An Inverse Scattering Construction of a Fuzzball

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ISM Puri 2014

Last sentence of my ISM 2012 talk was: it will be nice if in ISM 2014 I can report of the inverse scattering construction of the JMaRT fuzzball....

Outline

Motivation

Our Formalism

Inverse Scattering JMaRT

Future

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- Inverse scattering is the best developed technique for constructing novel solutions of vacuum gravity theories.

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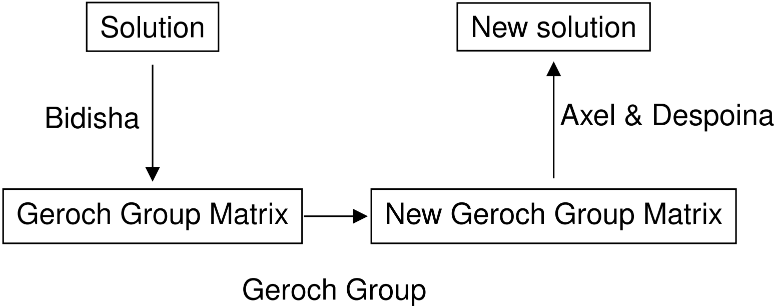
- Some three years back it was not clear how to apply such techniques to supergravities.
- Fuzzball proposal of Samir Mathur asks for certain smooth geometries.
- Only a very few fuzzballs are known for non-extremal black holes.

Summary of formalism (4 JHEP papers)



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- **Motivation**
- Our Formalism
- Inverse Scattering JMaRT
- Future

Motivation

- Dimensionally reduced gravity theories have large U-duality groups
- These symmetries have been used to study black holes.
- In many situations these symmetries are infinite dimensional.
  - Black holes/fuzzballs in 4d (5d) have 2 (3) commuting Killing vectors. Thus we have access to symmetries of theories reduced to 2d, which are infinite dimensional.

Motivation

- We want to understand and make use of these symmetries.
- Using these symmetries one can arrive at inverse scattering techniques for supergravities.
- I will review our formalism, and present JMaRT construction.

Outline

Geroch Group

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  - 5d gravity: affine SL(3)
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  - STU model: affine SO(4,4)
- Most natural way to think about affine $\mathfrak{sl}(2,\mathbb{R})$ is to think about two non-commuting $\mathfrak{sl}(2,\mathbb{R})$.

Ehlers and Matzner–Misner

- 4d metric
$$ds^2 = -e^{-\phi}(dt + \mathcal{A})^2 + e^{\phi} ds_3^2$$
with $\mathcal{A}_{(1)}$ is one form and ϕ the 3d dilaton. Define $d\chi = \star_3(e^{-2\phi}\mathcal{F})$. Axion-Dilaton (ϕ, χ) SL(2)/SO(2). Ehlers SL(2).

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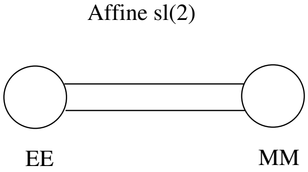
- Rewrite 4d metric
$$ds^2 = e^{2\nu}(d\rho^2 + dz^2) + g_{ab} dx^a dx^b, \tag{1}$$
use Einstein equations to impose
$$\det g_{ab} = -\rho^2.$$
By scaling by ρ and using the signature $\{-1, 1\}$, we construct a representative of SL(2)/SO(1,1). Matzner-Misner SL(2).

Dynkin Diagram

These SL(2)'s do not commute. They form affine SL(2).



[Geroch 1971, Julia 1980]

Key elements I, Ehlers symmetry

- Consider 3d Euclidean gravity-matter system G/K. Let V be the representative of G/K.

Key elements II, 2d reduction

- Now if the system admits an axial isometry ∂_ϕ we reduce the metric

$$ds_3^2 = f^2 ds_2^2 + \rho^2 d\phi^2. \tag{2}$$

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- We seek $\mathcal{V}(t)$—representative of the affine group—such that

$$\lim_{t\to 0} \mathcal{V}(t) = V$$

  that solves the Lax pair.

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- t: spacetime dependent spectral parameter.
  w: spacetime independent spectral parameter.

Key elements IV, monodromy

- The involution T extends to functions $\mathcal{V}(t)$ by

$$(\mathcal{V}(t))^T = \mathcal{V}^T\left(-\frac{1}{t}\right). \tag{4}$$

and one defines

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$$\partial_\pm \mathcal{M}(t,x) = 0 \Longrightarrow \mathcal{M}(t,x) = \mathcal{M}(w). \tag{6}$$

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- Under group transformations $\mathcal{M}(w)$ transforms as

$$\mathcal{M}(w) \to \mathcal{M}^g(w) = g^T(w)\mathcal{M}(w)g(w). \quad (7)$$

Key elements, summary

- Group action

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Main Technical Result (Katsimpouri, Kleinschmidt, AV)

We solve this Riemann-Hilbert problem for SL(2) matrices with simple poles in w with residues of rank one.

