Holographic chiral induced W-gravity.

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Introduction

- Polyakov's induced chiral $2d$ gravity ['87]
  - Induced gravity: found by integrating out scalars on worldsheet.
  - Is covariant and non-local.
  - Analysis in light-cone gauge reveals $sl(2,\mathbb{R})$ current algebra.
- Induced chiral W_3-gravity in $2d$
  - Higher spin extensions of $2d$ induced gravity.
  - Induced (matter fields integrated out) 2d gravities with W_3-symmetries.
  - Analysis in light-cone gauge similarly reveals an $sl(3,\mathbb{R})$ current algebra, just as above.
  - These were also derived from gauged WZNW models for the gauge group $SL(3,\mathbb{R})$.

- What is the holographic dual to such induced chiral W_3-gravity?
- Possible answer: massless spin-3 field in AdS_3
- AdS_3 gravity yielding $sl(2,\mathbb{R})$ Kač-Moody algebra with level $k = c/6$; along with a copy of Virasoro. [Avery, RP, Suryanarayana]
- There are boundary conditions on AdS_3 which yielding $U(1)$ Kač-Moody with central extension; along with a copy of Virasoro. [Compère, Song, Strominger]

Results

- Higher-spin boundary conditions yielding an $sl(3,\mathbb{R})$ or $su(1,2)$ or $u(1)\oplus u(1)$ Kač-Moody algebra,
- and a copy of classical $\mathcal{W}_3$.
- Choice depends on values of certain parameters in the general ansatz for the flat gauge connection used.
- $sl(3,\mathbb{R})$ case may correspond to the induced W_3-gravity studied in the early nineties by Ooguri *et al* and Verlinde.
  [Ooguri, Schoutens, Sevrin, Van Neuwenhuizen][Verlinde]
- W_3 gravities with symmetries of the other kind need to be studied further.

Plan

- We would like to generalize the boundary conditions on the AdS_3 metric which yield an $sl(2,\mathbb{R})$ current algebra to include a massless spin-3 field.
- Higher-spin analysis is better done in Chern-Simons(CS) formalism for AdS_3.
- Hence, we cast the metric formulation which yields $sl(2,\mathbb{R})$ currents, in the CS form.
- Try and generalise the $sl(2,\mathbb{R})$ guage fields to $sl(3,\mathbb{R})$ gauge fields.
- Compute the Poisson brackets between parameters of the space of solutions.

Metric to CS formulation for $sl(2,\mathbb{R})$ currents

- Boundary conditions on AdS_3 metric which yield an $sl(2,\mathbb{R})$ current algebra and a Virasoro [Avery, RP, Suryanarayana]:

$$\begin{aligned} g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}(r^{-4}), \;\; g_{r+} = \mathcal{O}(r^{-1}), \;\; g_{r-} = \mathcal{O}(r^{-3}), \\ g_{+-} &= -\frac{r^2}{2} + \mathcal{O}(r^0), \;\; g_{--} = \mathcal{O}(r^0), \\ g_{++} &= r^2 F(x^+, x^-) + \mathcal{O}(r^0), \end{aligned} \tag{1}$$

- Can be completely solved for order by order in r in Fefferman-Graham(FG) gauge:

$$\begin{aligned} g^{(0)}_{++} &= F(x^+, x^-), \;\; g^{(0)}_{+-} = -\frac{1}{2}, \;\; g^{(0)}_{--} = 0, \\ g^{(2)}_{++} &= \kappa(x^+, x^-), \;\; g^{(2)}_{+-} = \sigma(x^+, x^-), \;\; g^{(2)}_{--} = \tilde{\kappa}(x^+, x^-), \\ g^{(4)}_{ab} &= \frac{1}{4} g^{(2)}_{ac} g^{cd}_{(0)} g^{(2)}_{db}\,, \end{aligned} \tag{2}$$

- The EOM further impose a constraint which looks like the Virasoro Ward identity of the boundary theory:

$$2\,(\partial_+ + 2\,\partial_- F + F\,\partial_-)\,\tilde{\kappa} = \partial_-^3 F\,. \tag{3}$$

- One can get specific bulk solutions by imposing $\tilde{\kappa} = -1/4$ and solve the above diff. eq. for F

$$\begin{aligned} \tilde{\kappa} &= -1/4 \\ \Longrightarrow\ F &= f(x^+) + g(x^+)\,e^{i\,x^-} + \bar{g}(x^+)\,e^{-i\,x^-} \end{aligned}$$

- These become the $sl(2,\mathbb{R})$ currents.

