Generalized Superconductors and Holographic Optics

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Introduction

- The strong-weak nature of the gauge-gravity duality can be and has been exploited to compute useful quantities in a strongly coupled field theory from relatively simpler calculations in its dual classical gravity theory. The correspondence has been successfully applied to gain useful insights into a number of fields like entanglement entropy ,hydrodynamics, superconductivity and so on.

- In this talk, we will discuss another direction of this duality: Optics

- Our primary objective here is use gauge/gravity duality to study the electro-magnetic response of strongly coupled superconducting media which appear as boundary theories of the AdS-Schwarzschild and R-charged black holes in 4d with full backreaction.

- This can be carried out via the computation of the retarded correlators of the theory and then extracting the response functions of the media from these correlators.

Earlier Works…

- This was first carried out by Policastro and his collaborators using gauge/gravity duality for boundary theory of 5-dimensional charged AdS black hole (JHEP04(2011)036).

- They were able to show that at small enough frequencies, the boundary media exhibited negative refraction.

- Work has been, later, generalized to the cases of 4d RN-AdS black hole [Ge, Jo, Sin], holographic superconductors in 5d. [Gao & Zhang, Amariti *et al*.] and for holographic superconductors in 4d in the probe limit [Dey *et al*.].

- In all these examples with the exception of the probe limit study on 5d holographic superconductors, negative refraction has been a generic feature of the strongly coupled media at small frequencies.

- This was quite interesting in light of the construction of a class of such negative refractive index materials (also known as metamaterials) in early 2000. By now large number of materials which support negative refraction are known.

Metamaterials

- Metamaterials are a class of artificially engineered materials in which phase velocity of an EM wave propagates in a direction opposite to the direction of to energy flux or poynting vector.

- The direction of phase velocity is determined by the sign of Re[n] and that of energy flow is determined by the sign of Re[n/μ].

- For Metamaterials

  Re[n] < 0 and Re[n/μ]>0 Should be satisfied simultaneously

- With $n^2 = \varepsilon\mu$

- These conditions lead to the following simple form for negative refraction

$$n_{DL} = \mathrm{Re}[\varepsilon]|\mu| + \mathrm{Re}[\mu]|\varepsilon| < 0$$

- Above condition is strictly based on the assumptions that Im[ε]>0 and Im[μ]>0. The condition Im[μ]<0 can occur in the probe limit.

Holographic set up

Einstein + Maxwell + scalar field action

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}m^2\tilde{\psi}^2-\frac{1}{2}\left|\partial\tilde{\psi}-iA\tilde{\psi}\right|^2\right)$$

Gubser, Hartnoll *et al*. (2008)

Writing the scalar field as $\tilde{\psi}=\psi e^{i\alpha(x)}$

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}m^2\psi^2-\frac{\psi^2}{2}(\partial\alpha-A)^2\right)$$

Gauge symmetry: $\begin{cases} A\rightarrow A+\partial\lambda \\ \alpha\rightarrow\alpha+\lambda \end{cases}$

Generalize the model in gauge invariant way (Franco *et al*. 2009)

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2}m^2\psi^2-\frac{|G(\psi)|}{2}(\partial\alpha-A)^2\right)$$

Holographic superconducting solution

Ansatz: $\psi = \psi(z), \quad A = \phi(z)dt$

With Metric:

$$ds^2 = -\frac{1}{z^2}e^{2\chi(z)}g(z)dt^2 + \frac{1}{z^2 g(z)}dz^2 + \frac{1}{z^2}\left(dx^2 + dy^2\right)$$

Equations of motion

$$\psi'' + \psi'\left(-\frac{2}{z} + \frac{g'}{g} + \chi'\right) - \frac{m^2\psi}{z^2 g} + \frac{e^{-2\chi(z)}\phi^2}{2g^2}\frac{dG(\psi)}{d\psi} = 0 \qquad (1)$$

$$\phi'' - \chi'\phi' - \frac{G(\psi)\phi}{z^2 g} = 0 \qquad (2)$$

$$g' - \kappa^2 z\left(\frac{e^{-2\chi}\phi^2 G(\psi)}{2g} + \frac{1}{2}g\psi'^2 + \frac{m^2\psi}{2z^2} + \frac{1}{2}e^{-2\chi}z^2\phi'^2\right) + \frac{3}{z}(1-g) = 0 \qquad (3)$$

$$\chi' + \frac{1}{2}z\kappa^2\psi'^2 + \frac{z\kappa^2 e^{-2\chi}\phi^2 G(\psi)}{2g^2} = 0 \qquad (4)$$

