Supergravity, Non-Geometric Fluxes and Double Field Theory

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The Problem

supergravity is not complete!

The 'good' sector

- there is no problem with the supergravity fields that describe physical degrees of freedom

- for instance, in each dimension $D \leq 11$ maximal supergravity describes $128 + 128$ degrees of freedom

The 'problematic' sector

- D-dimensional maximal supergravity also contains (D-1)-form and D-form potentials that are not controlled by the representation theory of the supersymmetry algebra and that are not related to each other via dimensional reduction. These potentials couple to domain walls and space-filling branes

- Note: (D-1)-form potentials ⇔ gauged supergravities

Outline

An Update on Branes

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Brane Classification $\rightarrow$ Exotic Branes $\Leftrightarrow$ Non-Geometric Fluxes

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Branes

The NS-NS 2-form B_2 suggests a half-supersymmetric string

Similarly, the 3-form C_3 of 11D sugra couples to a half-susy M2-brane

Branes are extended objects with a number of worldvolume and transverse directions. They are an essential part of (non-perturbative) string theory

sugra potential $\leftrightarrow$ *half-supersymmetric brane*

Does it always work as simple as that?

Strings and T-duality

In $D < 10$ we have a singlet NS-NS 2-form B_2 as well as 1-forms $B_{1,A}$ $(A = 1, \ldots, 2d)$ that transform as a vector under the T-duality group SO(d,d) with $d = 10 - D$

To construct a gauge-invariant WZ term

$$\mathcal{L}_{\text{WZ}}(D < 10) = B_2 + \eta^{AB} \mathcal{F}_{1,A} B_{1,B}$$

we need to introduce "extra scalars" $b_{0,A}$ via $\mathcal{F}_{1,A} = d\, b_{0,A} + B_{1,A}$

Counting the Bosonic Worldvolume D.O.F.

$$D = 10 \quad : \qquad (10-2) = 8\,,$$

$$D < 10 \quad : \qquad (D-2) + 2(10-D) \neq 8\,!$$

Twice too many 'extra scalars' $b_{0,A}$ $\rightarrow$ 'doubled geometry'

Hull, Reid-Edwards (2006-2008)

Self-duality conditions on the extra scalars $b_{0,A}$ give correct counting

‘Wess-Zumino term requirement’

the construction of a gauge-invariant WZ term may require, besides the embedding coordinates, the introduction of a number of extra worldvolume p-form potentials

worldvolume supersymmetry requires that these worldvolume fields fit into a multiplet with 16 supercharges

Does the ‘WZ term requirement’ always lead to the rule that

potential ⇔ half-susy brane ?

'New' supergravity development

- The T-duality representations of all high-rank form potentials have been determined using three different techniques:

    - closure of the supersymmetry algebra

      de Roo, Hartong, Howe, Kerstan, Ortín, Riccioni + E.B. (2005-2010)

    - using the very extended Kac-Moody algebra E_{11}

      Riccioni, West (2007); Nutma + E.B. (2007)

      a similar analysis can be done for E_{10}, see e.g. Nicolai, Fischbacher (2002)

    - using the embedding tensor technique

      for a review, see de Wit, Nicolai, Samtleben (2008)

Outline

Question

given a $(p+1)$-form potential which components of its T-duality repres. couple to a half-supersymmetric brane ?

A scaling symmetry

All potentials transform as a representation of the T-duality group O(d,d;$\mathbb{Z}$) and scale under a $\mathbb{R}^+$ scaling symmetry

The scaling weight α determines the dependence of the brane tension T on the string coupling constant g_s via

$$\mathrm{T} \sim (g_s)^{\alpha}$$

This scaling weight is invariant under dimensional reduction

A universal pattern arises

α	potentials	branes
$\alpha = 0$	$B_{1,A}\ , B_2$	fundamental
$\alpha = -1$	$C_{2n+1,a}\ , C_{2n,\dot{a}}$	Dirichlet
$\alpha = -2$	$D_{D-4},\ D_{D-3,A},\ D_{D-2,A_1A_2},\ D_{D-1,A_1\cdots A_3},\ D_{D,A_1\cdots A_4}$	solitonic
$\vdots$	$\vdots$	$\vdots$

A $(a, \dot{a})$ are vector (spinor)-indices of T-duality

10D : $\alpha = 0, -1, -2, -3, -4$ universal behaviour for $-4 < \alpha \leq 0$

Outcome Wess-Zumino Term Requirement

Riccioni + E.B. (2010)

There is a simple group-theoretical characterization of which components of the T-duality representation couple to a half-supersymmetric brane

- the (group-theoretical) details can be found in our papers

for an alternative E_{11} derivation, see Kleinschmidt (2011)

- Comparing branes in different dimensions an interesting patterns arises ...

