Stress tensor correlators from holography

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Outline

• Motivation
• Review of relative entropy for $\langle T_{a_1 b_1} \rangle$
• $\langle T_{a_1 b_1} \cdots T_{a_n b_n} \rangle$ for n=1,2,3 for theories with arbitrary curvature invariants.
• Hofman-Maldacena bounds
• The inevitability of string theory--Camanho, Edelstein, Maldacena, Zhiboedov
Motivation

\[ T_{a_1 b_1}(x)T_{a_2 b_2}(0) \sim \ldots + \sum_{k=1}^{3} c_k t^{(k)}_{a_1 b_1 a_2 b_2 a_3 b_3}(x)T^{a_3 b_3}(0) + \ldots \]

OPE coefficients

- Are the OPE coefficients bounded?
- Reasons (a) Conformal bootstrap  
  (b) Positive energy fluxes.
Motivation

• Conformal bootstrap intuition arises from work of Caracciolo-Rychkov, Rattazzi et al.

• Using scalar four point function bootstrap arguments, OPE coefficients appearing via $\langle \phi \phi O_{\Delta,l} \rangle$ are bounded.

• May be similar feature for conserved currents (namely replace scalar with conserved currents and ask if $\langle J J O_{\Delta,l} \rangle$ OPEs are bounded--very hard and till now open question).
Motivation

• Positive energy flux argument due to Hofman and Maldacena

\[ \int \langle 0 \left| T_{a_1 b_1}(x_1)T_{tt}(x_2)T_{a_3 b_3}(x_3) \right| 0 \rangle > 0 \]

• Leads to inequalities for OPE coefficients.
How does one calculate \( \langle T_{a_1 b_1}(x_1) \cdots T_{a_n b_n}(x_n) \rangle \)?

For this talk we focus on the holographic dual side of the story. 3pt functions fixed by conformal symmetry upto numbers. We will focus on how to get these numbers.

I will explain how to calculate \( n=1,2,3 \) for any theory

\[
\mathcal{L}(g^{ab}, R_{abcd}, \nabla_e R_{abcd})
\]
• Even the one point function appears very difficult.

• Reason is that we usually need to start with an action which includes the surface term and counterterms.

• Not known except for very special theories like Lovelock theories.
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• Step 2, we need a general and useful procedure to compute stress tensor correlation functions in arbitrary higher derivative gravity.
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Step 2, we need a general and useful procedure to compute stress tensor correlation functions in arbitrary higher derivative gravity.

Step 3, we need an algorithm which is feasible within the “life-span” of a graduate student!
...AND I HAVE FOUND THIS ONE WORKS A LOT BETTER.
• To accomplish step 1, we appeal to entanglement entropy.

• Recently Faulkner et al showed that for spherical entangling surfaces, using positivity of relative entropy, one recovers the linearized (general) gravity equations. As a by product they obtain a simple way to compute the renormalized holographic stress tensor for a general theory of gravity. [Faulkner, Guica, Hartman, Myers, Raamsdonk, 2013]
Relative entropy

Relative Entropy follows

\[ S(\rho | \sigma) = \Delta \langle H \rangle - \Delta S \]

From Klein's Inequality

\[ S(\rho | \sigma) \geq 0 \]

We get

\[ \Delta \langle H \rangle \geq \Delta S \]

“1st law of entanglement” when saturated

From Holography we can compute \[ \Delta S \]

\[ \Delta S = S(\rho) - S(\sigma) \]

Area functional - Area functional
We can compute $S(\rho|\sigma)$ holographically provided we know $H$ holographically [Blanco, Casini, Hung, Myers, 2013]

We know $H$ in some special cases.

For a spherical entangling surface

$$H = 2\pi \int_{|x|<R} d^{d-1}x \frac{R^2-r^2}{2R} T_{00}(\vec{x})$$

If we knew how to calculate $\Delta S$ we would be able to infer the stress tensor.

Can compute using holography

We can compute $H$
- The causal development of a ball is mapped to the evolution generated by ordinary $H$ in the hyperboloid. [Casini, Huerta, Myers]

- EE gets mapped to thermal entropy. [Myers, AS; Casini, Huerta, Myers]

- Can use Wald entropy to calculate the latter. [Myers, AS]
Holographic stress tensor from $\Delta S=\Delta H$

- More precisely we have

\[
\delta T_{tt}^{\text{grav}}(x_0) = \frac{d^2 - 1}{2\pi \Omega_{d-2}} \lim_{R \to 0} \left( \frac{1}{R^d} \delta S_B^{\text{wald}} \right)
\]

renormalized stress tensor: want to know

Can calculate using linearization of Wald entropy functional. Small sphere limit makes integral local.
• If we think about it, calculation of correlation functions will be simpler if we do a background field expansion of the bulk lagrangian such that on the AdS background $\Delta Riemann$ is zero. [Sen,AS]
• So schematically we have the bulk lagrangian to have terms like $(\Delta \text{Riemann})^n$

• This suggests that the bulk action can be rearranged in terms of stress tensor correlation functions. For instance if we wanted n-point functions we only consider upto power n.

