuum shift

LSZ formula for S-matrix elements in QFT

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2)\}$$

$G^{(n)}$: n-point Green's function

$a_1, \cdots a_n$: quantum numbers, $k_1, \ldots k_n$: momenta

$m_{i,p}$: physical mass of the i-th external state

– given by the locations of the poles of two point function in the $-k^2$ plane.

Z_i: wave-function renormalization factors, given by the residues at the poles.

In contrast, string amplitudes compute 'truncated Greens function on classical mass-shell'

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2)\,.$$

m_i: <u>tree level</u> mass of the i-th external state.

$k_i^2 \to -m_i^2$ condition is needed to make the vertex operators conformally invariant.

String amplitudes:

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2)\,,$$

The S-matrix elements:

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2)\}$$

The effect of Z_i can be taken care of. Witten

The effect of mass renormalization is more subtle.

$\Rightarrow$ String amplitudes compute S-matrix elements directly if $m_{i,p}^2 = m_i^2$ but not otherwise.

– Includes BPS states, massless gauge particles and all amplitudes at tree level.

Problem with vacuum shift

Example: In many compactifications of SO(32) heterotic string theory on Calabi-Yau 3-folds, one loop correction generates a Fayet-Ilioupoulos term.

Effect: Generate a potential of a charged scalar ϕ of the form

$$\mathbf{c}(\phi^*\phi - \mathbf{K}\,\mathbf{g_s}^2)^2$$

c, K: positive constants, $\mathbf{g_s}$: string coupling

Dine, Seiberg, Witten; Atick, Dixon, A.S.; Dine, Ichinose, Seiberg
Atick, A.S.; Witten; D'Hoker, Phong; Berkovits, Witten

Correct vacuum: $|\phi| = \mathbf{g_s}\sqrt{\mathbf{K}}$

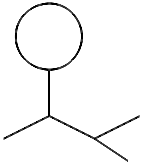
– not described by a world-sheet CFT

– conventional perturbation theory fails.

Even in absence of mass renormalization and vacuum shift we have to deal with infrared divergences in the integration over moduli space at intermediate stages.

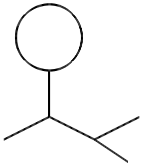
Witten

Consider a tadpole diagram in a QFT:



This diverges if a massless state propagates along the vertical propagator.

Solution: Do the loop integral first and hope that the result vanishes (possibly due to SUSY).



In string theory, this translates to a specific regularization procedure for integration over moduli spaces of Riemann surfaces.

1. Put an upper cut-off L on certain moduli corresponding to the Schwinger parameter of the vertical propagator.

2. Do integration over the other moduli first.

3. Then let L go to infinity.

Witten

How do we circumvent these difficulties / ad hoc prescription?

Go off-shell.

Off-shell amplitudes

Nelson; Zwiebach; Pius, Rudra, A.S.

1. Relax the constraint of conformal invariance on the vertex operators

– result will depend on the world-sheet metric around the punctures where the vertex operators are inserted.

2. Choose a local coordinate system w_i around the i-th puncture for each i and take the metric around the puncture $w_i = 0$ to be $|dw_i|^2$.

A different choice of the local coordinate system e.g. $y_i = f(w_i) \Rightarrow$ different metric $|dy_i|^2 = |f'(w_i)|^2 \, |dw_i|^2$
$\Rightarrow$ different off-shell amplitudes for the same external states.

For superstring theories we need insertion of picture changing operators (PCO) on the Riemann surface.

(Superstring refers to both heterotic and type II string theories)

Off-shell amplitudes depend not only on the choice of local coordinates at the punctures but also on the locations of the PCO's.

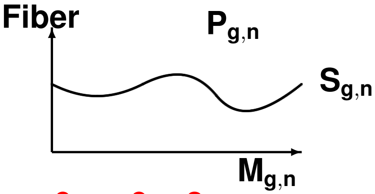
Are the physical quantities computed from off-shell amplitudes independent of the choice of local coordinates and PCO locations?

Some notations:

$M_{g,n}$: moduli space of genus g Riemann surfaces with n punctures.

$P_{g,n}$: A fiber bundle with $M_{g,n}$ as the base and the choice of local coordinates at punctures and PCO locations as fibers (infinite dimensional).

A choice of local coordinate system and PCO locations corresponds to a section $S_{g,n}$ of this fiber bundle.