'Wrapping Rules'

the wrapping rules of 'standard geometry'

$$\text{any brane} \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

only works for D-branes !

Counting D-branes

Dp-brane	IIA/IIB	9	8	7	6	5	4	3
0	1/0	1	2	4	8	16	32	64
1	0/1	1	2	4	8	**16**	**32**	64
2	1/0	1	2	4	8	**16**	32	64
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
8	1/0	1						
9	0/1							

spinors $(\mathrm{Dp})_\alpha\,,\qquad \alpha = 1 \cdots 2^{9-D}$

Fundamental Branes

the wrapping rules of fundamental branes are given by

$$\mathrm{T}_{\mathsf{F}} \sim 1 \ : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{undoubled} \end{cases}$$

the extra input comes from pp-waves

Two points of view:

'new objects' (pp-waves) or 'string geometry' (doubled geometry)

Counting Fundamental Branes

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	**10**	**12**	14
1	1/1	1	1	1	1	**1**	1	1

$(F0)_A$ and F1

$A = 1, \ldots, 2(10 - D)$

Solitonic Branes with $T \geq 3$

the wrapping rules of solitonic branes are given by

$$\mathrm{T_S} \sim (g_s)^{-2} \; : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{undoubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

For instance, in 9D we have two solitonic 5-branes coming from an un-wrapped NS5-brane and a KK monopole

$$\text{10D KK monopole}: \quad \begin{cases} 5+1 \text{ worldvolume directions} \\ 1 \text{ isometry direction} \\ 3 \text{ transverse directions} \end{cases}$$

Counting Solitonic Branes with $T \geq 3$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						**1**	**12**	
1					1	**10**		
2				1	8			
3			1	6				
4		1	4					
5	1/1	2						

S(D-5)-brane and S(D-4)-brane$_A$

Solitonic Branes with $T \leq 2$

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

The red numbers follow from imposing the Wess-Zumino term requirement

Solitonic Branes with $T \leq 2$

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2				1	8	40	**160**	**560**
3			1	6	24	80	**240**	
4		1	4	12	32	80		
5	1/1	2	4	8	16			

Precisely the same numbers are reproduced by the solitonic wrapping rule !

Question

what is the 10D origin of the solitonic branes with $T \leq 2$?

needed for T-duality!

Exotic Branes

- T-duality can be recovered by assuming that supergravity can be extended with a set of mixed-symmetry potentials with an underlying E_{11}-symmetry structure

- These mixed symmetry potentials couple to extended objects called 'generalized monopoles' or 'exotic branes'. These extended objects have worldvolume, transverse and special isometry directions

see, e.g., Lozano-Tellechea, Ortín (2001)

see also work by de Boer and Shigemori (2010, 2012)

The general picture

The branes with fixed $\alpha = 0, -1, -2, -3, -4$ satisfy specific wrapping rules

$$\mathrm{T} \sim (g_s)^{-3} \; : \quad \begin{cases} \text{wrapped} & \rightarrow \quad \text{doubled} \\ \text{unwrapped} & \rightarrow \quad \text{doubled} \end{cases}$$

$$\mathrm{T} \sim (g_s)^{-4} \; : \quad \left\{ \text{wrapped} \quad \rightarrow \quad \text{doubled} \right.$$

The branes with $\alpha < -4$ do not have a higher-dimensional brane origin. They belong to 'non-geometric' orbits

see also: Dibitetto, Fernandez-Melgarejo, Marqués and Roest (2012)

A Relation

exotic branes $\Leftrightarrow$ non-geometric fluxes

An Example: 7D Solitonic Gaugings

The 7D solitonic 6-form potentials $D_{6,ABC}$ $(A = 1, \ldots, 6)$ transform as **20** under $SO(3,3)$. These 6-forms are dual to constant fluxes θ_{ABC}