• The coefficients of these terms themselves would involve ALL higher derivative terms in the bulk lagrangian we started with.
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• This suggests that the bulk action can be rearranged in terms of stress tensor correlation functions. For instance if we wanted n-point functions we only consider upto power n.

• The coefficients of these terms themselves would involve ALL higher derivative terms in the bulk lagrangian we started with.
There are some immediate advantages of doing this background field expansion. In TT gauge:

\[
\Delta R_{abcd} = R_{abcd} - \bar{R}_{abcd}
\]

\[
(\Delta R_{abcd})^{AdS} = 0
\]

\[
(\Delta R_{ab})^{linearized} = 0
\]

\[
(\Delta R)^{linearized} = 0
\]
• Consider \((\Delta R)^2\) or \((\Delta R_{ab})^2\) in the action.

• To compute the effect of these terms on the \(n\)-point function, we have to expand this around AdS upto \(n\)'th order in the perturbation.

• This immediately tells us that these will start contributing only to 4-point functions. This is the reason for the simple expressions we will find for one, two and three point functions.
Background field Lagrangian from

$$\mathcal{L}(g^{ab}, R_{cdef})$$

$$\mathcal{L} = (c_0 + c_1 \Delta R + \frac{c_4}{2} \Delta R^2 + \frac{c_5}{2} \Delta R^{ab} \Delta R_{ab} + \frac{c_6}{2} \Delta R^{abcd} \Delta R_{abcd} + \sum_{i=1}^{8} \tilde{c}_i \Delta K_i + \cdots)$$

Two point function

$$\langle T_{ab}(x) T_{cd}(x') \rangle = \frac{c_T}{|x - x'|^{2d}} T_{ab,cd}(x - x')$$

$$C_T = 2 \frac{d + 1}{d - 1} \frac{\Gamma[d + 1]}{\pi^{d/2} \Gamma[d/2]} \tilde{L}^{d - 1} [c_1 + 2(d - 2)c_6]$$

Simple!!

Stress tensor and geometry

$$\langle T_{\mu\nu} \rangle = \frac{\pi^{d/2}}{2 \tilde{L}^2} \frac{d - 1}{d + 1} \frac{\Gamma[d/2]}{\Gamma[d]} C_T h^{(d)}_{\mu\nu}$$

$$ds^2 = \tilde{L}^2 \frac{dz^2}{z^2} + \frac{1}{z^2} (g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + \cdots z^d h_{\mu\nu}^{(d)} + \cdots) dx^\mu dx^\nu$$

Trace anomaly

$$\langle T^a_a \rangle \sim A_d \underbrace{E_d}_{\text{Euler}} + \sum_i B_i \underbrace{I_i}_{\text{Weyl}}$$
Background field Lagrangian from \( \mathcal{L}(g^{ab}, R_{cdef}) \)

\[
\mathcal{L} = (c_0 + c_1 \Delta R + \frac{c_4}{2} \Delta R^2 + \frac{c_5}{2} \Delta R^{ab} \Delta R_{ab} + \frac{c_6}{2} \Delta R^{abcd} \Delta R_{abcd} + \sum_{i=1}^{8} \tilde{c}_i \Delta K_i + \cdots)
\]

A-type anomaly: \( \alpha \)-theorems??

Two point function

\[
\langle T_{ab}(x)T_{cd}(x') \rangle = \frac{c_T}{|x-x'|^{2d}} T_{ab,cd}(x-x')
\]

\[
C_T = 2 \frac{d+1}{d-1} \frac{\Gamma[d+1]}{\pi^{d/2} \Gamma[d/2]} \tilde{L}^{d-1} [c_1 + 2(d-2)c_6]
\]

Stress tensor and geometry

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\]

**A-type anomaly:** a-theorems??

Two point function

\[
\langle T_{ab}(x)T_{cd}(x') \rangle = \frac{c_T}{|x-x'|^{2d}} \mathcal{I}_{ab,cd}(x-x')
\]

\[
C_T = 2 \frac{d+1}{d-1} \frac{\Gamma[d+1]}{\pi^{d/2} \Gamma[d/2]} \tilde{L}^{d-1} \left[ c_1 + 2(d-2)c_6 \right]
\]

**B-type anomaly**

Simple!!