Boundary Conditions:

At the horizon (z=1)

$$\phi(1) = 0, \quad \psi'(1) = \frac{m^2 \psi(1)}{g'(1)}, \quad g(1) = 0$$

At the boundary (z=0)

$$\phi = \mu - \rho z + \ldots, \qquad \psi = \psi_1 z + \psi_2 z^2 + \ldots$$

$$g = 1 + \ldots, \qquad \chi = 0$$

Results

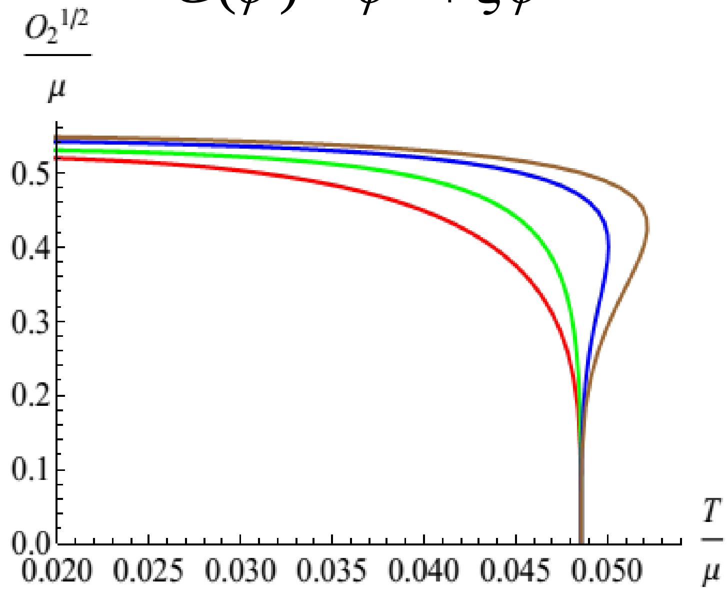
- Condensate as a function of T/μ for $\kappa = 0.3$. The red, green, blue and brown curves correspond to ξ= 0, 0.2, 0.5 and 0.7 respectively.

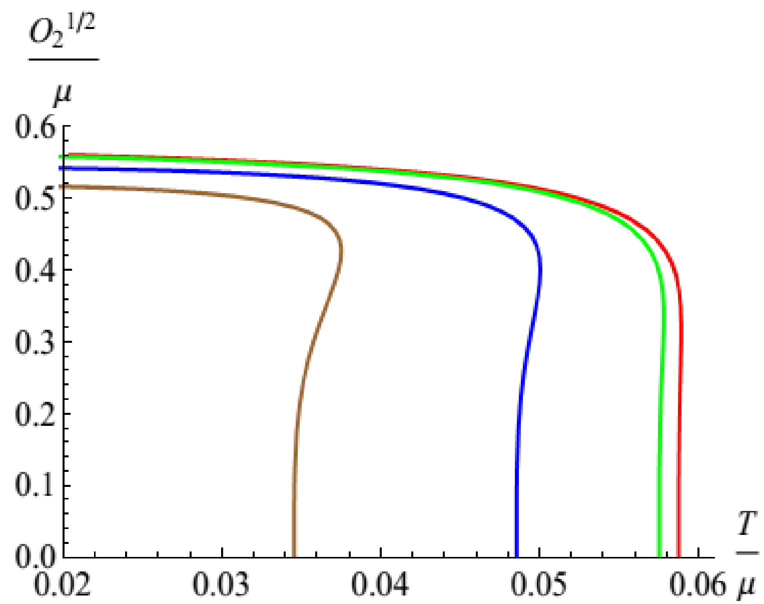
- Exists critical ξ_c above which the transition from normal to superconducting phase is first order. Below ξ_c transition is Second order. The critical temperature does not depend on ξ .

- Condensate for different values of κ with $\xi = 0.5.$ The red, green, blue, brown and black curves correspond to κ =10^(-10), 0.1, 0.3 and 0.5 respectively.

