Backreaction effects of matter coupled higher derivative gravity

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(Based on arXiv:1409.8019, work done with Ramadevi)

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Introduction

Strongly coupled field theories

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- **Specific limit:**

Fluid-Gravity Correspondence

Hydrodynamics: Large length scales in field theories.

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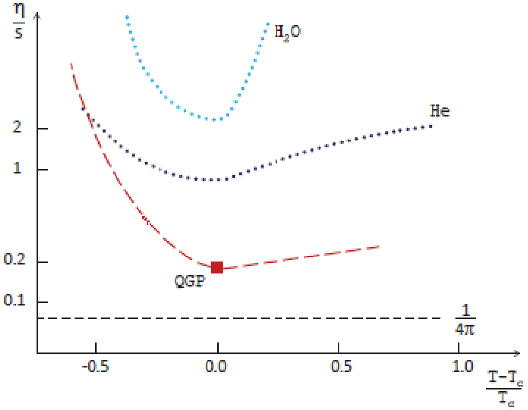

Figure: Sera Cremonini et al. arXiv:1206.3581

- **The figure shows expectation taking analogy with H_2O and He plots.**

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Use AdS-CFT correspondence

$$\left\langle e^{i\int_{\partial M}\phi_0\tilde{O}}\right\rangle = e^{iS_{cl}[\phi]}$$

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Giving,

$$\langle \tilde{O}(x_1)\tilde{O}(x_2)\rangle = i^2 \frac{\delta^2}{\delta\phi_0(x_1)\delta\phi_0(x_2)} e^{iS_{cl}[\phi_0]} \qquad \textbf{at } \phi_0 \to 0$$

Note: We need to know the background geometry for the exact form of above correlator.

Background solutions in gravity

Einstein gravity with higher derivatives

$$S = \int d^D x \sqrt{-g}\left(R - 2\Lambda + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\right)$$

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with
$$\begin{aligned} \Lambda &= -\frac{(D-1)(D-2)}{2} \\ ds^2 &= \frac{1}{r^2}\left((-f(r)dt^2 + (d\vec{x}^2)) + \frac{dr^2}{f(r)} \right) \end{aligned}$$

$$\text{where, } f(r) = 1 - r^{D-1} + \delta + \kappa r^{2(D-1)}$$

δ and κ being dependent on D, c_1, c_2 and c_3
(Kats and Petrov: arXiv 0712.0743)

Matter coupling

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Consider the action

$$S = \frac{1}{2\kappa_D^2}\int d^D x[R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r))]\ ;\ V(\Phi(r)) = 2\Lambda e^{\alpha\Phi(r)}$$

The already known solution to this action shows effects of matter on the background metric

It is interesting to look upon the case of matter coupled higher derivative gravity

$$S = \frac{1}{2\kappa_D^2}\int d^D x(R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r)) + \beta G(\Phi(r))R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho})$$

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Varying the action with respect to metric field and scalar field gives,

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= T^{matter}_{\mu\nu} + T^{hd}_{\mu\nu} \\ \frac{8}{D-2}\Box\Phi(r) &= \partial_\Phi\left(\beta G(\Phi(r)) R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - V(\Phi(r))\right) \end{aligned}$$

$$\text{where}, T_{\mu\nu} = -\frac{1}{\sqrt{-g}}\frac{\partial\sqrt{-g}L_M}{\partial g^{\mu\nu}}$$

- **Choosing an ansatz is the most crucial part in solving the above equation.**

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- **To find the solution up to $\mathcal{O}(\beta)$, we choose the ansatz as:**

$$
\begin{aligned}
ds^2 &= -r^{-2a}(1 - r^{c(\beta,r)})dt^2 + \frac{dr^2}{r^{-2a}(1 - r^{c(\beta,r)})r^4} + r^{-2a}d\vec{x}^2 \\
\Phi(r) &= m\mathrm{Log}(r) + \beta\Phi_1(r)
\end{aligned}
$$

where $c[\beta, r]$ is chosen as:

$$
c[r] = c + \frac{Log(1 - \beta\kappa(r))}{Log(r)}
$$

The first order correction $\kappa(r)$, is obtained as:

$$-(D-4)\left(4+\frac{(D-4)(D-1)16^{D-\frac{6\left((-1)^D+1\right)m}{D-2}}(g\times r)^{-aD+1}}{(D-2)\left((D-2)^2\alpha^2-16(D-1)\right)}\right)+\frac{2\Lambda r^{\frac{m}{2}(\alpha+\gamma)}}{(D-2)}\times$$

$$\left(r^{m\gamma+a(D-1)}\frac{\left((D-2)^2\alpha\left(3(D-2)\alpha+4(D-3)\gamma\right)-16(D-4)(D-3)\right)}{\left((D-2)^2\alpha\gamma+16(D-1)\right)}\right.$$

$$\left.-r^{m\alpha}\frac{\left(4(D-1)(D-2)^2\alpha^2+8(D-3)(D-2)^2\alpha\gamma+64(D-4)\right)}{\left((D-2)^2\alpha(\alpha+2\gamma)+16(D-1)\right)}\right)$$

Horizon gets a linear order correction as: $\mathbf{r}_h = 1+\beta r_1$.