Dimension of $S_{g,n} = 6g - 6 + 2n$.

General procedure for constructing an off-shell amplitude

For a given set of external off-shell states collectively called ϕ, construct p-forms $\omega_p(\phi)$ on $P_{g,n}$ satisfying

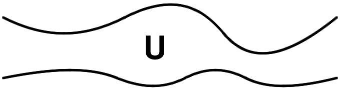
$$\omega_p(\sum_i Q_B^{(i)}\phi) \propto d\omega_{p-1}$$

$Q_B^{(i)}$: BRST charge acting on i-th state

ω_p is constructed from approprate correlation functions of off-shell vertex operators and ghost insertions on the Riemann surface.

Genus g, n-point amplitude

$$\int_{S_{g,n}} \omega_{6g-6+2n}$$



If U is the region bounded by the two sections then the difference in the integral over the two sections is

$$\int_{\mathbf{U}} \mathbf{d}\omega_{\mathbf{6g-6+2n}} \propto \int_{\mathbf{U}} \omega_{\mathbf{6g-5+2n}} \Big(\sum_{\mathbf{i}} \mathbf{Q_B^{(i)}} \phi\Big)$$

– vanishes for on-shell states for which $\mathbf{Q_B^{(i)}} |\phi\rangle = \mathbf{0}$.

However it does not vanish for off-shell states.

$\Rightarrow$ the off-shell amplitudes depend on the choice of the local coordinates at the punctures and PCO locations.

Questions

1. Are the renormalized masses and S-matrix elements independent of the choice of the section $S_{g,n}$?

2. Can we use this formalism to describe string perturbation theory around the shifted vacuum when vacuum shift is necessary?

3. Are the renormalized masses and S-matrix elements at the shifted vacuum independent of the choice of $S_{g,n}$?

We address these questions for a restricted class of sections $S_{g,n}$ which we shall call gluing compatible sections.

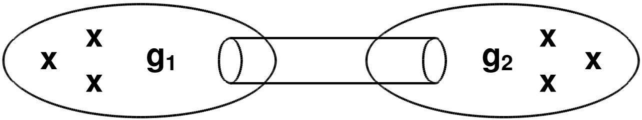
Consider a genus g_1, m-punctured Riemann surface and a genus g_2, n-punctured Riemann surface.

Take one puncture from each of them, and let w_1, w_2 be the local coordinates around the punctures at $w_1 = 0$ and $w_2 = 0$.

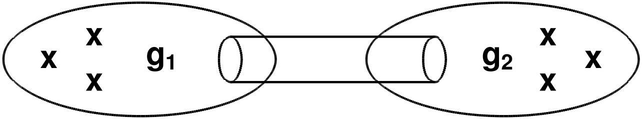
Glue them via the identification (plumbing fixture)

$$w_1 w_2 = e^{-s+i\theta}, \quad 0 \leq s < \infty, \quad 0 \leq \theta < 2\pi$$

– gives a family of new Riemann surfaces of genus $g_1 + g_2$ with (m+n-2) punctures.



Gluing compatibility: Choice of local coordinates at the punctures and the PCO locations on the genus $g_1 + g_2$ Riemann surface must agree with the one induced from the local coordinates at the punctures and PCO locations on the original Riemann surfaces.



Gluing compatibility allows us to divide the contributions to off-shell Green's functions into <u>1-particle reducible</u> (1PR) and <u>1-particle irreducible (1PI)</u> contributions.

Two Riemann surfaces joined by plumbing fixture

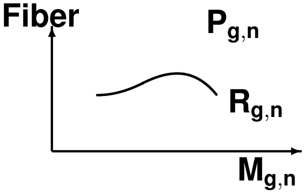
$\Updownarrow$

Two amplitudes joined by a propagator

Riemann surfaces which <u>cannot</u> be obtained by plumbing fixture of other Riemann surfaces contribute to <u>1PI</u> amplitudes.

1PI amplitudes do not include degenerate Riemann surfaces and hence are free from poles.

Put another way, for a gluing compatible choice of sections, we can identify a subspace $R_{g,n}$ of the full section $S_{g,n}$ which we can call the 1PI subspace.



All the Riemann surfaces corresponding to the full section $S_{g,n}$ are given by the Riemann surfaces in $R_{g,n}$ and their plumbing fixture in all possible ways.