- AdS_3 gravity as difference of Chern-Simons:

$$\begin{aligned} S[A] &= \frac{k}{4\pi}\int \mathrm{tr}(A\wedge A+\frac{2}{3}A\wedge A\wedge A) \\ S &= S[A]-S[\tilde{A}]+S_{bdy} \\ A=\omega+e/\ell \qquad & \tilde{A}=\omega-e/\ell. \end{aligned} \tag{4}$$

- Gauge fields ansatz corresponding to the above metric:

$$A=b^{-1}\mathrm{d}b+b^{-1}a\,b \qquad \tilde{A}=b\,\mathrm{d}b^{-1}+b\,\tilde{a}\,b^{-1},$$

$$\begin{aligned} a &= (L_1+a_+^{(-)}\,L_{-1}+a_+^{(0)}\,L_0)\,\mathrm{d}x^+ + (a_-^{(-)}\,L_{-1})\,\mathrm{d}x^-, \\ \tilde{a} &= (\tilde{a}_+^{(0)}\,L_0+\tilde{a}_+^{(+)}\,L_1+\underline{\tilde{a}_+^{(-)}\,L_{-1}})\,\mathrm{d}x^+ + (\tilde{a}_-^{(+)}\,L_1-L_{-1})\,\mathrm{d}x^-. \end{aligned} \tag{5}$$

where $b=e^{(r/l)L_0}$ is an $SL(2,\mathbb{R})$ element can be used to absorb away all the r dependence.

- Wherein we have allowed for a function $\tilde{a}_+^{(-)}$ to have a positive power in r.

- The eom:

$$da + [a \wedge a] = 0 \qquad d\tilde{a} + [\tilde{a} \wedge \tilde{a}] = 0 \tag{6}$$

with Fefferman-Graham gauge condition $a_+^{(0)} = \tilde{a}_+^{(0)}$ implies

- The left gauge field components are solved upto a function $\kappa(x^+)$
- The right gauge field components are solved for in terms of $\tilde{a}_+^{(-)}$ and $\tilde{a}_-^{(+)}$ with Virasoro constraint:

$$(\partial_+ + 2\,\partial_- \tilde{a}_+^{(-)} + \tilde{a}_+^{(-)}\,\partial_-)\,\tilde{a}_-^{(+)} = \frac{1}{2}\partial_-^3\,\tilde{a}_+^{(-)} \tag{7}$$

- For $\tilde{a}_-^{(+)} = -1/4$

$$\tilde{a}_+^{(-)} \;\approx F \;=\; f(x^+) + g(x^+)\,e^{i\,x^-} + \bar{g}(x^+)\,e^{-i\,x^-} \tag{8}$$

- The above functions give rise to an $sl(2,\mathbb{R})$ current algebra with Virasoro parametrized by $\kappa(x^+)$.

$sl(3,\mathbb{R})$ CS formulation for spin-3

- Gauge algebra

$$\begin{aligned} L_{-1} &= \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}, L_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, L_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \\ W_2 &= 2\alpha L_1^2 \quad , \qquad [L_i, W_m] = (2i-m)W_{i+m}, \quad m \in \{-2, ..., 2\}. \end{aligned}$$

- Where, $\alpha = 1 \implies sl(3,\mathbb{R})$ and $\alpha = i \implies su(1,2)$.
  $\eta_{ab} =$ Killing metric, $f_{abc} =$ structure consts.

- Choose anzats and solve for components by imposing flatness:

$$\begin{aligned} a &= L_1 - \kappa L_{-1} - \omega W_{-2}\, dx^+ \\ \tilde{a} &= (-L_{-1} + \tilde{\kappa}\, L_1 + \tilde{\omega}\, W_2)\; dx^- + \left(\sum_{a=-1}^{1} f^{(a)} L_a + \sum_{i=-2}^{2} g^{(i)} W_i \right) dx^+ \\ A &= b^{-1}\partial_r b\, dr + b^{-1} a\, b, \quad \tilde{A} = b\,\partial_r b^{-1}\, dr + b\,\tilde{a}\, b^{-1}. \end{aligned}$$

  where $b = e^{(r/l)L_0}$ is an $SL(3,\mathbb{R})$ element.