**Stress tensor and geometry**

\[
\langle T_{\mu\nu} \rangle = \frac{\pi^{d/2}}{2 \tilde{L}^2} \frac{d-1}{d+1} \frac{\Gamma[d/2]}{\Gamma[d]} C_T h^{(d)}_{\mu\nu}
\]

\[
ds^2 = \tilde{L}^2 \frac{dz^2}{z^2} + \frac{1}{z^2} (g_{\mu\nu}^{(0)} + z^2 g_{\mu\nu}^{(2)} + \cdots z^d h_{\mu\nu}^{(d)} + \cdots) dx^\mu dx^\nu
\]

**Trace anomaly**

\[
\langle T_a^a \rangle \sim A_d \underbrace{E_d}_{Euler} + \sum_i B_i \underbrace{I_i}_{Weyl}
\]

[Fefferman, Graham; de Haro, Skenderis, Solodukhin]

Deser, Schwimmer; Henningson-Skenderis; Miao

Tuesday, 16 December 14
Example

New massive gravity (Bergshoeff, Hohm, Townsend)

\[ \mathcal{L}_{D=3} = R + \frac{2}{L^2} + 4L^2 \lambda \left( R_{ab} R^{ab} - \frac{3}{8} R^2 \right) \]

\[ R = \Delta R - \frac{6}{\tilde{L}^2} \]

\[ R_{ac} = \Delta R_{ac} - \frac{2}{\tilde{L}^2} g_{ac} \]

\[ c_1 = \frac{1}{2\ell_P} \left( 1 + 2\lambda \frac{L^2}{\tilde{L}^2} \right) \] This is the expected answer.
• In even dimensions $c_T$ is related to a B-anomaly coefficient. Let me sketch the argument. RG equation gives [Erdmenger, Osborn; Osborn, Petkou]

$$ (\mu \partial_\mu + 2 \int d^d x g^{\mu\nu} \frac{\delta}{\delta g_{\mu\nu}}) W = 0. $$

Quantum effective action

• Hit twice more with $\frac{\delta}{\delta g_{\mu\nu}}$

$$ \mu \partial_\mu \langle T_{ab}(x)T_{cd}(0) \rangle = -2 \int d^d x \frac{\delta^2 A_{\text{anomaly}}}{\delta g_{ab} \delta g_{cd}}. $$
• Since RHS is to be computed around flat space, it is easy to see that we need terms in the anomaly that have at most 2 Riemann curvatures. In 4 dimensions an explicit calculation picks out the c-anomaly coefficient.

• In 6 dimensions the only anomaly term with 2 Riemann’s is $B_3$. Hence this coefficient is picked out. All this we have explicitly check directly in holography.
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We can also compute 3 point functions!
"Conformal Collider Physics"

- Consider scattering "experiments" in d=4 CFT's
- Insert disturbance with stress tensor
- Measure energy flux at infinity

\[ \mathcal{E}(\vec{n}) = \lim_{r \to \infty} r^2 \int_{-\infty}^{\infty} dt \, n^i T_i^0(t, r n^i) \]

\[ \langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon_{ij}^* T_{ij} \mathcal{E}(\vec{n}) \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon_{ij}^* T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle} \]

Controlled by 3-pt function!!
“Conformal Collider Physics”

- result fixed by symmetry of “experiment”:

\[
\langle \mathcal{E}(\vec{n}) \rangle = \frac{\langle 0 | \epsilon_{ij}^* T_{ij} \mathcal{E}(\vec{n}) \epsilon_{kl} T_{kl} | 0 \rangle}{\langle 0 | \epsilon_{ij}^* T_{ij} \epsilon_{kl} T_{kl} | 0 \rangle}
\]

\[
= \frac{E}{4\pi} \left[ 1 + t_2 \left( \frac{\epsilon_{ij}^* \epsilon_{il} n_j n_l}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{1}{3} \right) + t_4 \left( \frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon_{ij}^* \epsilon_{ij}} - \frac{2}{15} \right) \right]
\]

**Comments:**

- only three independent parm’s amongst \( a, c, t_2, t_4 \):

\[
\frac{c - a}{c} = \frac{1}{6} t_2 + \frac{4}{45} t_4
\]

