BCS Instability and Finite Temperature Corrections to Tachyon Mass in intersecting D-branes

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S.P.C., B. Sathiapalan , S. Sarkar, JHEP, 2014
S.P.C, S. Sarkar, ongoing

ISM, 2014, Puri.

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PLAN

- MOTIVATION: Sakai Sugimoto model...BCS instability and tachyonic instability
- INTERSECTING D1-BRANES: A simpler problem with relevant features
- TACHYONIC INSTABILITY: the zero temperature spectrum
- FINITE TEMPERATURE CORRECTIONS: two-point functions, UV, IR divergences
- CONCLUSION

MOTIVATION: Sakai-Sugimoto

- Sakai-Sugimoto model: A string theoretic model for Quantum Chromodynamics (QCD) giving a holographic description.
- Construction:
- A background consisting of N_c number of overlapping $D4$-branes compactified on an S^1(Witten,'98).
- Imposing anti-periodic boundary conditions on the S^1 breaks Supersymmetry. Scalars and fermions are massive at 1-loop and low energy theory is $SU(N_c)$ pure YM. (boundary)
- Insertion of N_f number of probe $D8$ and $\bar{D}8$-branes on the background (transverse to the S^1)(bulk).

MOTIVATION: Chiral symmetry breaking in SS-model

- The SS-model demonstrates Chiral Symmetry Breaking geometrically.
- Holography: $U(N_f)_L \times U(N_f)_R$ symmetry of QCD $=$ gauge symmetry of the N_f number of $D8 - \bar{D}8$ pairs in the bulk.
- There is an upper cutoff for the radial direction(SUGRA) to the S^1 transverse to the $D8 - \bar{D}8$. As the radial coordinate approaches this cutoff the size of the S^1 shrinks.
- $D8 - \bar{D}8$-pair merges into $D8$-brane,
- $U(N_f)_L \times U(N_f)_R \to U(N_f)_L$.

MOTIVATION

- In conventional QCD The Nambu, Jona-Lasinio-model of chiral symmetry breaking elucidates certain apparent similarities between chiral symmetry breaking and the BCS instability in superconductors.
- Inspired by this similarity, a holographic model of BCS superconductivity has been proposed within the broken chiral symmetric scenario in the Sakai Sugimoto model.(N. Sarkar, S. Sarkar, B. Sathiapalan, K. Rama)
- proposal: BCS instability (Cooper pairing between Baryons) in the boundary($D4$ wrapped on S^1) corresponds to tachyonic instability in the bulk ($D8$).

MOTIVATION: INTERSECTING $D8$-BRANES

- The formation of Cooper pairs in the boundary: introduce a finite Baryon number density on the boundary theory i.e. a Chemical Potential for Baryon number.
- How?: A point source of Baryon number in the bulk which creates a cusp singularity in the bulk. For two $D8$-branes, $SU(2)$ is broken and the branes intersect at one angle between them.(Bergman, Lifschytz, Lippert)
- In the SS-model a configuration of two intersecting $D8$-branes were found to have a tachyonic instability in the bulk spectrum which is proposed to correspond to Cooper pairing instability in the boundary theory.(B. Sathiapalan, et.al.)

INTERSECTING $D8$-BRANES

- The tachyon mode is identified as the lowest mode in the open string excitation between the intersecting branes. (B Sathiapalan et. al., K. Hashimoto & Nagaoka, A. Hashimoto & Taylor)
- There is a stable minimum in the presence of electric field.(B.Sathiapalan et.al.)
- Another way of stabilizing: Finite temperature field theory.
- Computation: Finite temperature one-loop mass-squared corrections to the tree-level tachyon.
- Finite temperature effects : Existence of T_c at which the effective mass-squared of the tachyon vanish. Our main goal is to calculate the T_c.

INTERSECTING $D8$-BRANES

- However this problem is difficult to handle in the case of $D8$-branes on a curved $D4$-background. But many of the technical features are captured by a much simpler set-up consisting of two intersecting $D1$-branes on a flat background.

- We choose to study the finite temperature effects in this simpler set-up. We are able to do so because the tachyon dynamics is a local phenomenon and not influenced significantly by curvature effects.