- The critical temperature decreases with higher backreaction parameter, making it harder for condensate harder to form.

$$G(\psi) = \psi^2 + \xi\psi^4$$





Momentum dependent vector type perturbations

- $g_{tx} \sim g_{tx}(z)e^{-i\omega t+iky}$
- $g_{xy} \sim g_{xy}(z)e^{-i\omega t+iky}$
- $A_x \sim A_x(z)e^{-i\omega t+iky}$

Linearized Einstein and Maxwell equations of motion

$$A_x'' + A_x'\left(\frac{g'}{g}+\chi'\right) + A_x\left(\frac{e^{-2\chi}\omega^2}{g^2} - \frac{k^2}{g} - \frac{G(\psi)}{z^2 g}\right) + \frac{e^{-2\chi}\phi' g_t^{x\prime}}{g} = 0 \qquad (5)$$

$$g_t^{x\prime\prime} - g_t^{x\prime}\left(\frac{2}{z}+\chi'\right) + 2\kappa^2 z^2\phi' A_x' + \frac{2\kappa^2\phi G(\psi)A_x}{g} - \frac{k^2 g_t^x}{g} = 0 \qquad (6)$$

$$g_y^{x\prime\prime} - g_y^{x\prime}\left(-\frac{2}{z}+\frac{g'}{g}+\chi'\right) + \frac{e^{-2\chi}\omega^2}{g^2}g_y^x + \frac{k\omega e^{-2\chi}}{g^2}g_t^x = 0 \qquad (7)$$

$$\omega g_t^{x\prime} + kge^{2\chi}g_y^{x\prime} + 2\kappa^2 z^2\omega\phi' A_x = 0 \qquad (8)$$

Eqs. (5)-(8) are not independent, eqs. (5), (6) and (8) implies eq. (7).

Boundary Action

$$S_{on-shell} = \int d^3x \left(\frac{1}{4z^2\kappa^2} \left(g_t^x g_t^{x\prime} - g g_y^x g_y^{x\prime} \right) - \frac{g}{2} \left(A_x A_x{}' - \rho g_t^x \right) \right) \Bigg|_{z=0}^{z=1}$$

With near boundary $z = \alpha$ expansion

$$g_t^{x\prime}(\alpha) = \frac{1}{A_{xtxt}} \left(G_{xtxt} g_t^x(0) - G_{xtxy} g_y^x(0) + G_{xtx} A_x(0) \right)$$

$$g_y^{x\prime}(\alpha) = \frac{1}{A_{xyxy}} \left(-G_{xtxy} g_t^x(0) + G_{xyxy} g_y^x(0) - G_{xyx} A_x(0) \right)$$

$$A_x{}'(\alpha) = \frac{1}{A_{xx}} \left((G_{xtx} - \rho A_{xx}) g_t^x(0) - G_{xyx} g_y^x(0) + G_{xx} A_x(0) \right)$$

Here $A_x(0)$, $g_t^x(0)$ and $g_y^x(0)$ are the boundary values of $A_x(z)$, $g_t^x(z)$ and $g_y^x(z)$ respectively. G_{xx} , G_{xtxt} ,G_{xyx} etc are correlators for current and energy momentum tensor. $A_{xtxt} = -A_{xyxy} = 1/(4\kappa^2\alpha^2)$.

- These coupled differential equations are extremely difficult to solve even numerically. However, for normal phase $\psi = 0$ analytic solution in the hydrodynamic limit is possible.

- We use another technique in which we solve eqs. (5), (6) ,(7) simultaneously, and then treat eq. (8) separately as the constraint equation on the various correlators.

$$\omega G_{xtxt} + kG_{xtxy} = 0$$
$$\omega G_{xtxy} + kG_{xyxy} = 0$$
$$\omega G_{xtx} + kG_{xyx} + 2\kappa^2 \omega z^2 \phi'(z) A_{xtxt} = 0$$

- In terms of transverse current current Correlators, the boundary response functions are

