Modified Channel Crossing Symmetry in Large N U(N) Chern-Simons Matter Theories

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Introduction

- Chern-Simons theory is a gauge theory in 2+1 dimensions given by

$$S_{CS} = \int d^3x \left[i\varepsilon^{\mu\nu\rho} \frac{k}{4\pi} Tr(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho) \right]$$

- Pure Chern-Simons theory is a topological theory; it has no propagating degrees of freedom.
- However when coupled to matter, it has interesting dynamics, like a duality between bosonic and fermionic theories, and violation of channel crossing symmetry.
- Has interesting applications in planar condensed matter physics. Provides a very good model for anyonic systems.

Theories

Chern-Simons coupled to fundamental fermions:

$$S=\int d^3x\left[i\varepsilon^{\mu\nu\rho}\frac{k_F}{4\pi}Tr(A_\mu\partial_\nu A_\rho-\frac{2i}{3}A_\mu A_\nu A_\rho)+\bar{\psi}\gamma^\mu D_\mu\psi+m_F\bar{\psi}\psi\right]$$

is dual to Chern-Simons coupled to critical bosons, i.e.

$$S=\int d^3x\left[i\varepsilon^{\mu\nu\rho}\frac{k_B}{4\pi}Tr(A_\mu\partial_\nu A_\rho-\frac{2i}{3}A_\mu A_\nu A_\rho)+D_\mu\bar{\phi}D^\mu\phi\right.$$
$$\left.+\sigma\bar{\phi}\phi+N_B\frac{m_B^2}{b_4}\sigma-N_B\frac{\sigma^2}{2b_4}\right].$$

in the Wilson-Fischer limit

$$b_4\to\infty,\quad m_B\to\infty,\quad \frac{4\pi m_B^2}{b_4}=m_B^{\rm cri}=\text{fixed}.$$

Duality

- These theories map to each other under following parameter identifications:

$$
\begin{aligned}
k_F &= -k_B, \\
N_F &= |k_B| - N_B, \\
\lambda_B &= \lambda_F - \mathrm{sgn}(\lambda_F), \\
m_F &= -m_B^{\mathrm{cri}} \lambda_B.
\end{aligned}
$$

- Where we work in 't Hooft coupling limit:
  $k_B, k_F, N_B, N_F \to \infty$ with $\lambda_B = \frac{N_B}{k_B}$ and $\lambda_F = \frac{N_F}{k_F}$ held fixed

Check for duality

- This duality has been checked at many places.
  - The spectra and 3-point functions of single trace operators match ([1110.4382], [1110.4386], [1112.1016])
  - Thermal partition functions match ([1211.4843], [1301.6169])
- In [1404.6373] we aimed at proving that the exact S-matrix for $2 \to 2$ scattering follows this duality.
- While working on this problem we encountered many unusual features of these theories.

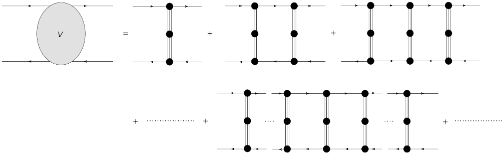
Process and Channels

We will evaluate the exact S-matrix for $2 \to 2$ scattering of bosons minimally coupled to Chern-Simons theory in all the channels which are

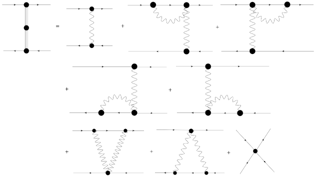
- S-channel, i.e. particle-antiparticle singlet scattering channel
  $P_i(p_1) + A^i(p_2) \to P_j(-p_3) + A^j(-p_4)$
- T-channel, i.e. particle-antiparticle adjoint scattering channel
  $P_i(p_1) + A^j(p_2) \to P_i(-p_3) + A^j(-p_4)$
- U-channel, i.e. particle-particle scattering channel
  $P_i(p_1) + P^j(p_2) \to P_i(-p_3) + P^j(-p_4)$

Method

The all-order 4-point function in Euclidean lightcone gauge $(A_- = \frac{A_1 - iA_3}{\sqrt{2}} = 0)$ for the bosonic theory for general ϕ^4 coupling in 't Hooft limit is diagrammatically represented below: ([1110.4382],[1404.6373])

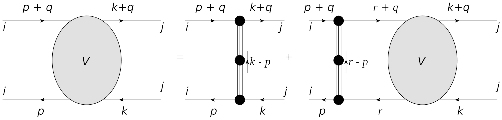


where



Method

In 't Hooft large N limit this is just a sum over ladder diagrams which takes the form of an integral equation called 'Schwinger-Dyson equation', which is diagrammatically represented as:



In Euclidean lightcone co-ordinates, this translates to

$$V(p,k,q) = V_0(p,k,q) + \int \frac{d^3r}{(2\pi)^3} \frac{V_0(p,r,q_3) N V(r,k,q_3)}{\left(r^2+c_B^2\right)\left((r+q)^2+c_B^2\right)}$$

$$NV_0(p,k,q_3) = -4\pi i \lambda_B q_3 \frac{(k+p)_-}{(k-p)_-} + \tilde{b_4}$$

where $k_\pm = \frac{k_1 \pm i k_3}{\sqrt{2}}$, $\qquad \tilde{b_4} = 2\pi\lambda_B^2 c_B - b_4.$

Solution

The Schwinger-Dyson equation can be easily solved using complex analytic techniques to yield ([1404.6373])

$$NV = e^{-2i\lambda_B\left(\tan^{-1}\left(\frac{2(a(k))}{q_3}\right)-\tan^{-1}\left(\frac{2(a(p))}{q_3}\right)\right)}\left(4\pi i\lambda_B q_3\frac{p_-+k_-}{p_--k_-}+j(q_3,\lambda_B)\right)$$

where

$$\frac{j(q_3,\lambda_B)}{4\pi i\lambda_B q_3} =$$
$$\left(\frac{\left(4\pi i\lambda_B q_3+\tilde{b_4}\right)e^{2i\lambda_B\tan^{-1}\left(\frac{2c_B}{q_3}\right)}+\left(-4\pi i\lambda_B q_3+\tilde{b_4}\right)e^{\pi i\lambda_B \mathrm{sgn}(q_3)}}{\left(4\pi i\lambda_B q_3+\tilde{b_4}\right)e^{2i\lambda_B\tan^{-1}\left(\frac{2c_B}{q_3}\right)}-\left(-4\pi i\lambda_B q_3+\tilde{b_4}\right)e^{\pi i\lambda_B \mathrm{sgn}(q_3)}}\right)$$

$$a(p)=\sqrt{2p_+p_-+c_B^2}$$

T and U channel results

When this Euclidean result is analytically continued to T and U channels, and the onshell limit is taken we get the scattering matrix in those channels:

$$
\begin{aligned}
&T_T^B(p_1, p_2, p_3, p_4, k_B, \lambda_B, \tilde{b}_4, c_B) \\
=&E(p_1, p_2, p_3) \frac{4i\pi}{k_B} \sqrt{\frac{u\ t}{s}} \\
&- \frac{4\ i\pi}{k_B} \sqrt{-t} \quad \frac{(\tilde{b}_4 - 4\pi i \lambda_B \sqrt{-t}) + (\tilde{b}_4 + 4\pi i \lambda_B \sqrt{-t}) e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}{-(\tilde{b}_4 - 4\pi i \lambda_B \sqrt{-t}) + (\tilde{b}_4 + 4\pi i \lambda_B \sqrt{-t}) e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}
\end{aligned}
$$

T and U channel results

and

$$
\begin{aligned}
&T_U^B(p_1, p_2, p_3, p_4, k_B, \lambda_B, \tilde{b}_4, c_B) \\
=&E(p_1, p_2, p_3)\frac{4i\pi}{k_B}\sqrt{\frac{s\ t}{u}} \\
&-\frac{4\ i\pi}{k_B}\sqrt{-t}\ \ \frac{(\tilde{b}_4 - 4\pi i\lambda_B\sqrt{-t}) + (\tilde{b}_4 + 4\pi i\lambda_B\sqrt{-t})e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}{-(\tilde{b}_4 - 4\pi i\lambda_B\sqrt{-t}) + (\tilde{b}_4 + 4\pi i\lambda_B\sqrt{-t})e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}
\end{aligned}
$$

Duality of T and U channel results

When we take Wilson-Fischer limit, i.e. $b_4 \to \infty$, of these results,

$$T_T^{B\infty}(p_1, p_2, p_3, p_4, k_B, \lambda_B, c_B) = E(p_1, p_2, p_3)\frac{4i\pi}{k_B}\sqrt{\frac{u\ t}{s}} - \frac{4\ i\pi}{k_B}\sqrt{-t}\ \frac{1 + e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}{1 - e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}$$

$$T_U^{B\infty}(p_1, p_2, p_3, p_4, k_B, \lambda_B, c_B) = E(p_1, p_2, p_3)\frac{4i\pi}{k_B}\sqrt{\frac{s\ t}{u}} - \frac{4\ i\pi}{k_B}\sqrt{-t}\ \frac{1 + e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}{1 - e^{-2i\lambda_B \tan^{-1}\left(\frac{\sqrt{-t}}{2|c_B|}\right)}}$$

Which match with the fermionic results derived independently under duality mapping. This matching is very nontrivial.

Something wrong with S-channel?