Validity

- The low energy theory on the brane can be described by the DBI action for the massless fields on the brane. This is valid as long as only energies $<< \frac{1}{\alpha'}$ are being probed.
- We can study this as a quantum theory with a cutoff $\Lambda < \frac{1}{\sqrt{\alpha}'}$ and proceed to study the corrections due to the massless mode quantum *and* thermal fluctuations.
- The Yang-Mills action (in $D \leq 3+1$) is finite.
- Thermal corrections should be unambiguously finite.
- Supersymmetry ensures finiteness.

INTERSECTING Dp-BRANES

- Consider two Dp-branes: world-volume:
  $S_{p+1} =$
  $\frac{1}{g_{YM}^2}\text{tr}\int d^px\left[-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D_\mu\Phi_I D^\mu\Phi_I + \frac{1}{2}\left[\Phi_I,\Phi_J\right]^2\right] +$
  Fermions
- $a = \{1,2,3\}= SU(2)$ gauge index,
- $I = 1,\cdots,8=$ transverse directions.
- Background : $\langle\Phi_1^3\rangle = qx$, separation between branes;
- x : world-volume. (Hashimoto, Nagaoka, D.Lust et al, etc.)
- Slope: $q = \left(\frac{1}{\pi\alpha'}\right)\tan(\frac{\theta}{2})$.
- $q = 0$: Coincident branes.
- Putting $A_0^a = 0$ removes ghosts.

INTERSECTING Dp-BRANES: SPECTRUM OF BOSONS

- The background fields : $\{(\Phi^1_{1B}, A^2_{xB}); (\Phi^2_{1B}, A^1_{xB})\}$. The Lagrangian for the background fields decouple into two pieces, one for each of these doublets.
- Define bosonic doublets ($\tau = it$).

$$\zeta(x,\tau) = \left(\begin{array}{c} A^2_{xB}(x,\tau) \\ \Phi^1_{1B}(x,\tau) \end{array} \right), \qquad \zeta^{'}(x,\tau) = \left(\begin{array}{c} A^1_{xB}(x,\tau) \\ \Phi^2_{1B}(x,\tau) \end{array} \right)$$

- In each doublet the fields satisfy a set of coupled differential equations.
- There are two sectors of solutions: $m_n^2 = (2n-1)\frac{q}{g^2_{YM}}$, $m_n^2 = 0$. Two different sets of normalized eigenfunctions for each of these doublet fields.

INTERSECTING Dp-BRANES: SPECTRUM OF BOSONS

- $(A_x^2, \Phi_1^1), (A_x^1, \Phi_1^2)$:
- Normalized Eigenfunctions: $m_n^2 = \frac{(2n-1)q}{g_{YM}^2}$

$$A_n(x) = \pm \mathcal{N} e^{-\frac{qx^2}{2}} \left(H_n(\sqrt{q}x) + 2nH_{n-2}(\sqrt{q}x)\right) e^{ik_\alpha x^\alpha},$$
$$\Phi_n(x) = \mathcal{N} e^{-\frac{qx^2}{2}} \left(H_n(\sqrt{q}x) - 2nH_{n-2}(\sqrt{q}x)\right) e^{ik_\alpha x^\alpha},$$
$$n \neq 1.$$

Normalized Eigenfunctions: $m_n^2 = 0$

$$\tilde{A}_n(x) = \pm \mathcal{N}' e^{-\frac{qx^2}{2}} \left(H_n(\sqrt{q}x) - 2(n-1)H_{n-2}(\sqrt{q}x)\right) e^{ik_\alpha x^\alpha},$$
$$\tilde{\Phi}_n(x) = \tilde{\mathcal{N}} e^{-\frac{qx^2}{2}} \left(H_n(\sqrt{q}x) + 2(n-1)H_{n-2}(\sqrt{q}x)\right) e^{ik_\alpha x^\alpha},$$
$$n \neq 0.$$

INTERSECTING Dp-BRANES:SPECTRUM OF BOSONS

- We turn on all the other fields as fluctuations. For the other bosonic fields:

$$\Phi_{In}^{1,2}(x) = \mathcal{N}^{1,2} e^{-\frac{qx^2}{2}} H_n(\sqrt{q}x) e^{ik_\alpha x^\alpha}, \quad m_n^2 = (2n+1)\frac{q}{g_{YM}^2}$$

  where $I \neq 1$

- The third gauge components of all fields are massless
- $\{A_x^3, A_\alpha^3, \Phi_I^3\} \sim e^{ik.x}$