- Flatness for *a* implies : $\partial_- \kappa = 0$ and $\partial_- \omega = 0$

- Flatness for $\tilde{a}$ implies: Components can be solved for in terms
  . of $\{\tilde{\kappa}, \tilde{\omega}, f^{(-1)}, g^{(-2)}\}$.

  Further constraints among $\{\tilde{\kappa}, \tilde{\omega}, f^{(-1)}, g^{(-2)}\}$ are the Ward identities for the boundary theory

$$(\partial_+ + 2\,\partial_- f^{(-1)} + f^{(-1)}\,\partial_-)\,\tilde{\kappa} - \alpha^2\,(12\,\partial_- g^{(-2)} + 8\,g^{(-2)}\partial_-)\,\tilde{\omega} = \frac{1}{2}\partial_-^3 f^{(-1)},$$

$$\begin{aligned} &12\,(\partial_+ + 3\,\partial_- f^{(-1)} + f^{(-1)}\partial_-)\,\tilde{\omega} + (10\,\partial_-^3 g^{(-2)} + 15\,\partial_-^2 g^{(-2)}\,\partial_- \\ &+9\,\partial_- g^{(-2)}\partial_-^2 + 2\,g^{(-2)}\partial_-^3)\,\tilde{\kappa} - 16\,(2\,\partial_- g^{(-2)} + g^{(-2)}\,\partial_-)\,\tilde{\kappa}^2 = \frac{1}{2}\partial_-^5 g^{(-2)}. \end{aligned} \quad (9)$$

- Note: That α enters only in the first of the equations.

- So far we have done the following:
  - Ansatz for $sl(3, \mathbb{R})$ gauge field.
  - Soved for EOM in terms of $\{\tilde{\kappa}, \tilde{\omega}, f^{(-1)}, g^{(-2)}\}$.
  - These staisfy constraints, identical to Ward Identities on the boundary.
- Next, we would like these solutions to be variationally well defined by requiring the boundary theory have $f^{(-1)}$ & $g^{(-2)}$ as the dynamical fields.

Boundary terms for the action

- For the solution to be variationally well defined, one needs to add boundary terms to the bulk action.

- Boundary term in action for fixed $\tilde{\kappa}$ and $\tilde{\omega}$

$$S_{bdy} = \frac{k}{4\pi}\int d^2x\, tr\left(-L_0[\tilde{a}_+, \tilde{a}_-] + 2\,\tilde{\kappa}_0\, L_1\, \tilde{a}_+ + \frac{1}{2\,\alpha}\, W_0\{\tilde{a}_+, \tilde{a}_-\} + \frac{1}{3}\tilde{a}_+\,\tilde{a}_- + 2\,\tilde{\omega}_0\, W_2 \tilde{a}_+\right). \tag{10}$$

The above form is demanded by requiring:

$$\delta S_{total} = -\frac{k}{2\pi}\int d^2x\, \left[(\tilde{\kappa} - \tilde{\kappa}_0)\,\delta f^{(-1)} + 4\,\alpha^2\,(\tilde{\omega} - \tilde{\omega}_0)\,\delta g^{(-2)}\right] \tag{11}$$

- Cases studied for fixed and constant values $\tilde{\kappa} = \tilde{\kappa}_0$ and $\tilde{\omega} = \tilde{\omega}_0$

- Solve the Ward identities for $f^{(-1)}$ and $g^{(-2)}$ for constants $\tilde{\kappa}_0$ & $\tilde{\omega}_0$

$$\begin{aligned} \partial_-^3 f^{(-1)} + 24\,\alpha^2\,\tilde{\omega}_0\,\partial_- g^{(-2)} - 4\,\tilde{\kappa}_0\,\partial_- f^{(-1)} &= 0 \\ \partial_-^5 g^{(-2)} - 20\,\tilde{\kappa}_0\,\partial_-^3 g^{(-2)} + 64\,\tilde{\kappa}_0^2\,\partial_- g^{(-2)} - 72\,\tilde{\omega}_0\,\partial_- f^{(-1)} &= 0 \end{aligned} \tag{12}$$

- 1. $\tilde{\kappa}_0 \neq 0$, $\tilde{\omega}_0 \neq 0$ & $\partial_-\{f^{(-1)}, g^{(-2)}\} = 0$:

$$f^{(-1)} = f(x^+), \quad g^{(-2)} = g(x^+). \tag{13}$$

This trivially solves the above equations. This can be viewed as a spin-3 generalisation of the boundary conditions of Compère *et al*.