However, when this result is naively analytically continued to S-channel,

$$T_S^{trial} = (\pi\lambda_B)\, 4i\sqrt{s}E(p_1,p_2,p_3)\sqrt{\frac{u}{t}}$$
$$+ (\pi\lambda_B)\, 4\sqrt{s}\left(\frac{\left(4\pi\lambda_B\sqrt{s}+\tilde{b_4}\right)+e^{i\pi\lambda_B}\left(-4\pi\lambda_B\sqrt{s}+\tilde{b_4}\right)\left(\frac{\frac{1}{2}+\frac{c_B}{\sqrt{s}}}{\frac{1}{2}-\frac{c_B}{\sqrt{s}}}\right)^{\lambda_B}}{\left(4\pi\lambda_B\sqrt{s}+\tilde{b_4}\right)-e^{i\pi\lambda_B}\left(-4\pi\lambda_B\sqrt{s}+\tilde{b_4}\right)\left(\frac{\frac{1}{2}+\frac{c_B}{\sqrt{s}}}{\frac{1}{2}-\frac{c_B}{\sqrt{s}}}\right)^{\lambda_B}}\right)$$

Unlike other channels this result is not consistent with unitarity due to the presence of a nontrivial cut in the S channel.

Remedy: the conjecture

Hence we propose a conjecture for the S-channel S-matrix:In the case of the bosonic theory we conjecture that S matrix in the S-channel is given by ([1404.6373])

$$T_S^B = -i\left(\cos(\pi\lambda_B) - 1\right) I(p_1, p_2, p_3, p_4) + \frac{\sin(\pi\lambda_B)}{\pi\lambda_B} T_S^{trial}$$

Which, in centre of mass frame can be written as:

$$T_S(\sqrt{s}, \theta) = 8\pi i\sqrt{s}(1 - \cos(\pi\lambda_B))\delta(\theta) + 4i\sqrt{s}\sin(\pi\lambda_B) Pv\left(\cot\left(\frac{\theta}{2}\right)\right)$$

$$+ 4\sqrt{s}\sin(\pi|\lambda_B|) j_M,$$

$$j_M = \left(\frac{\left(4\pi|\lambda_B|\sqrt{s} + \tilde{b_4}\right) + e^{i\pi|\lambda_B|}\left(-4\pi|\lambda_B|\sqrt{s} + \tilde{b_4}\right)\left(\frac{\frac{1}{2} + \frac{c_B}{\sqrt{s}}}{-\frac{1}{2} + \frac{c_B}{\sqrt{s}}}\right)^{|\lambda_B|}}{\left(4\pi|\lambda_B|\sqrt{s} + \tilde{b_4}\right) - e^{i\pi|\lambda_B|}\left(-4\pi|\lambda_B|\sqrt{s} + \tilde{b_4}\right)\left(\frac{\frac{1}{2} + \frac{c_B}{\sqrt{s}}}{-\frac{1}{2} + \frac{c_B}{\sqrt{s}}}\right)^{|\lambda_B|}}\right)$$

This conjectured S-matrix is manifestly unitary.

Salient features of the conjecture

- It is compatible with the duality, i.e. it is dual to a similar conjecture on the fermionic side.
- It proposes a δ function in forward scattering. So it is non-analytic. Perturbatively, it should start showing up at every odd loop order from the 1st loop order.
- Its analytic piece differs from the result for other channels by the factor of $\frac{\sin(\pi\lambda)}{\pi\lambda}$. This should show up in perturbative expansion from second loop order.

Connection with Aharonov-Bohm scattering

- The equation of motion for the gauge field in Chern-Simons theory is

$$F_{\mu\nu} = -\frac{2\pi}{k}\varepsilon_{\mu\nu\rho}J^{\rho}$$

- F_{ij} component of this equation says that every charge traps a magnetic flux.
- i.e. a charged particle going around another should behave like a charged particle going around a magnetic flux.
- So the $2 \to 2$ scattering in this theory in nonrelativistic limit and in centre of mass frame becomes the quantum mechanical problem with a charge moving around a point magnetic flux at origin.
- This is the Aharonov-Bohm scattering problem.

Aharonov-Bohm scattering as the nonrelativistic limit

When we take "near-threshold" nonrelativistic limit ([1407.1322]) of the conjecture

$$\frac{\delta b_4}{c_B} \to 0, \quad \frac{k}{c_B} \to 0, \quad \frac{k}{c_B}\left(\frac{c_B}{\delta b_4}\right)^{\frac{1}{2|\lambda_B|}} = \text{ fixed.}$$

$$\delta b_4 = \tilde{b_4} - \tilde{b}_4^{crit}, \quad \tilde{b}_4^{crit} = 8\pi c_B|\lambda_B|$$

we find that the result is a one parameter set of solutions

$$T_S(\sqrt{s}, \theta) = -16\pi i c_B(\cos(\pi\lambda_B) - 1)\delta(\theta) + 8ic_B \sin(\pi\lambda_B) Pv\left(\cot\left(\frac{\theta}{2}\right)\right)$$

$$+ 8c_B|\sin(\pi\lambda_B)|\frac{1 + e^{i\pi|\lambda_B|}\frac{A_R}{k^{2|\lambda_B|}}}{1 - e^{i\pi|\lambda_B|}\frac{A_R}{k^{2|\lambda_B|}}},$$

$$A_R = \left[\frac{\delta b_4\,(2c_B)^{2|\lambda_B|}}{16\pi|\lambda_B|c_B.}\right]$$