INTERSECTING $D1$-BRANES:FERMIONS

- The fermions in the picture play a crucial role in ensuring the UV finiteness of one-loop computations.
- We shall restrict our discussion to only $D1$-branes now. We have a complete calculation for this case. For $D2$ and $D3$-branes the work is still in progress.
- The fermions: sixteen left and sixteen right moving Majorana-Weyl fermions, grouped into two different sets of eight pairs distinguished by their e.o.m.

$$(\partial_0 + \partial_x)L \pm qxR = 0$$
$$(-\partial_0 + \partial_x)R \pm qxL = 0$$

INTERSECTING $D1$-BRANES:SPECTRUM:FERMIONS

- The Eigenfunctions for the Fermions: $m_n = \pm\sqrt{2nq}$

$$L_n = \mathcal{N}_f e^{-\frac{qx^2}{2}} \left(-\frac{i}{\sqrt{2n}} H_n(\sqrt{q}x) + H_{n-1}(\sqrt{q}x) \right)$$
$$R_n = \pm\mathcal{N}_f e^{-\frac{qx^2}{2}} \left(-\frac{i}{\sqrt{2n}} H_n(\sqrt{q}x) - H_{n-1}(\sqrt{q}x) \right)$$

  and their complex conjugates.
- L_i^3 and R_i^3 are massless fermions(plane waves).

TACHYON INSTABILITY

- The bosonic doublets ζ_k are eigenvectors corresponding to the mass squared eigenvalue:

$$m_k^2 = \lambda_k = \frac{(2k-1)q}{g_{YM}^2}$$

where $k = 0$ corresponds to tachyonic modes.

FINITE TEMPERATURE CORRECTIONS

- Main Idea: We implement the background field method
- Fluctuations participate only at the level of loop.
- We use perturbation theory to construct the full spectrum for the fluctuations.
- Technical Difficulties:
- Harmonic Oscillator basis
- Bosonic amplitudes (two-point functions for tachyon): problem of IR + UV divergences
- IR div. occur due to massless fields in the loops

$$\text{UV div} \sim \sum_n \frac{1}{\sqrt{n}} \quad \text{arise from Quantum Corrections}(T=0)$$

- Fermionic amplitudes: problem of UV divergences

FINITE TEMPERATURE CORRECTIONS

- IR problem: A two-step Resolution
- Step 1: Calculate the finite T 1-loop mass-corrections for the massless fields namely, Φ_1^3, $\Phi_I^3, (I \neq 1)$ and A_x^3.
- $m_n^2 = 0$: Infinitely degenerate massless modes corresponding to the zero eigenvalue sector: diagonalized mass matrices as a function of temperature (numerically).
- Step 2: These temperature dependent masses modify the propagators in the tachyonic amplitudes.
- The tachyon two-point functions are computed self-consistently (numerical computation).

FINITE TEMPERATURE CORRECTIONS

- UV problem: for all fields.
- Finite T 1-loop bosonic and fermionic amplitudes: Each term is UV divergent.
- Sum over discrete momemtum n (fields coupled to the background are massive)
- integral over continuous momemtum (massless modes).
- Compute the integrals involved in the vertices and expand the sums over n about $n = \infty$: leading order $\frac{1}{\sqrt{n}}$.
- Cancellation between Bosonic and fermionic terms yeilds finite answer.

FINITE TEMPERATURE CORRECTIONS

- No divergence from temp-dependent part.
- One-loop corrections to the tachyon mass term: set all external momenta in the Feynman diagrams = 0 and integrate/sum over the loop momenta.
  One-loop diagrams:

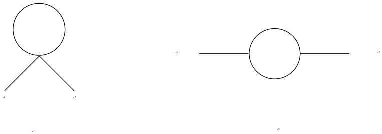


Figure : one-loop bosons

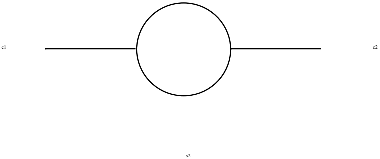


Figure : one-loop fermions

FINITE TEMPERATURE CORRECTIONS

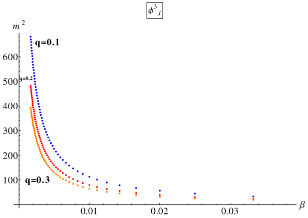
- What do we expect?: Tachyon: tree-level mass squared $= -\frac{q}{g_{YM}^2}$.
- Corrections: Quantum Corrections$(T=0)$ + Thermal Corrections $(T \neq 0)$.
- Expand the finite temperature integrands and summands about $\beta = 0$: Leading order behaviour is given by $\frac{1}{\beta\sqrt{q}}$.
- The parameter q provides a scale for supersymmetry breaking. The effective mass of the tree-level tachyon

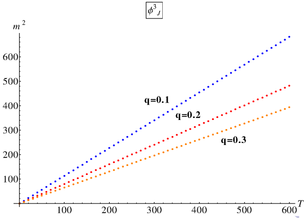
$$m^2(q,T) = -\frac{q}{g_{YM}^2} + \left(m_0^2 + \frac{T}{\sqrt{q}} \left(\sum_n \frac{1}{\sqrt{\lambda_n}} + \cdots \right) + \mathcal{O}\left(\frac{g_{YM}^2}{q}\right) \right)$$

- m_0^2: Quantum corrections $(T=0)$. Only true for $1+1$-dimensions.

FINITE TEMPERATURE CORRECTIONS (MASSLESS FIELDS)

Sample plot for massless field : Φ^3_I, $\qquad m_0^2 = 1.6 g^2_{YM}$





FINITE TEMPERATURE CORRECTIONS

- Numerical Plot: $m^2(q,T)$ vs T, $|g^2_{YM}| = 1/100$.

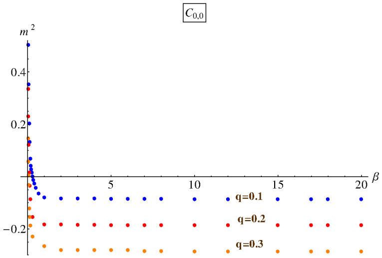


Figure : Mass-squared Vs β

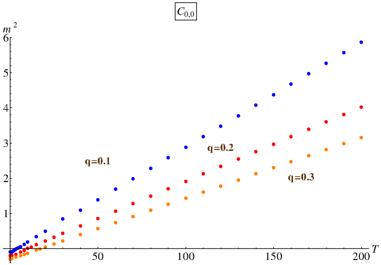


Figure : Mass-squared Vs T

FINITE TEMPERATURE CORRECTIONS

- $$T_c = \sum_n \left(\frac{1}{\sqrt{\lambda_n}} + \cdots \right) \left(\sqrt{q} \left(\frac{q}{g_{YM}^2} - \tilde{m}_0^2 \right) \right) \qquad (0.1)$$

  $\tilde{m}_0^2 = 1.6$ is the dimensionless zero temperature quantum correction.

q	T_c (leading order analytical)	T_c (numerical)
0.1	3.34	3.38
0.2	9.48	9.51
0.3	16.73	16.79

Table : Comparing between analytical and numerical values of T_c

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CONCLUSION

- The finite temperature effects remove the tachyon instability in intersecting $D1$-branes and stabilize the configuration.
- The effective mass-squared of the tree-level tachyon grows linearly with temperature as expected in $(1+1)$-dimensions.
- The zero temperature quantum corrections are independent of the parameter q $(1+1$-dim.$)$.
- At finite temperature the superconducting instability transits into a stable normal phase.
- This phenomenon bears the hallmark of a phase transition.

FUTURE DIRECTIONS

- To do the full stability analysis we must compute the full finite temperature effective action for the tachyon, which calls for computing higher point functions.
- Our results can be generalized to higher dimensional branes($D2$ and $D3$) without much difficulty. It will be interesting to study the issue of phase transition in higher dimensions. (ongoing)
- By scaling arguments(scaling the integrals by powers of β) we see that the finite temperature bhaviour in $p+1$-dims $(p>1)$is T^{p-1}.
- Question of adding α'-corrections in the loop may be interesting.
- Open string world-sheet perspective : calculating the annular amplitudes at finite T.

THANK YOU!

