Large N Yang-Mills Theories at Finite Density
(and holography)

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Collaboration

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References

1410.4466 with Anton, David & Javier

To appear, Ongoing etc ... with Anton, Christiana, David, Javier

Outline

Introduction and Motivation

conventional wisdom, gauge-string duality as a tool

Gauge-string duality and fundamental flavours

introducing a charge density

"Identifying the Ground State"

emergence of scaling geometries (HV-type) in the IR

Features of the IR-geometries

RG-flows, thermodynamics, etc..

Conclusions and Outlook

General lessons etc.

Motivation

The complete phase diagram of QCD at finite density

e.g. color-flavour locked (CFL) phase

Alford et. al.

Strong coupling posits a considerable challenge

Lattice calculations suffer from the "sign" problem

Representative soluble models

gauge-string duality provides a variety of examples

need to consider flavours ~ colours: backreaction & Veneziano limit

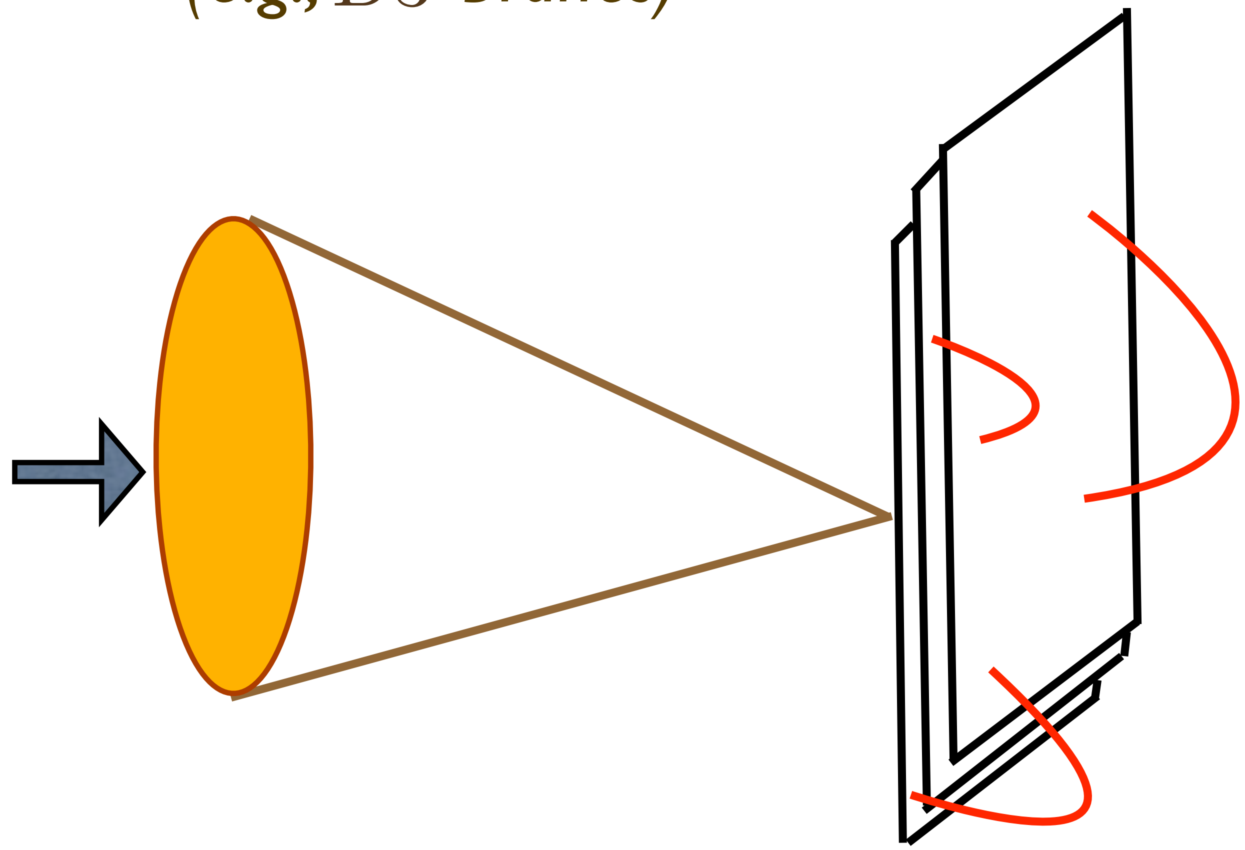
Schuster et. al.