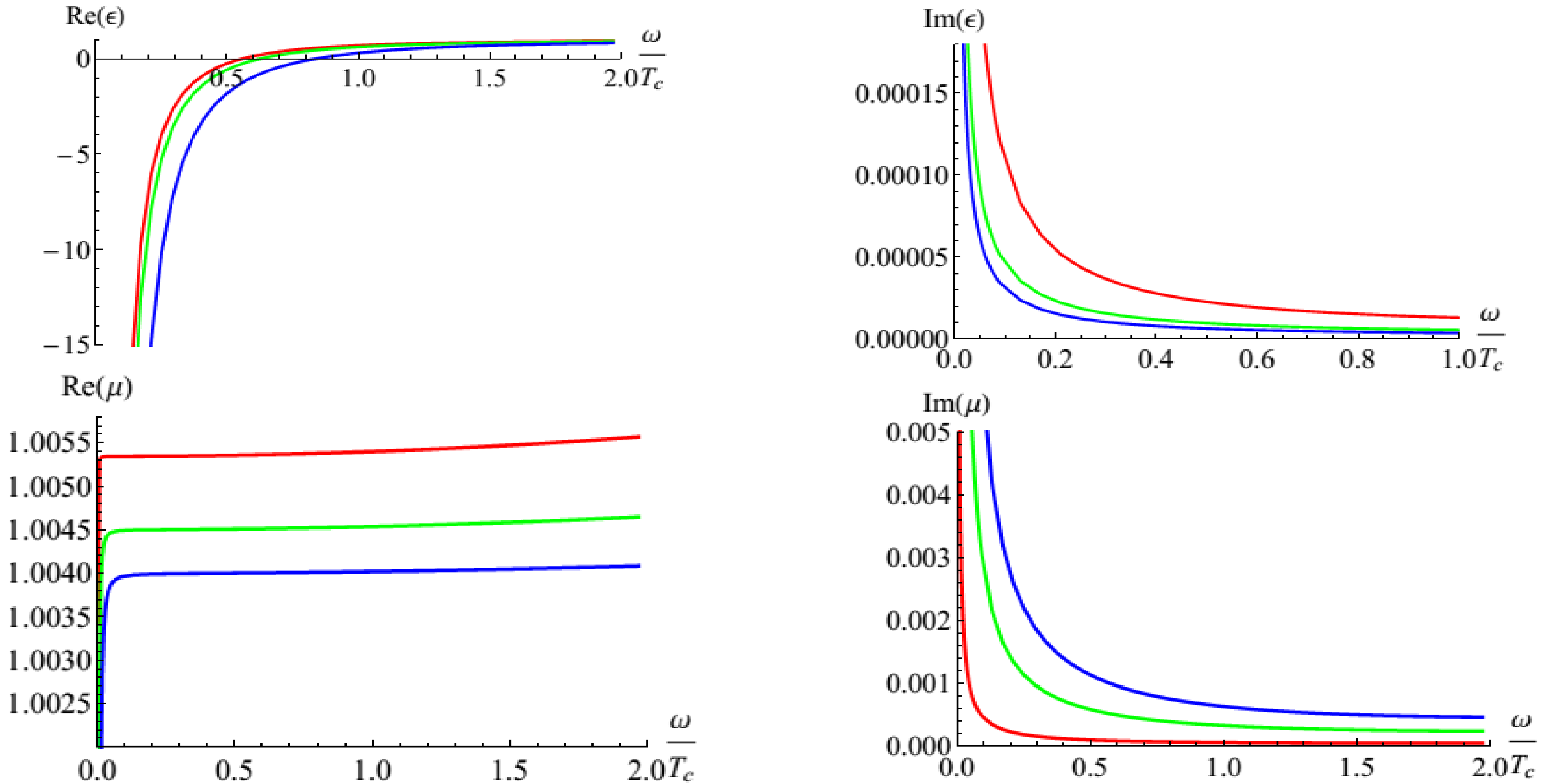
$$\varepsilon(\omega) = 1 - \frac{4\pi C_{em}^2 G_T^{(0)}(\omega)}{\omega^2}, \qquad \mu(\omega) = \frac{1}{1 - 4\pi C_{em}^2 G_T^{(2)}(\omega)}$$

- Where $G_T^{(0)}(\omega)$ and $G_T^{(2)}(\omega)$ are the coefficients of powers of the spatial momentum k, in the series expansion of the transverse current-current correlator

$$G_{xx} = G_T(\omega, k) = G_T^{(0)}(\omega) + k^2 G_T^{(2)}(\omega) + \ldots$$

It is to be noted that the boundary system doesn't have a dynamical photon. The strongly coupled field theory is assumed to be weakly coupled to a dynamical EM field at the boundary.