- The one-loop corrections from the bosonic diagrams with 4-pont vertex

$$\begin{aligned}
&\Sigma^1(w, w', k, k', \beta, q) \\
= \quad & \frac{1}{2} N \sum_m \left[\sum_n \left(\frac{F_1(k, k', n, n)}{\omega_m^2 + \lambda_n} + \frac{\tilde{F}_1(k, k', n, n)}{\omega_m^2} + \frac{7F_2(k, k', n, n)}{\omega_m^2 + \gamma_n} \right) \right. \\
+ \quad & \int \frac{dl}{(2\pi\sqrt{q})} \left(\frac{7F_2'(k, k', l, -l)}{\omega_m^2 + l^2} + \frac{F_3'(k, k', l, -l)}{\omega_m^2 + l^2} \right) \\
+ \quad & \left. \int \frac{dl}{2\pi\sqrt{q}} \frac{F_3(k, k', l, -l)}{\omega_m^2} \right] \delta_{w+w'}
\end{aligned} \tag{0.2}$$

- where F's denote the four point vertices in this expression.

$$V_i^4 = -\frac{N}{g^2} \mathcal{F}_i^4(k, k', n/l, n'/l') \delta_{w+w'+m+m'} \tag{0.3}$$

$N = \sqrt{q}/\beta$.

- The one-loop corrections from the bosonic diagrams with 3-pont vertex

$$\Sigma^2(w, w^{'}, k, k^{'}, \beta, q) = -\frac{1}{2} q N \sum_{m,n} \Bigg[\int \frac{dl}{2\pi\sqrt{q}} \frac{F_4(k,l,n) F_4^*(k^{'},l,n)}{(\omega_m^2 + \lambda_n)\omega^2_{m^{'}}}$$

$$+ \quad \int \frac{dl}{2\pi\sqrt{q}} \frac{\tilde{F}_4(k,l,n)\tilde{F}_4^*(k^{'},l,n)}{\omega_m^2 \omega^2_{m^{'}}}$$

$$+ \quad \int \frac{dl}{2\pi\sqrt{q}} \left(\frac{7F_5(k,l,n)F_5^*(k^{'},-l,n)}{(\omega_m^2 + \gamma_n)(\omega^2_{m^{'}} + l^2)} + \frac{F_5^{'}(k,l,n)F_5^{'*}(k^{'},-l,n)}{(\omega_m^2 + \lambda_n)(\omega^2_{m^{'}} + l^2)} \right)$$

$$+ \quad \int \frac{dl}{2\pi\sqrt{q}} \frac{\tilde{F}_5^{'}(k,l,n)\tilde{F}_5^{'*}(k^{'},-l,n)}{(\omega_m^2)(\omega^2_{m^{'}} + l^2)} \Bigg] \delta_{w+w^{'}} \tag{0.4}$$

$$V_i^3 = -\frac{N^{\frac{3}{2}}}{g^2} \mathcal{F}_i^3(k, k^{'}, n/l, n^{'}/l^{'}) \delta_{w+w^{'}+m+m^{'}} \tag{0.5}$$

- After performing the Matsubara sums, the mass correction for the four-point vertices become

$$
\begin{aligned}
\Sigma^1(k,k^{'},\beta,q) &= \frac{1}{2}\Bigg[\sum_n \frac{F_1(k,k^{'},n,n)}{\sqrt{(2n-1)}}\left(\frac{1}{2}+\frac{1}{e^{\beta\sqrt{(2n-1)q}}-1}\right) \\
&+ \sum_m \left(\frac{\tilde{F}_1(k,k^{'},n,n)}{\omega_m^2}+\int\frac{dl}{2\pi\sqrt{q}}\frac{F_3(k,k^{'},l-l)}{\omega_m^2}\right) \\
&+ \sum_n \left(\frac{7F_2(k,k^{'},n,n)}{\sqrt{(2n+1)}}\left(\frac{1}{2}+\frac{1}{e^{\beta\sqrt{(2n+1)q}}-1}\right)\right) \\
&+ \left(\int\frac{dl}{(2\pi\sqrt{q})}\frac{15N}{2l^2}\left((\beta l/2)\coth(\beta l/2)-1\right)\right)\Bigg] \qquad (0.6)
\end{aligned}
$$