Shear viscosity

Kubo formula for viscosity Low energy and low momentum limit of retarded Green's function of stress tensor in CFT.

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega} Im G^{R}_{xy;xy}(\omega, \mathbf{k}=0)$$

- Translate the calculation of the correlator to a holographic one

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- Translate the calculation of the correlator to a holographic one
- perturbation in the xy components of the metric.

$$g_{xy} = g^0_{xy} + \epsilon h_{xy}(r,x) = g^0_{xy} + \epsilon g_{ii}\phi(r,x), \qquad \left[x = (t, \overrightarrow{x})\right]$$

with

$$\phi(r,x) = \frac{1}{(2\pi)^4} \int d^4k e^{-ik.x} \phi(r,k),$$

$$\left[k = (\omega, \overrightarrow{k}), \qquad k.x = k_\mu x^\mu\right]$$

- Effective action up to order two for any power of curvature tensor under this perturbation:

$$S \sim \int \frac{d\omega d^{D-2}k}{(2\pi)^{D-1}} dr \left[A(r,k)\phi''(r,k)\phi(r,-k) + B(r,k)\phi'(r,k)\phi'(r,-k) \right.$$
$$+ C(r,k)\phi'(r,k)\phi(r,-k) + D(r,k)\phi(r,k)\phi(r,-k)$$
$$\left. + E(r,k)\phi''(r,k)\phi''(r,-k) + F(r,k)\phi''\phi'(r,-k) \right]$$

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- Corresponding to this metric,

$$\eta = \left[\sqrt{\frac{-g_{rr}}{g_{tt}}}\left(A - B + \frac{F'}{2}\right) + \left(E\left(\sqrt{\frac{-g_{rr}}{g_{tt}}}\right)'\right)'\right]\Bigg|_{r=r_H}$$

(Myers, Poulos and Sinha: arXiv 0903.2834)

Entropy density

Wald formula

$$S = -2\pi \int_{\Sigma} d^{D-2}x\sqrt{-h}\frac{\delta L}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

1. $\epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$
2. $\epsilon_{\mu\nu} = \xi_{\mu}\eta_{\nu} - \xi_{\nu}\eta_{\mu}$

The L above is chosen by writing the action as

$$S_{action} \sim \int d^5x\sqrt{-g}L$$

Backreaction effects on η and s

For the matter coupled higher derivative action the shear viscosity and entropy density is

$$\eta = \frac{1}{2\kappa_D^2}\left[r_h^P - \frac{4\beta(D-2)^2\alpha\gamma\left(\alpha^2(D-2)^2-16(D-1)\right)r_0^M}{\left(16+\alpha^2(D-2)^2\right)^2}\right]$$

$$s = \frac{1}{2\kappa_D^2}\left[4\pi r_h^P - \frac{128\pi\beta(D-4)\left(\alpha^2(D-2)^2-16(D-1)\right)r_0^N}{\left(16+\alpha^2(D-2)^2\right)^2}\right]$$

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Giving,

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{4\beta\left(-16(D-1)+(D-2)^2\alpha^2\right)\left(-8(D-4)+(D-2)^2\alpha\gamma\right)}{\left(16+(D-2)^2\alpha^2\right)^2}\right]$$

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Thank You

Anti de Sitter space

- Solution to Einstein equation with negative cosmological constant.
- Space of Lorentzian signature $(-, +, +..+)$, but of constant negative curvature
- On boundary

$$-x_0^2 + \sum_{i=1}^{d-1} x_i^2 - x_{d+1}^2 = R^2$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \frac{1}{2} g_{\mu\nu} \Lambda = 0 \tag{1}$$

$$\implies R = \Lambda \frac{(d+1)}{d-1} \tag{2}$$

Thus constant negative curvature

$$R_{\mu\nu} = \frac{1}{d-1} \Lambda g_{\mu\nu} \tag{3}$$

Parametrize,

$$\Lambda = -\frac{d(d-1)}{L^2}$$

$$\implies R = -\frac{d(d+1)}{L^2} \tag{4}$$

Back

Conformal Field Theory

- Guage field theory with enhanced symmetries.
- Poincare symmetry + dilatations + special conformal symmetry

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- Guage field theory with enhanced symmetries.
- Poincare symmetry + dilatations + special conformal symmetry
- Dilatations: $x_\mu \to \lambda x_\mu$
- special conformal: Translation+Inversion+Translation

Under special conformal: $x'^\mu = x^\mu + 2x^\mu b.x - x^2 b^\mu$

Back

$$\text{Translation:} P_\mu = -i\partial_\mu \tag{5}$$
$$\text{Dilatations:} D = -ix^\mu \partial_\mu \tag{6}$$
$$\text{Rotations:} L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu) \tag{7}$$
$$\text{SCT:} G_\mu = -2ix_\mu (x.\partial) + ix^2 \partial_\mu \tag{8}$$

Back