- \( t_4 = 0 \) for supersymmetric CFT’s
Consider scattering “experiments” in d=4 CFT’s

\[ \langle E(\vec{n}) \rangle = \frac{\langle 0| \epsilon^{*}_{i,j} T_{i,j} E(\vec{n}) \epsilon_{k,l} T_{k,l} |0 \rangle}{\langle 0| \epsilon^{*}_{i,j} T_{i,j} \epsilon_{k,l} T_{k,l} |0 \rangle} \]

\[ = \frac{E}{4\pi} \left[ 1 + t_2 \left( \frac{\epsilon^{*}_{i,j} \epsilon_{i,l} n_{j} n_{l}}{\epsilon^{*}_{i,j} \epsilon_{i,j}} - \frac{1}{3} \right) + t_4 \left( \frac{|\epsilon_{i,j} n_{i} n_{j}|^2}{\epsilon^{*}_{i,j} \epsilon_{i,j}} - \frac{2}{15} \right) \right] \]

Comments:

- Demanding \( \langle E(\vec{n}) \rangle \geq 0 \) imposes nontrivial constraints

\[ d = 4 \]

\[ t_2 \text{ absent in 3d} \]

- Spin 2:
  \[ 1 - \frac{1}{3} t_2 - \frac{2}{15} t_4 \geq 0 \]

- Spin 1:
  \[ 1 + \frac{1}{6} t_2 - \frac{2}{15} t_4 \geq 0 \]

- Spin 0:
  \[ 1 + \frac{1}{3} t_2 + \frac{8}{15} t_4 \geq 0 \]

(Latorre & Osborn, hep-th/9703196)
\[ t_2 = \frac{d(d-1)}{c_1 + 2(d-2)c_6} [2c_6 - 12(3d+4)\tilde{c}_7 + 3(7d+4)\tilde{c}_8], \]
\[ t_4 = \frac{6d(d^2-1)(d+2)}{c_1 + 2(d-2)c_6} (2\tilde{c}_7 - \tilde{c}_8). \]

- When \( t_4 = 0 \) (\( \& \) \( c_7 = c_8 = 0 \))

- Thus KSS bound is violated whenever we have \( t_2 > 0 \) in a perturbative expansion.

- For a general four derivative theory (non-unitary) we find \( \eta/s \rightarrow 0 \)
HM bounds from entanglement

\[ \sum_{i=0}^{\infty} c_i (z - z_h)^\alpha \]

\[ 0 < \alpha < 1 \]

\[ c_0 \in \text{Reals} \]

see Arpan’s talk

Banerjee, Bhattacharyya, Kaviraj, Sen, AS, 1401.5089; Ogawa-Takayanagi 1107.4363
Demanding a smooth surface puts bound on GB coupling.
Is string completion necessary?

- In IIB supergravity where the leading correction begins at 8-derivative order, both $t_2$ and $t_4$ are zero.

- $t_4$ needs six derivative terms which are absent in supersymmetric theories.

- $t_2$ could be present in supersymmetric theories.

- Shapiro time delay calculation by Camanho, Edelstein, Maldacena and Zhiboedov show that causality is always violated in a truncated higher derivative theory!!
So for instance Gauss-Bonnet gravity in spite of having two derivative eoms will always lead to a time advance.

To fix such problems in theories with non-zero $\frac{t_2}{t_4}$ it seems that we will either need an infinite set of higher derivative corrections or we need to invoke some new physics involving an infinite set of higher spin massive particles. (String theory is inevitable!)
Allowed $t_2, t_4$ shrinks as we go inside the bulk. Disappears for a finite value of the coordinate.
• Allowed $t_2, t_4$ region shrinks to zero size at some value of the radial coordinate (impact parameter). $\rho_c \approx \sqrt{\alpha_2 \alpha_3}$

• Fix causality problem by adding more higher derivative terms? This is not possible since each factor of Riemann introduces $(1 - \rho)$. Easy to see using our background field approach.

• Thus will need new physics at $\alpha_2, \alpha_3 \sim \frac{1}{\Delta g}$

• Shapiro time delay gets mapped to anomalous dimensions of operators in N=4 CFT. Starting with SUGRA spectrum Alday et al found that the conformal dimension could violate the unitarity bound unless the spectrum includes higher spin operators.
• Having an infinite tower of higher spin massive operators appears inevitable for non-zero $t_2/t_4$.

• Points at the fact that string theory may be necessary to talk about the duals of CFTs which have unequal $c,a$. 
Gamow-Ivanenko-Landau physics cube--holographic version
Gamow-Ivanenko-Landau physics cube--holographic version

Is Interstellar possible?
• Take home message: If string theory is right, Interstellar is impossible!