Ingredients from string theory

Dp-branes: $(p+1)$-dim extended object where a string ends

(e.g., D3-branes)

Sasaki-Einstein 5-Manifold (SUSY)
Einstein 5-Manifold (no SUSY)



SU(N)-quiver gauge theories for N coincident branes

$\parallel\!\!|$

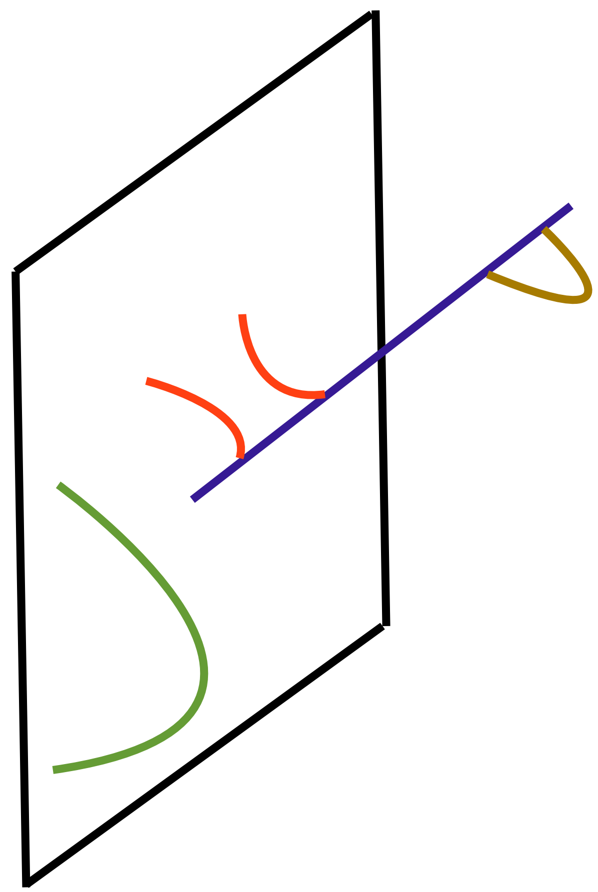
$\mathrm{AdS}_5 \times X^5$- background geometry

(e.g., $\mathcal{N}$=4 SYM $\equiv \mathrm{AdS}_5 \times S^5$)

Adding flavours

Background geometry is made of N D3 -branes

Add N_f D7-branes



3-3 strings: adjoint sector

3-7 strings: fundamental matter

7-7 strings: global symmetry

$U(N_f)$

D7-branes are typically probes of the geometry

Our goal is to go beyond the probe approximation

Karch & Katz

General comments

We can take N coincident $\mathrm{D}p$-branes $(p < 5)$ → $\mathrm{SU}(N)$ Yang-Mills theory

||||

→ conformal $\mathrm{AdS}_{p+2} \times X^{8-p}$ geometry

↑ *Einstein manifold (Sasaki, nearly-Kahler etc.)*

(described by supergravity)

+

$\mathrm{D}(p+4)$-flavour branes → fundamental degrees of freedom

wraps ~ $\mathrm{AdS}_{p+2} \times X^3$

Veneziano limit: $N \to \infty \;, \quad N_f \to \infty \;, \quad \dfrac{N_f}{N} = \text{fixed}$