The mass correction for the three-point vertices after the Matsubara sum assumes the form

$$
\begin{aligned}
&\Sigma^2(k, k^{'}, \beta, q)\\
=\quad & -\frac{1}{2}\sum_n \left[\int \frac{dl}{2\pi\sqrt{q}} \frac{F_4(k,l,n)F_4^*(k^{'},l,n)}{2n-1}\left[\left(\frac{\sqrt{q}}{\beta\omega_m^2} - \frac{1}{\sqrt{2n-1}}\left(\frac{1}{2} + \frac{1}{e^{\sqrt{(2n-1)q}\beta}-1}\right)\right)\right]\right.\\
+\quad & \int \frac{dl}{2\pi\sqrt{q}} \sum_m \frac{\tilde{F}_4(k,l,n)\tilde{F}_4^*(k^{'},l,n)}{\omega_m^4}\\
+\quad & \int \frac{dl}{2\pi\sqrt{q}} \left[\frac{7F_5(k,l,n)F_5^*(k^{'},-l,n)}{l^2-(2n+1)q}\left(\frac{1}{\sqrt{2n+1}}\left(\frac{1}{2} + \frac{1}{e^{\sqrt{(2n+1)q}\beta}-1}\right)\right.\right.\\
-\quad & \left.\left.\frac{1}{l}\left(\frac{1}{2} + \frac{1}{e^{l\beta}-1}\right)\right)\right]\\
+\quad & \int \frac{dl}{2\pi\sqrt{q}} \left[\frac{F_5^{'}(k,l,n)F_5^{'*}(k^{'},-l,n)}{l^2-(2n-1)q}\left(\frac{1}{\sqrt{2n-1}}\left(\frac{1}{2} + \frac{1}{e^{\sqrt{(2n-1)q}\beta}-1}\right)\right.\right.\\
-\quad & \left.\left.\frac{1}{l}\left(\frac{1}{2} + \frac{1}{e^{l\beta}-1}\right)\right)\right]\\
+\quad & \left.\int \frac{dl}{2\pi\sqrt{q}} \frac{\tilde{F}_5^{'}(k,l,n)\tilde{F}_5^{'*}(k^{'},-l,n)}{l^2}\left(\frac{\sqrt{q}}{\beta\omega_{m^{'}}^2} - \frac{1}{l}\left(\frac{1}{2} + \frac{1}{e^{l\beta}-1}\right)\right)\right]\delta_{w+w^{'}}
\end{aligned}
\tag{0.7}
$$

- The fermionic corrections are accompanied with diagrams with only 3-point vertices.

$$
\begin{aligned}
\Sigma^3(w, w^{'}, k, k^{'}, \beta, q) =&(8N) \sum_{n,m,m^{'}} \int \frac{dl}{2\pi\sqrt{q}} \frac{1}{(i\omega_m + \sqrt{\lambda^{'}_n})} \\
&\times [\frac{F_6^R(k,n,l)F_6^{R*}(k^{'},n,l)}{(i\omega_m + l)} + \frac{F_6^L(k,n,l)F_6^{L*}(k^{'},n,l)}{(i\omega_m - l)} \\
&+ \frac{F_7^L(k,n,l)F_7^{L*}(k^{'},n,l)}{(i\omega_m + l)} + \frac{F_7^R(k,n,l)F_7^{R*}(k^{'},n,l)}{(i\omega_m - l)} \\
&+ \frac{F_6^R(k,n,l)F_7^{L*}(k^{'},n,-l)}{(i\omega_m + l)} + \frac{F_7^L(k,n,l)F_6^{R*}(k^{'},n,-l)}{(i\omega_m - l)} \\
&+ \frac{F_7^R(k,n,l)F_6^{L*}(k^{'},n,-l)}{(i\omega_m + l)} + \frac{F_6^L(k,n,l)F_7^{R*}(k^{'},n,-l)}{(i\omega_m - l)}]\delta_{w+w^{'}}
\end{aligned}
$$

where $\omega_m = \frac{(2m+1)\pi}{\beta}$

- The fermionic corrections are accompanied with diagrams with only 3-point vertices.