Aharonov-Bohm scattering as the nonrelativistic limit

which matches with the selfadjoint extension of the Aharonov - Bohm - Ruijsenaars scattering matrix ([hep-th/9406213])

$$
\begin{aligned}
T_{NR} &= -16\pi i c_B \left(\cos\left(\pi\lambda_B\right) - 1\right)\delta(\theta) + 8 i c_B \sin(\pi\lambda_B)\mathrm{Pv}\left(\cot\frac{\theta}{2}\right), \\
&+ 8 c_B |\sin\pi\lambda_B| \frac{1 + e^{i\pi|\lambda_B|}\frac{A_{NR}}{k^{2|\lambda_B|}}}{1 - e^{i\pi|\lambda_B|}\frac{A_{NR}}{k^{2|\lambda_B|}}} \\
A_{NR} &= \frac{-1}{w}\left(\frac{2}{R}\right)^{2|\lambda_B|}\frac{\Gamma(1+|\lambda_B|)}{\Gamma(1-|\lambda_B|)}.
\end{aligned}
$$

under following parameter map:

$$
-w\left(c_B R\right)^{2|\lambda_B|} = \frac{c_B}{\delta b_4}\left(16\pi|\lambda_B|\frac{\Gamma(1+|\lambda_B|)}{\Gamma(1-|\lambda_B|)}\right).
$$

Non-analyticity from Feynman diagrams?

- Thus, this conjecture is the relativistic generalization of Aharonov-Bohm scattering, which retains the characteristic δ function in forward scattering.
- There are no textbook examples of such non-analytic piece coming from onshell Feynman diagram computation.
- However when we compute the nonrelativistic Coulomb gauge bosonic scattering diagrams in Chern-Simons theory, at first two loop order they reduce to the Lippmann-Schwinger diagrams obtained from the anyonic Aharonov-Bohm scattering.
- These Lippmann-Schwinger diagrams on evaluation do give the desired delta function at one loop, provided a particular onshell limit is taken when we consider the near forward scattering region.
- Also they give the conjectured answer at second loop.

Landau gauge calculation

- The modified channel crossing symmetry is evident if the calculations are done in a manifestly covariant gauge. Here we work in Landau gauge.
- Unlike lightcone gauge, in Coulomb and Landau gauges obtaining the exact result by solving an integral equation is not feasible due to nonabelian gauge coupling. So we work perturbatively.
- We find that at one loop in the offshell calculation, we can divide the result into two parts:
  - one that onshell goes smoothly to the analytic piece
  - the other that goes to delta function, i.e. it is zero for nonzero scattering angles, but blows up in forward scattering.
- If 'the other' piece is evaluated near forward direction in an onshell limit analogous to the coulomb gauge nonrelativistic calculation, we find that it gives rise to the desired δ function in the S-channel, while in other channels it vanishes.

Summary

- The lightcone gauge calculation for $2 \to 2$ singlet scattering amplitude in Chern-Simons coupled to fundamental matter (boson/fermion) is found to violate unitarity.
- A conjecture for the S-matrix in this channel is made which is unitary, and reduces to Aharanov-Bohm-Ruijsenaars anyonic scattering in the nonrelativistic limit.
- This conjecture has nonanalyticity in the form of a δ function in forward scattering direction, and its analytic piece is different from the analytic continuation of other channels.
- These features can be seen even in perturbative Feynman diagram calculation in a particular onshell limit, which is currently being analysed.

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Major references:

- S. Jain, M. Mandlik, S. Minwalla, T. Takimi, S. R. Wadia and S. Yokoyama, [1404.6373]
- Y. Dandekar, M. Mandlik, S. Minwalla, [1407.1322]
- O. Aharony, G. Gur-Ari, and R. Yacoby, [1110.4382]
- S. Giombi, S. Minwalla, S. Prakash, S. P. Trivedi, S. R. Wadia and Xi Yin, [1110.4386]
- J. Maldacena and A. Zhiboedov, [1112.1016]
- O. Aharony, S. Giombi, G. Gur-Ari, J. Maldacena, and R. Yacoby, [1211.4843]
- S. Jain, S. Minwalla, T. Sharma, T. Takimi, S. R. Wadia and S. Yokoyama, [1301.6169]
- G. Amelino-Camelia and D. Bak, [hep-th/9406213]