Introducing charge density

Dynamics is described by Dirac-Born-Infeld

Excite a $\mathrm{U}(1)$ -field on the "flavour"-brane $\quad F_2 = A_t'(r)\, dt \wedge dr$

Mateos et. al.

$$A_t = \mu + \frac{\rho}{u^2} + \dots$$

chemical potential (μ) *charge density* (ρ)

"Large charge" limit: $\quad S_{\mathrm{DBI}} \Longrightarrow S_{\mathrm{NG}}$

(non-dynamical external quarks)

Replace the "flavour brane + flux" by an explicit external "String-sources"

The action

We want to consider the following system:

$$S_{\text{total}} = S_{\text{IIA/IIB}} + S_{\text{Strings}}$$

Strings act as localized sources $\Longrightarrow$ coupled non-linear PDEs

Hard to solve!!

So we smear (in a particular way)

Prem Kumar

	t	r	$\vec{x}_p$	Ω_{8-p}
Dp	—	•	—	•
F1	—	—	----	----

The UV-behaviour

Thus:

$$S_{\text{Strings}} = \frac{N_q}{2\pi\alpha'} \int \left(\sqrt{-G_{tt} G_{rr}} dt \wedge dr - B \right) \wedge \Xi_8$$

$$\Xi_8 \sim dx^1 \wedge \ldots \wedge dx^p \wedge \omega_{8-p}$$

Schematically:

$$G_{\mu\nu} = G_{\mu\nu}^{(\mathrm{D}p)} \left(1 + \chi \, N_q \left(\frac{L}{u} \right)^{6-p} + \ldots \right)$$

The asymptotic $\mathrm{D}p$ *geometry*

the asymptotic structure is preserved for

$p < 6$

a set of constants, fully determined by the eom

Note, $N_q \sim \rho$

Faedo et. al.

The IR-solutions

Dimensionally reducing on the compact manifold: hyperscaling violating-Lifshitz type background

$$ds^2 = r^{\frac{-2\theta}{p}} \left[-r^{2z} dt^2 + r^2 d\vec{x}_p^2 + \frac{dr^2}{r^2} \right]$$

$$e^\phi = Q^{\frac{p-7}{2}} r^{\frac{p(p-7)}{2(p-4)}} \,, \qquad G_{\Omega\Omega} \sim Q^{\frac{3-p}{4}} r^{\frac{p(3-p)}{4(p-4)}} \,, \qquad Q \sim \frac{N_q}{N^2}$$

$$z = \frac{16-3p}{4-p} \,, \qquad \theta = \frac{p(3-p)}{4-p}$$

Faedo et. al.

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The $Q \to 0$ limit is singular for $p < 7$

$$z = \frac{16-3p}{4-p} \;, \qquad \theta = \frac{p(3-p)}{4-p}$$

what is happening at $p = 4$?

Faedo et. al.

The special case

The $p = 4$ case yields $\mathrm{AdS}_2 \otimes_w \mathbb{R}^4$

$$ds^2 = r^{1/2}\left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\vec{x}_4^2\right] \ , \quad e^{\phi} \sim Q^{-3/2} r^{3/2}$$

It can also be obtained from the general solution by setting:

$$z \to \infty \ , \quad \theta \to \infty \ , \quad z/\theta = -1$$

This has an M-theory uplift of the form: $\mathrm{AdS}_3 \times \mathbb{R}^4$

Faedo et. al.

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Faedo et. al.

The HV-Lif geometry

The metric: $$ds^2 = r^{\frac{-2\theta}{p}} \left[-r^{2z} dt^2 + r^2 d\vec{x}_p^2 + \frac{dr^2}{r^2} \right]$$

Covariance under scaling: $t \to \Lambda^z t \ , \quad \vec{x} \to \Lambda \vec{x}$

Huijse et. al.