Once this division has been made, we can define the 1PI amplitudes as

$$\int_{R_{g,n}} \omega_{6g-6+2n}$$

Generating function of these amplitudes is 1PI effective action.

<u>Tree amplitudes</u> computed from 1PI action

= full off-shell string amplitude including loop corrections, given by integrals over the whole section $S_{g,n}$

One finds that this action automatically has infinite dimensional gauge invariance.

– includes general coordinate transformation, local supersymmetry etc.

What is the effect of using a different gluing compatible section $S_{g,n}$?

$\Rightarrow$ a different choice of $R_{g,n}$ and hence different 1PI action

Result: 1PI effective action for different choices of $R_{g,n}$ are related to each other by <u>field redefinition</u>

– generalizes old result of Hata and Zwiebach in string field theory.

$\Rightarrow$ renormalized masses and S-matrix elements remain invariant under this change.

Furthermore in the 1PI effective string field theory we always have to first determine the vacuum by solving classical equations of motion, and then do perturbation expansion around the vacuum.

As a result the perturbation expansion is free from any IR divergence associated with tadpoles.

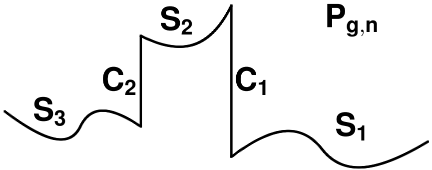
– no need to regulate infrared divergences even at intermediate stages of the calculation.

– perfectly suitable for dealing with the vacuum shift.

Some technical issues

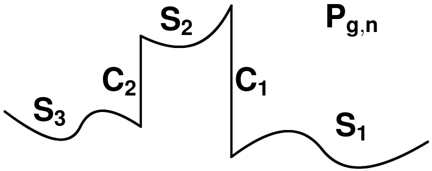
1. It is not always possible to choose $S_{g,n}$ and $R_{g,n}$ to be regular sections.

They contain vertical segments.



– location of one of the PCO's jump over a codimension one subspace of the base $M_{g,n}$.

– needed to avoid 'spurious singularities'.



Since the differential form ω_p is defined over the whole of $P_{g,n}$ there is no difficulty in integrating ω_p over these vertical segments.

2. For Ramond sector states it is not possible to write down an action with local kinetic term.

We can only write down the equation of motion.

– related to the fact that Ramond sector states carry picture number $-1/2$ and the inner product between two such states vanish by ghost number conservation.

For a string field theory this would be problematic since we would not know how to quantize the theory.

However for 1PI theory this is not a problem since we only need to work at the tree level.

General structure:

A general string field configuration corresponds to a state $|\Psi\rangle$ of ghost number 2 and picture number $(-1,-1/2)$ in (NS,R) sector.

1PI equation of motion:

$$Q_B|\psi\rangle + \sum_{n=1}^{\infty} \frac{1}{(n-1)!} G[\psi^{n-1}] = 0$$

Q_B: BRST operator

G: identity in NS sector
$\oint z^{-1} dz\, X(z)$ in R sector   X(z): PCO

Erler, Konopka, Sachs

$$\langle\chi|c_0^-|[A_1 A_2 \cdots A_{n-1}]\rangle \equiv \sum_g {g_s}^{2g} \int_{R_{g,n}} \omega_{6g-6+2n}(|\chi\rangle, |A_1\rangle, \cdots |A_{n-1}\rangle)$$

for any state $|\chi\rangle$.

Gauge transformations

The infinitesimal gauge transformation parameters correspond to states $|\lambda\rangle$ of ghost number 1 and picture number $(-1, -1/2)$ in (NS,R) sector.

Gauge transformation law

$$\delta|\psi\rangle = \mathbf{Q_B}|\lambda\rangle + \sum_{\mathbf{n}=\mathbf{0}}^{\infty} \frac{\mathbf{1}}{\mathbf{n}!}\mathbf{G}[\psi^{\mathbf{n}}\,\lambda]$$

Future prospects

1. Demand of infinite dimensional gauge invariance more or less fixes the perturbative scattering amplitude

– integral over the full integration cycle $S_{g,n}$.

Could it constrain the structure of non-perturbative corrections to the 1PI effective action?

2. Can we use the off-shell action to study string theory in weak RR background field perturbatively?