10D potential	10D brane	7D 6-form potential	flux
B_6	NS5 (5_2)	1	H_{abc}
$h_{7,1}$	KK5 (5_2^1)	9	$f^a{}_{bc}$
$B_{8,2}$	5_2^2	9	$Q^{ab}{}_c$
$D_{9,3}$	5_2^3	1	R^{abc}

a=1,2,3

see also Haßler, Lüst (2013); Kimura, Sasaki (2013); Chatzistavrakidis, Gautason, Moutsopoulos, Zagermann (2013)

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An Update on Branes

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The relation with Double Field Theory

Summary and Open Issues

A very brief review of bosonic DFT

In DFT one extends the usual spacetime coordinates x^μ with a set of winding coordinates $\tilde{x}_\mu$: $X^M = (x^\mu, \tilde{x}_\mu)$

The metric $g_{\mu\nu}$ and the NS-NS 2-form $B_{\mu\nu}$ get combined into a generalized metric $\mathcal{H}_{MN}(X)$

The relation with an action and transformation rules of supergravity can only be made after imposing the strong constraint which can be solved by assuming that all fields only depend on x^μ

Siegel (1993); Hohm, Hull, Zwiebach (2009 - 2014)

Supergravity and T-duality

D	$\alpha = -2$ potentials	branes
10	B_6	solitonic
7	$B_3\,,\ B_{4,A}\,,\ D_{5,AB}\,,\ D_{6,ABC}\,,\ D_{7,ABCD}$	solitonic

To achieve T-duality we are forced to extend B_6 as follows:

$$B_6 \quad \Rightarrow \quad B_6\,,\ h_{7,1}\,,\ D_{8,2}\,,\ D_{9,3}\,,\ D_{10,4}$$

cp. to $B_2 \quad \Rightarrow \quad B_2\,,\ g_{1,1}$

Question

Why is it that DFT gets away with T-duality without introducing mixed-symmetry potentials?

DFT does not cover all sectors!

So far we have $\alpha = 0$ DFT (Siegel; Hohm, Hull, Zwiebach) and $\alpha = -1$ DFT (Hohm, Kwak)

Where does B_6 fits into DFT ? $\Rightarrow$ we need $\alpha = -2$ DFT!

In SUGRA one can dualize B_2 into B_6 without dualizing the metric tensor $g_{\mu\nu}$ but in DFT B_2 is part of the generalized metric $\mathcal{H}_{MN}$!

A DFT 4-form Potential D_{MNPQ}

One should look for a duality relation between the generalized metric $\mathcal{H}_{MN}$ and B_6 plus all the solitonic mixed-symmetry potentials

This suggests a DFT 4-form potential D_{MNPQ} $\qquad M = ({}^{\mu}, {}_{\mu})$

$$
\begin{aligned}
D^{\mu_1\cdots\mu_4} &\rightarrow B_6 \\
D^{\mu_1\cdots\mu_3}{}_{\mu_4} &\rightarrow h_{7,1} \\
D^{\mu_1\mu_2}{}_{\mu_3\mu_4} &\rightarrow D_{8,2} \\
D^{\mu_1}{}_{\mu_2\cdots\mu_4} &\rightarrow D_{9,3} \\
D_{\mu_1\cdots\mu_4} &\rightarrow D_{10,4}
\end{aligned}
$$

Poincare Duality

A possible duality relation between $\mathcal{H}_{MN}$ and D_{MNPQ} is given by

$$\partial^M D_{MNPQ} = \mathcal{H}_{M[N} \left(\partial_P \mathcal{H}_{Q]}{}^M\right)$$

Outline

Summary

- In this talk I reviewed the classification of the branes of maximal supergravity and showed how, for the solitonic sector, this suggests the introduction of exotic branes ⇔ non-geometric fluxes

- Next, I discussed the relation between exotic branes/non-geometric fluxes and double field theory and speculated on the introduction, within DFT, of a 4-form potential D_{MNPQ}

Open Issue

Is there a 'complete' description of supergravity ?

Take-Home Message

We need to understand mixed-symmetry potentials better !

See, e.g., Bunster, Henneaux (2013)