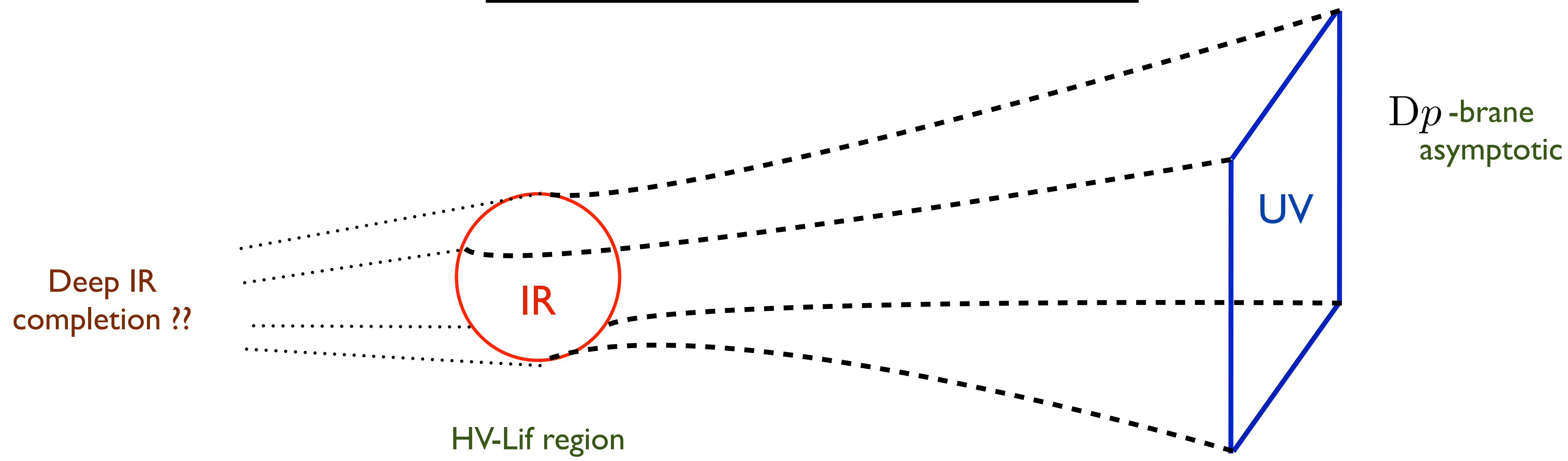
$$\Longrightarrow \qquad ds^2 \to \Lambda^{2\theta/p} ds^2$$

$z \neq 1 \quad \Longrightarrow$ broken Lorentz symmetry (Lifshitz)

$\theta \neq 1 \quad \Longrightarrow$ non-trivial scaling in thermodynamics: $s \sim T^{\frac{p-\theta}{z}}$

These geometries lack IR-completion: curvature singularity, infinite tidal forces etc..

A flow that connects



IR-data:

$$z_{\rm IR} = \frac{16-3p}{4-p}$$

$$\theta_{\rm IR} = \frac{p(3-p)}{4-p}$$

UV-data:

$$z_{\rm UV} = 1$$

$$\theta_{\rm UV} = p - \frac{9-p}{5-p}$$

Dong et. al.

$\exists$ smooth numerical interpolating solutions, for all cases

Faedo et. al.