- 2. $\tilde{\omega}_0 = 0$ and $\tilde{\kappa}_0 \neq 0$:

$$
\begin{aligned}
f^{(-1)} &= f_\kappa(x^+) + g_\kappa(x^+)\, e^{2\sqrt{\tilde{\kappa}_0}\, x^-} + \bar{g}_\kappa(x^+)\, e^{-2\sqrt{\tilde{\kappa}_0} x^-}, \\
g^{(-2)} &= f_\omega(x^+) + g_\omega(x^+)\, e^{2\sqrt{\tilde{\kappa}_0}\, x^-} + \bar{g}_\omega(x^+)\, e^{-2\sqrt{\tilde{\kappa}_0}\, x^-} + h_\omega(x^+)\, e^{4\sqrt{\tilde{\kappa}_0}\, x^-} \\
&\quad + \bar{h}_\omega(x^+)\, e^{-4\sqrt{\tilde{\kappa}_0}\, x^-}
\end{aligned}
\tag{14}
$$

$\tilde{\kappa}_0 = -1/4 \implies$ boundary of global AdS_3

- 3. $\tilde{\omega}_0 = 0$ and $\tilde{\kappa}_0 = 0$ gives:

$$
\begin{aligned}
f^{(-1)} &= f_{-1}(x^+) + x^-\, f_0(x^+) + (x^-)^2 f_1(x^+), \qquad (15)\\
g^{(-2)} &= g_{-2}(x^+) + x^-\, g_{-1}(x^+) + (x^-)^2\, g_0(x^+) + (x^-)^3\, g_1(x^+) \\
&\quad + (x^-)^4\, g_2(x^+)
\end{aligned}
$$

Suitable for non-compact x^+ and x^- (boundary of Poincare or Euclidean AdS_3)

Charge and asymptotic symmetry

- We would like to compute the Poisson brackets between the functions parametrizing the space of solutions.
- To this effect we would have to compute the residual gauge parameters which keeps the form of the gauge field same.
- And also compute the charge Q_Λ, associated with the these residual gauge transformations.

- Change in the asymptotic charge in CS theory associated with residual gauge transformation Λ: [Barnich, Brandt]

$$\not{\delta} Q_{\Lambda} \quad = \quad -\tfrac{k}{2\pi}\int_0^{2\pi} d\phi \; tr[\Lambda\,\delta A_{\phi}]. \tag{16}$$

These are trivially integrable.

- Construct Poisson brackets by demanding above charges generate required variation of solution space parameters $\mathcal{F}(x)$:

$$\delta_{\Lambda}\mathcal{F}(x) = \{Q_{\Lambda}, \mathcal{F}(x)\}, \tag{17}$$

where $\delta_{\Lambda}\mathcal{F}$ is read from $\delta_{\Lambda}A \ = \ \mathrm{d}\Lambda + [A, \Lambda]$

- Left sector: for $\Lambda = \lambda^{(i)} L_i + \eta^{(m)} W_m, \quad \delta a = d\Lambda + [a, \Lambda]$

$$Q_{(\lambda,\eta)} = -\frac{k}{2\pi} \int_0^{2\pi} d\phi \, [\lambda \, \kappa - 4\, \alpha^2 \, \eta \, \omega] \tag{18}$$

where $\lambda = \lambda^{(1)}$ & $\eta = \eta^{(2)}$ are gauge transformation parameters.

- This implies the following Poisson brackets:

$$\begin{aligned}
-\tfrac{k}{2\pi} \left\{\kappa(x^+), \kappa(\tilde{x}^+)\right\} &= -\kappa'(x^+)\, \delta(\Delta x^+) - 2\, \kappa\,(x^+)\, \delta'(\Delta x^+) + \tfrac{1}{2}\, \delta'''(\Delta x^+), \\
-\tfrac{k}{2\pi} \left\{\kappa(x^+), \omega(\tilde{x}^+)\right\} &= -2\, \omega'(x^+)\, \delta(\Delta x^+) - 3\, \omega(x^+)\, \delta'(\Delta x^+), \\
-\tfrac{2k\alpha^2}{\pi} \left\{\omega(x^+), \omega(\tilde{x}^+)\right\} &= \frac{8}{3}[\kappa^2(x^+)\, \delta'(\Delta x^+) + \kappa(x^+)\, \kappa'(x^+)\delta(\Delta x^+)] \\
&\quad -\frac{1}{6}[5\kappa(x^+)\delta'''(\Delta x^+) + \kappa'''(x^+)\delta(\Delta x^+)] \\
&\quad -\frac{1}{4}[3\kappa''(x^+)\delta'(\Delta x^+) + 5\kappa'(x^+)\delta''(x^+ - \tilde{x}^+)] + \frac{1}{24}\delta^{(5)}(\Delta x^+)
\end{aligned} \tag{19}$$

- this is the $\mathcal{W}_3$ algebra. [Campoleoni, Fredenhagen, Pfenninger, Theisen]

- Right sector: Solve for residual gauge transformation parameters $\tilde{\Lambda}$:

$$\tilde{\Lambda} \quad = \quad \tilde{\lambda}^i L_i + \tilde{\eta}^m W_m, \tag{20}$$

which keep $\tilde{a}$ form-invariant and $\delta\tilde{\kappa} = 0$ & $\delta\tilde{\omega} = 0$

- 1.$\tilde{\kappa}_0 \neq 0$, $\tilde{\omega}_0 \neq 0$, $\partial_- f^{(-1)} = \partial_- g^{(-2)} = 0$

  Residual gauge transformation parameters are

$$\tilde{\lambda}^{(-1)} = \tilde{\lambda}(x^+), \quad \tilde{\eta}^{(-2)} = \tilde{\eta}(x^+). \tag{21}$$

  These induce:

$$\delta f^{(-1)} = \partial_+ \tilde{\lambda}, \;\; \delta g^{(-2)} = \partial_+ \tilde{\eta}. \tag{22}$$

- Charge under above Λ :

$$Q_{\tilde{a}} \;=\; \tfrac{k}{2\pi}\int_0^{2\pi} d\phi\, 2\,[\tilde{\lambda}\,(\tilde{\kappa}_0\, f - 6\,\alpha^2\,\tilde{\omega}_0\, g) + \tilde{\eta}\, 2\,\alpha^2\,(\tfrac{8}{3}\tilde{\kappa}_0^2\, g - 3\,\tilde{\omega}_0\, f)] \quad (23)$$

$$\{f(x^+),\, f(\tilde{x}^+)\} = -\tfrac{\pi}{k}\,\tfrac{\tilde{\kappa}_0^2}{\Delta}\,\delta'(\Delta x^+), \quad \{g(x^+),\, g(\tilde{x}^+)\} = -\tfrac{\pi}{k}\,\tfrac{3\,\tilde{\kappa}_0}{16\,\Delta\alpha^2}\,\delta'(\Delta x^+),$$

$$\{f(x^+),\, g(\tilde{x}^+)\} \;=\; -\tfrac{\pi}{k}\,\tfrac{9\,\tilde{\omega}_0}{8\,\Delta}\,\delta'(\Delta x^+) \quad (24)$$

where

$$\Delta \;=\; \tilde{\kappa}_0^3 - \tfrac{27}{4}\,\alpha^2\,\tilde{\omega}_0^2 \quad (25)$$

- $\implies$ $u(1) \oplus u(1)$ Kač-Moody current for suitable combination of f & g.

- 2.$\tilde{\kappa}_0 = -\frac{1}{4}$ and $\tilde{\omega}_0 = 0$ residual gauge transformation parameters are

$$
\begin{aligned}
\tilde{\lambda}^{(-1)} &= \lambda_f(x^+) + \lambda_g(x^+)\, e^{i\,x^-} + \bar{\lambda}_{\bar{g}}(x^+)\, e^{-i\,x^-} \\
\tilde{\eta}^{(-2)} &= \eta_f(x^+) + \eta_g(x^+) e^{i\,x^-} + \bar{\eta}_{\bar{g}}(x^+) e^{-i\,x^-} + \eta_h(x^+) e^{2i\,x^-} \\
&\quad + \bar{\eta}_{\bar{h}}(x^+) e^{-2i\,x^-} \qquad (26)
\end{aligned}
$$