Numerical Results for κ = 0.1, 0.3, 0.5

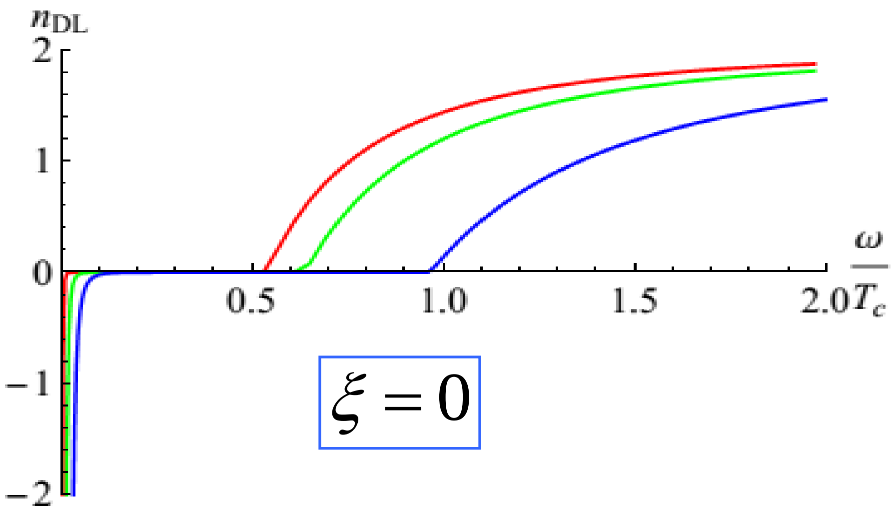


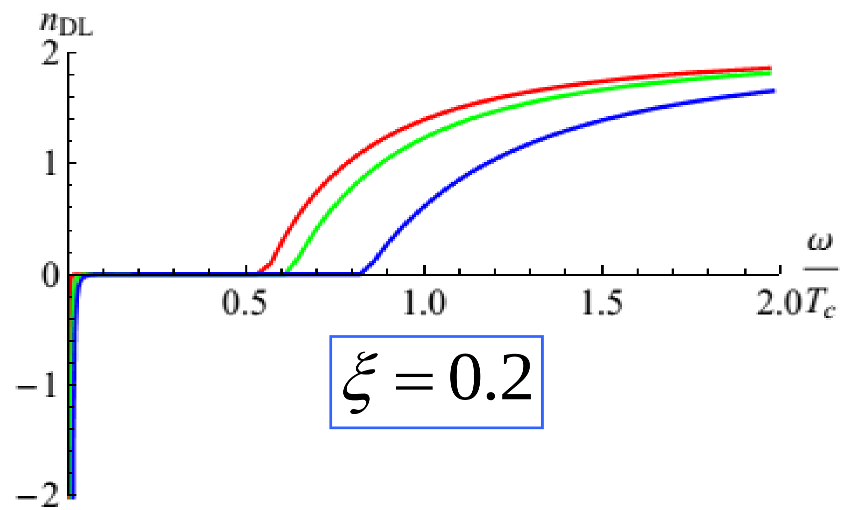
Responce functions as a function ω/T_c for $\xi = 0.2$ at T=0.5Tc.

With backreaction, Im(μ) is always positive and has a new diffusive pole at ω=0 which was absent in the probe limit case.

<u>Numerical Results</u> for κ = 0.1, 0.3, 0.5

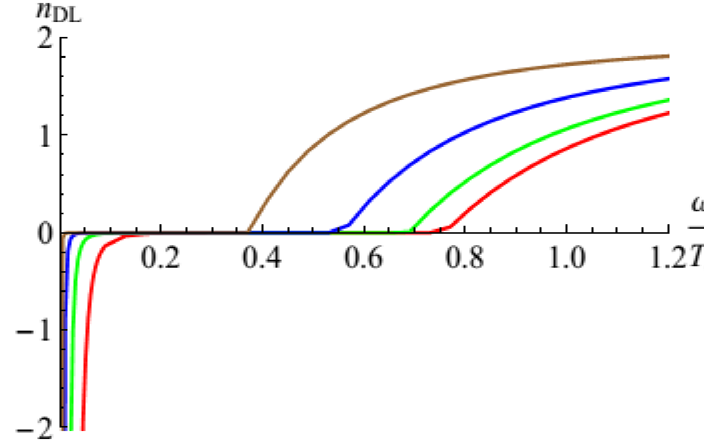
$$n_{DL} = \mathrm{Re}[\varepsilon]|\mu| + \mathrm{Re}[\mu]|\varepsilon|$$