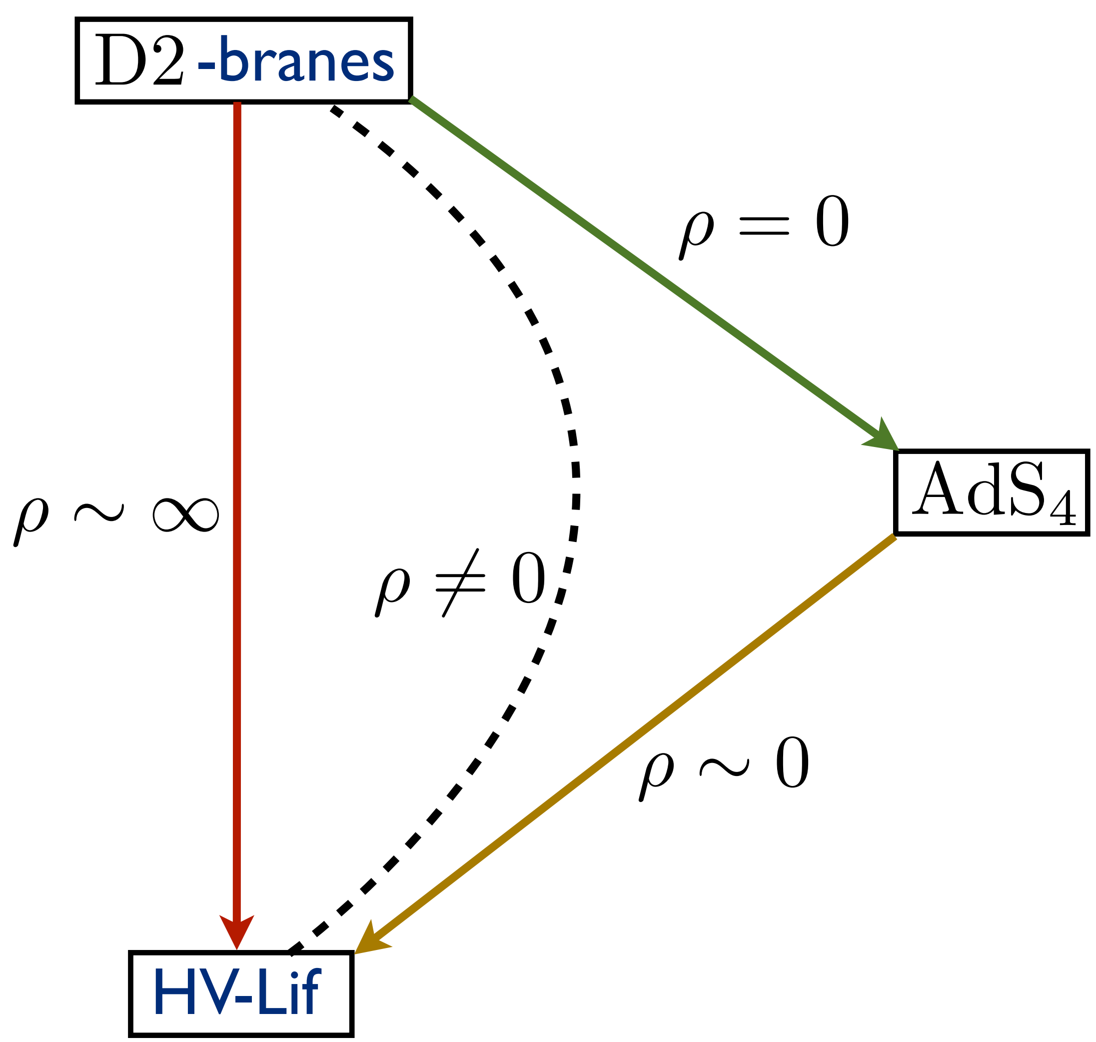
With dynamical quarks

The example of: $p = 2\ , \quad z_{\mathrm{IR}} = 5\ , \quad \theta_{\mathrm{IR}} = 1$

Making the flavours dynamical: N number of $\mathrm{D2}$-branes + N_f $\mathrm{D6}$-branes with flux

$(N \sim N_f)$

$$S_{\mathrm{total}} = S_{\mathrm{IIA}} + S_{\mathrm{D6}}$$



To appear, Ongoing

Conclusions

The HV-Lif type geometries naturally emerge in the IR, *Universality*

The "IR-completion" may be physically very interesting

A more "physical" case is the $(3+1)$ -dimensional one

Typically, issues with Landau pole etc ...

Provides a good starting point to explore further

Looking for the right ground state: properties of the IR-geometry, exploring instabilities etc ...

Perhaps towards the breaking of the gauge symmetry

Towards the effect of color superconductivity