$$
\begin{aligned}
&\Sigma^3(w,w^{'},k,k^{'},\beta,q) = \\
&(8N)\sum_n\left[\int\frac{dl}{2\pi\sqrt{q}}\left(\frac{-\beta\tanh\left(\frac{\beta l}{2}\right)+\beta\tanh\left(\frac{1}{2}\beta\sqrt{2nq}\right)}{2\left(l-\sqrt{2nq}\right)}\right)\right. \\
&\left[(F_6^R(k,n,l)F_6^{R*}(k^{'},n,l)+F_7^L(k,n,l)F_7^{L*}(k^{'},n,l)\right. \\
&\left.+F_6^R(k,n,l)F_7^{L*}(k^{'},n,l)+F_6^{R*}(k,n,l)F_7^L(k^{'},n,l))\right] \\
+\quad&\int\frac{dl}{2\pi\sqrt{q}}\left(\frac{-\beta\tanh\left(\frac{\beta l}{2}\right)-\beta\tanh\left(\frac{1}{2}\beta\sqrt{2nq}\right)}{2\left(l+\sqrt{2nq}\right)}\right) \\
&+\left[(F_6^L(k,n,l)F_6^{L*}(k^{'},n,l)+F_7^R(k,n,l)F_7^{R*}(k^{'},n,l)\right. \\
&\left.\left.+F_6^L(k,n,l)F_7^{R*}(k^{'},n,l)+F_6^{L*}(k,n,l)F_7^R(k^{'},n,l))\right]\right]\delta_{w+w^{'}} \qquad (0.8)
\end{aligned}
$$

$$\underbrace{\frac{1}{2}\sum_{m,n}\frac{1}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\lambda_n}+\frac{1}{2}\sum_{m,n}\frac{7\times 2}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\gamma_n}}_{\text{amplitudes for } F_1(0,0,n,n)+F_2(0,0,n,n)}+\underbrace{\sum_m\frac{1}{2\omega_m^2}}_{\text{from } \tilde{F}_1(0,0,n,n)}$$

$$-\underbrace{\int\frac{dl}{2\pi\sqrt{q}}\left(\sum_m\frac{1}{2\omega_m^2}+\frac{1}{2}\sum_{m,n}\frac{1}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\lambda_n}\right)}_{\text{from } \tilde{F}_4(0,l,n)}+\underbrace{\int\frac{dl}{2\pi\sqrt{q}}\sum_m\frac{1}{2\omega_m^2}}_{\text{from } F_3(0,0,l,-l)}$$

$$+\underbrace{\left(\frac{1}{2}(7)\int\frac{dl}{2\pi\sqrt{q}}\sum_m\frac{1}{\omega_m^2+l^2}+\frac{1}{2}\times\frac{1}{2}\int\frac{dl}{2\pi\sqrt{q}}\sum_m\frac{1}{\omega_m^2+l^2}\right)}_{\text{amplitudes for } F_2'(0,0,n,n)+F_3'(0,0,n,n)}$$

$$+\underbrace{\frac{1}{2}\times\frac{1}{2}\int\frac{dl}{2\pi\sqrt{q}}\sum_m\frac{1}{\omega_m^2+l^2}-\sum_m\frac{1}{2\omega_m^2}}_{\text{from } \tilde{F}_5'(0,l,n)}$$

$$-\underbrace{\left(\frac{1}{2}(7)\sum_{m,n}\frac{4}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\gamma_n}+\frac{1}{2}\sum_{m,n}\frac{4}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\lambda_n}\right)}_{\text{amplitudes for } F_5(0,l,n)+F_5'(0,l,n)} \quad (0.9)$$

- $$\underbrace{-\frac{1}{2}(16)\sum_{m,n}\frac{1}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\lambda_n'}}_{\text{1st term in } \Sigma^3(w,\,w,\,0,\,0,\,\beta,\,q)}\underbrace{-\frac{1}{2}(8)\int\frac{dl}{2\pi\sqrt{q}}\sum_{m}\frac{1}{\omega_m^2+l^2}}_{\text{2nd term in } \Sigma^3(w,\,w,\,0,\,0,\,\beta,\,q)}$$
$$\underbrace{+\frac{1}{2}(8)\sum_{m,n}\frac{4}{2\pi\sqrt{2n}}\frac{1}{\omega_m^2+\lambda_n'}}_{\text{3rd term in } \Sigma^3(w,\,w,\,0,\,0,\,\beta,\,q)} \tag{0.10}$$

- The asymptotic expansions of the one-loop corrections about $n=\infty$ gives the above terms. There is cancellation between the leading order Bosonic and Fermionic contributions.