- One can collect the solution space parameters and the (residual) gauge transformation parameters in:

$$
\begin{aligned}
\{J^a, a = 1, \cdots, 8\} &= \{\bar{g}_\kappa,\, f_\kappa,\, g_\kappa,\, \bar{h}_\omega,\, \bar{g}_\omega,\, f_\omega,\, g_\omega,\, h_\omega\} \\
\{\lambda^a, a = 1, \cdots, 8\} &= \{\bar{\lambda}_{\bar{g}},\, \lambda_f,\, \lambda_g,\, \bar{\eta}_{\bar{h}},\, \bar{\eta}_{\bar{g}},\, \eta_f,\, \eta_g,\, \eta_h\}
\end{aligned}
$$

- Notice that the parameters 8 in each set.

- In this notation, the action of the residual gauge parameters λ on the sol. space parameters J can be summarized:

$$\begin{aligned} \delta J^a &= \partial_+\lambda^a - i\,\hat{f}^a{}_{bc}J^b\lambda^c \\ Q[\lambda^a] &= -\tfrac{k}{4\pi}\int_0^{2\pi} d\phi\,\hat{\eta}_{ab}\,\lambda^a J^b \end{aligned} \tag{27}$$

  the expression for the charge can also be writen compactly as done above.

- The associated Poisson bracket algebra is:

$$\{J^a(x^+), J^b(\tilde{x}^+)\} = i\hat{f}^{ab}_{\ \ c}\,J^c(x^+)\,\delta(\Delta x^+) + \frac{k}{4\pi}\hat{h}^{ab}\delta'(\Delta x^+). \tag{28}$$

  where $\hat{\eta}_{ab}$ & $\hat{f}_{abc}$ are the ones obtained from η_{ab} & f_{abc} by replacing:

$$\alpha^2 \to -\alpha^2.$$

- $\Longrightarrow$ Had we started with a $sl(3,\mathbb{R})$ gauge group, we would end up with an $su(1,2)$ Kač-Moody current with central extension $k = c/6$.(and *vice versa*)

- 3. $\tilde{\kappa}_0 = 0$ & $\tilde{\omega}_0 = 0$

$$\begin{aligned}
\tilde{\lambda}^{(-1)} &= \lambda_{-1}(x^+) + x^- \, \lambda_0(x^+) + (x^-)^2 \, \lambda_1(x^+) \\
\tilde{\eta}^{(-2)} &= \eta_{-2}(x^+) + x^- \, \eta_{-1}(x^+) + (x^-)^2 \, \eta_0(x^+) + (x^-)^3 \, \eta_1(x^+) \\
&\quad + (x^-)^4 \, \eta_2(x^+) \qquad (29)
\end{aligned}$$

- Collecting the parametrizing functions as before:

$$\begin{aligned}
\{J^a, a = 1, \cdots, 8\} &= \{f_{-1}, f_0, f_1, g_{-2}, g_{-1}, g_0, g_1, g_2\} \\
\{\lambda^a, a = 1, \cdots, 8\} &= \{\lambda_{-1}, \lambda_0, \lambda_1, \eta_{-2}, \eta_{-1}, \eta_0, \eta_1, \eta_2\}
\end{aligned}$$

The expression for the change in the sol. space parameters and the associated charge simplifies to:

$$\begin{aligned}
\delta J^a &= \partial_+ \lambda^a - f^a{}_{bc} J^b \lambda^c \\
Q[\tilde{\lambda}] &= \frac{k}{4\pi} \int dx^+ \, \eta_{ab} J^a \lambda^b
\end{aligned} \qquad (30)$$

- The Poisson brackets are:

$$\{J^a(x^+), J^b(\tilde{x}^+)\} = f^{ab}{}_c\, J^c(x^+)\, \delta(\Delta x^+) - \frac{k}{4\pi}\, \eta^{ab}\, \delta'(\Delta x^+) \quad (31)$$

- $\Longrightarrow$ One gets the same current algebra of that of the gauge group *i.e.* $sl(3, \mathbb{R})$ Kač-Moody current for an $sl(3, \mathbb{R})$ gauge group; with central extension $k = c/6$.

Further directions

- It would be interesting to see what current algebras does one get once more spins are included.
- Different choices of asymptotic symmetry algebra $\implies$ diff. chiral induced W-gravities; further study of this analogy is required.
- Generalisations to include super-gravities and higher-spin supergravity of these considerations.

. . . .Thank You.