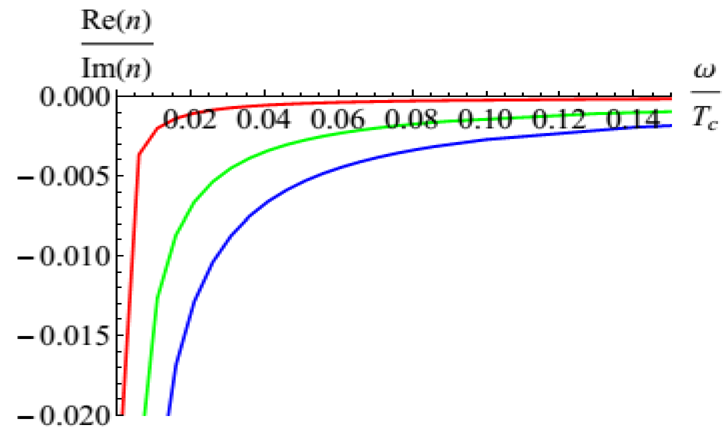
- superconducting system makes the transition from positive n_{DL} to negative n_{DL} as we decrease ω.
- The magnitude of cutoff ω_c increases with increase in backreaction, which implies that the superconducting phase can support negative refraction for relatively higher frequencies with higher backreaction.
- The transition from positive refraction to negative refraction with frequency is almost independent of ξ.

Temperature dependence of n_{DL} with ω for T/Tc= 0.8, 0.6, 0.4 and 0.2 . We find that negative refraction is present for all temperatures. This is another distinct result from the probe limit case where n_{DL} was found to be negative only within a window of temperatures.



Dissipation Effects

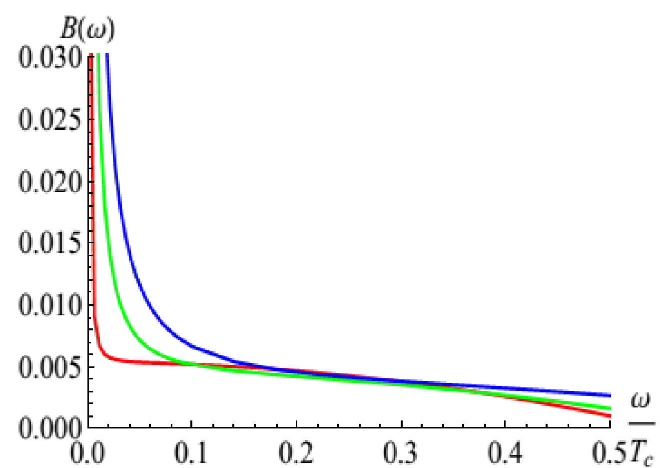
- Propagation to dissipation ratio in the region of negative n_{DL} is small and negative. This is not very uncommon among the isotropic metamaterials.