Riemann-Hilbert Factorization

Let us look at symmetric SL(2) matrices with simple poles in w with residues of rank one

$$\mathcal{M}(w) = \mathbb{1} + \sum_{i=1}^{N} \frac{A_i}{w - w_i}, \tag{9}$$

with

$$A_i = a_i \alpha_i a_i^T, \tag{10}$$

Ansatz for $\mathcal{V}(t,x)$

The above $\mathcal{M}(w)$ justifies the ansatz ($t_i = t(w_i)$)

$$\mathcal{V}(t,x) = V(x)A_+(t,x). \tag{11}$$

$$A_+(t) = \mathbb{1} - \sum_i \frac{c_i t a_i^T}{1+tt_i}. \tag{12}$$

Riemann Hilbert Factorization, group G

SL(2)	G
solitons: w_k, $k = 1, \ldots, N$	solitons: w_k, $k = 1, \ldots, N$
vectors: a_k	vectors: a_k^{α}, $\alpha = 1, \ldots, r$
matrix: Γ_{kl}	matrix: $\Gamma_{kl}^{\alpha\beta}$

Everything boosted up to rN dimensional space.

Example: Kerr solution in STU gravity

$$\mathcal{M}(w) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+\frac{2m(m-w)}{w^2-c^2} & 0 & 0 & 0 & 0 & \frac{2am}{w^2-c^2} \\ 0 & 0 & 0 & 1+\frac{2m(m-w)}{w^2-c^2} & 0 & 0 & -\frac{2am}{w^2-c^2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2am}{w^2-c^2} & 0 & 0 & 1+\frac{2m(m+w)}{w^2-c^2} & 0 \\ 0 & 0 & \frac{2am}{w^2-c^2} & 0 & 0 & 0 & 0 & 1+\frac{2m(m+w)}{w^2-c^2} \end{pmatrix}$$

STU supergravity

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Simple poles. $w = \pm c = \pm\sqrt{m^2 - a^2}$. Rank of residue matrices in two! And need to take into account SO(4,4) group structure.

Outline

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- JMaRT is the best understood non-supersymmetric fuzzball Jejjala, Madden, Ross, Titchener

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- Microscopic description known as a state in the D1-D5 CFT
- Obtained by studying over-rotating limit of 5d Cvetic-Youm metrics
- Extensively studied in the Fuzzball literature Chowdhry, Avery, Mathur et al

Single rotation, 2 charges

$$
\begin{aligned}
ds^2 = \;& \ldots \\
& \left[-f_s(dt - f_s^{-1} M c_1 c_5 a_1 \cos^2\theta d\phi)^2 + f(dy + f^{-1} M s_1 s_5 a_1 \sin^2\theta d\psi)^2\right] \\
& \ldots
\end{aligned}
$$

- $\{t, \phi\}$ and $\{y, \psi\}$ terms have similar structure
- When charges go to zero, we get over-rotating Myers-Perry

A discrete symmetry

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\delta_i &\rightarrow i\frac{\pi}{2} - \delta_i \\
a_1 &\rightarrow -ia_1 \\
M &\rightarrow -M \\
r^2 &\rightarrow r^2 - M + a_1^2
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A new view on JMaRT

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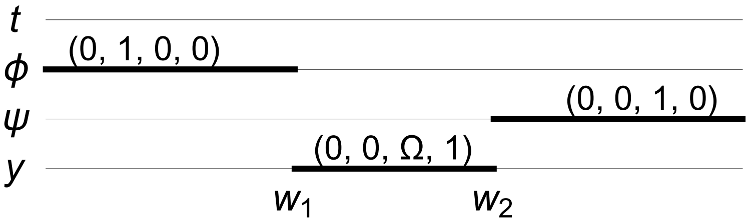
- There is no under-rotating version of the instanton.
- Systematic construction using inverse scattering easier.
- In STU theory (SO(4,4) group) we construct MP instanton and then do appropriate charging.

A bit more detail, rod diagrams

The rod diagram (interval structure) of JMaRT is like



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- We have given special emphasis on the SO(4,4) theory, and have obtained the 4-charge rotating black hole as an example.
- ...and also the JMaRT fuzzball.

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Thanks for your attention.