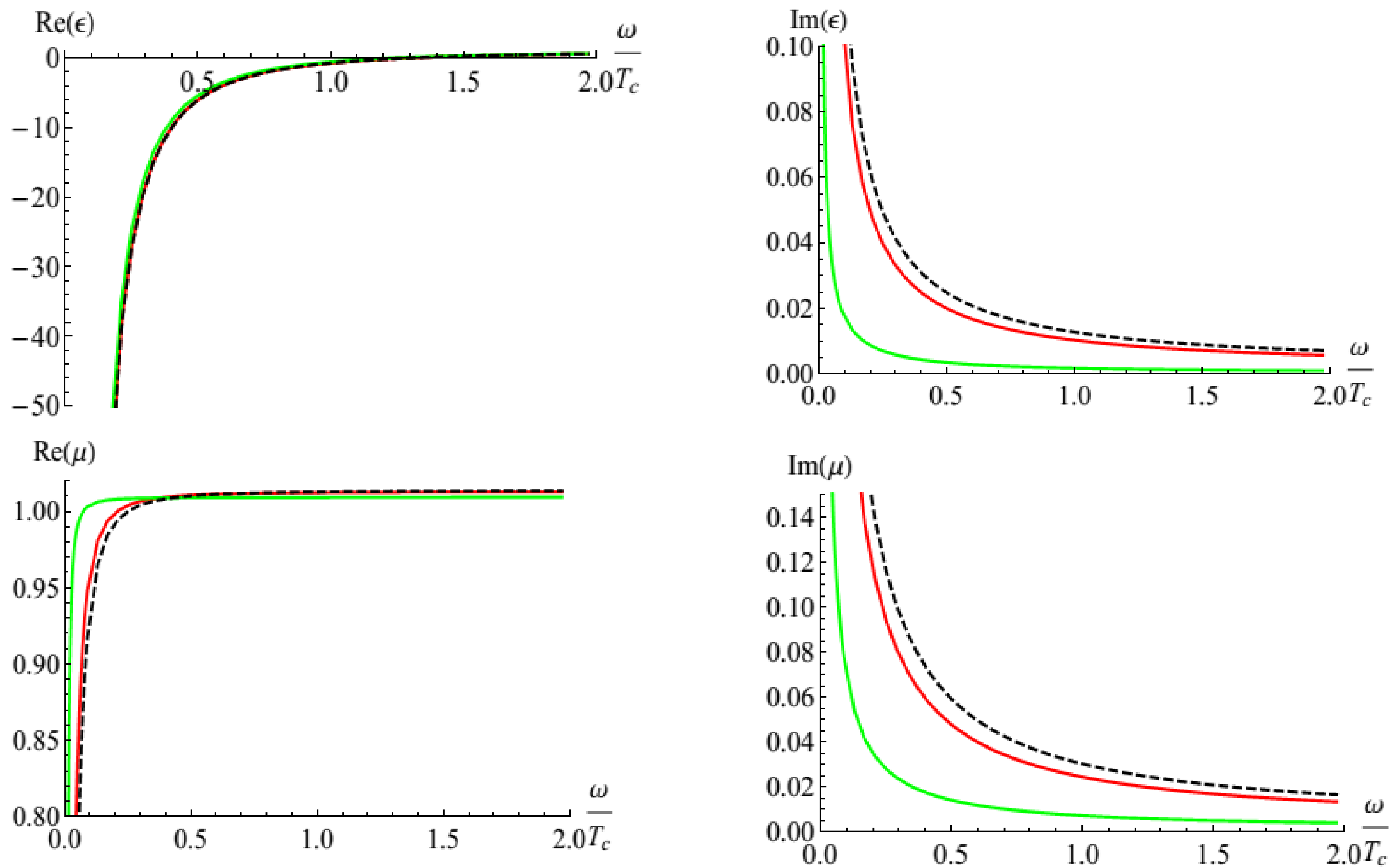
- Higher backreaction enhances the propagation. Unfortunately the propagation, on the other hand, decreases with higher values of ξ .

- Within the plotted frequency range, the

constraint $\left|\frac{k^2 G_T^{(2)}(\omega)}{G_T^{(0)}(\omega)}\right| = B(\omega) \ll 1$ holds true.



Response functions in the **normal**, superconducting and metastable regions



- At a fixed temperature, for (2 + 1) dimensional systems that show a first order transition from the normal to the superconducting phase, our results suggest that the imaginary part of the permittivity is always smaller in the superconducting phase compared with the normal phase.

Summary

- We have calculated the electro-magnetic response of strongly coupled superconducting boundary systems whose dual gravitational descriptions AdS-Schwarzschild black hole with the backreaction.
- Boundary system shows a superconducting phase transition above μ_c and that the nature of phase transition changes with ξ.
- Using the tools of AdS-CFT correspondence, we have computed the retarded correlators of the boundary theory.
- Numerical computations of the response functions ε and μ have been performed. It confirms the existence of negative refractive index in these boundary media at small enough frequencies.
- We also performed a comparative analysis of the response functions in the normal, superconducting and metastable regions of the phase diagram for holographic superconductors and our results suggest that the imaginary part of the permittivity is always smaller in the superconducting phase compared with the normal phase.
- We did the same analysis with the R-charged black hole background. Results involving R-charged examples indicate that the essential features remain the same as in the AdS-Schwarzschild black hole case.

Thank you