Resolving the information paradox

Work done with:

Avery, Chowdhury, Giusto, Lunin, Saxena, Srivastava

Many fuzzball results obtained by

Bena-Warner et. al.

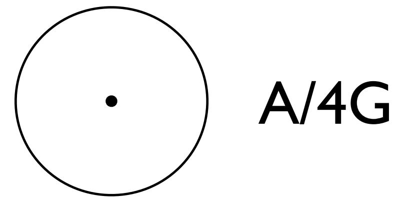
Balasubramanian, Gimon, Levi ....

Skenderis,Taylor et. al.

and others ...

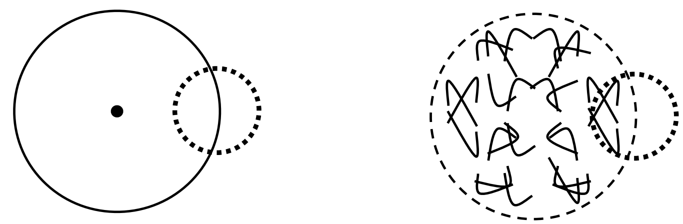
Puzzles with black holes:

(a) The entropy puzzle: *Does the `Area entropy' correspond to a 'count of states' for the black hole ?*

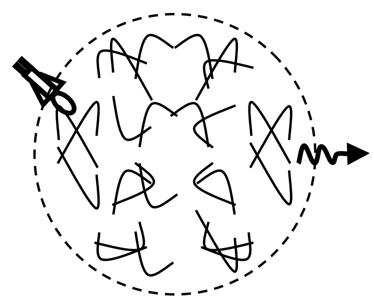


(b) The information paradox: *How can the Hawking radiation quanta carry the information in the hole ?*

i.e. Can general relativity and quantum mechanics co-exist ?



(b) The infall problem: *What does an infalling observer feel ?*



Plan

(a) What is the information paradox ?

(b) Results on fuzzballs: summary

2-charge, 3 charge, 4-charge extremal states
Nonextremal states: Can see explicitly information preserving 'Hawking emission' from one particular microstate

(c) Dynamical questions:

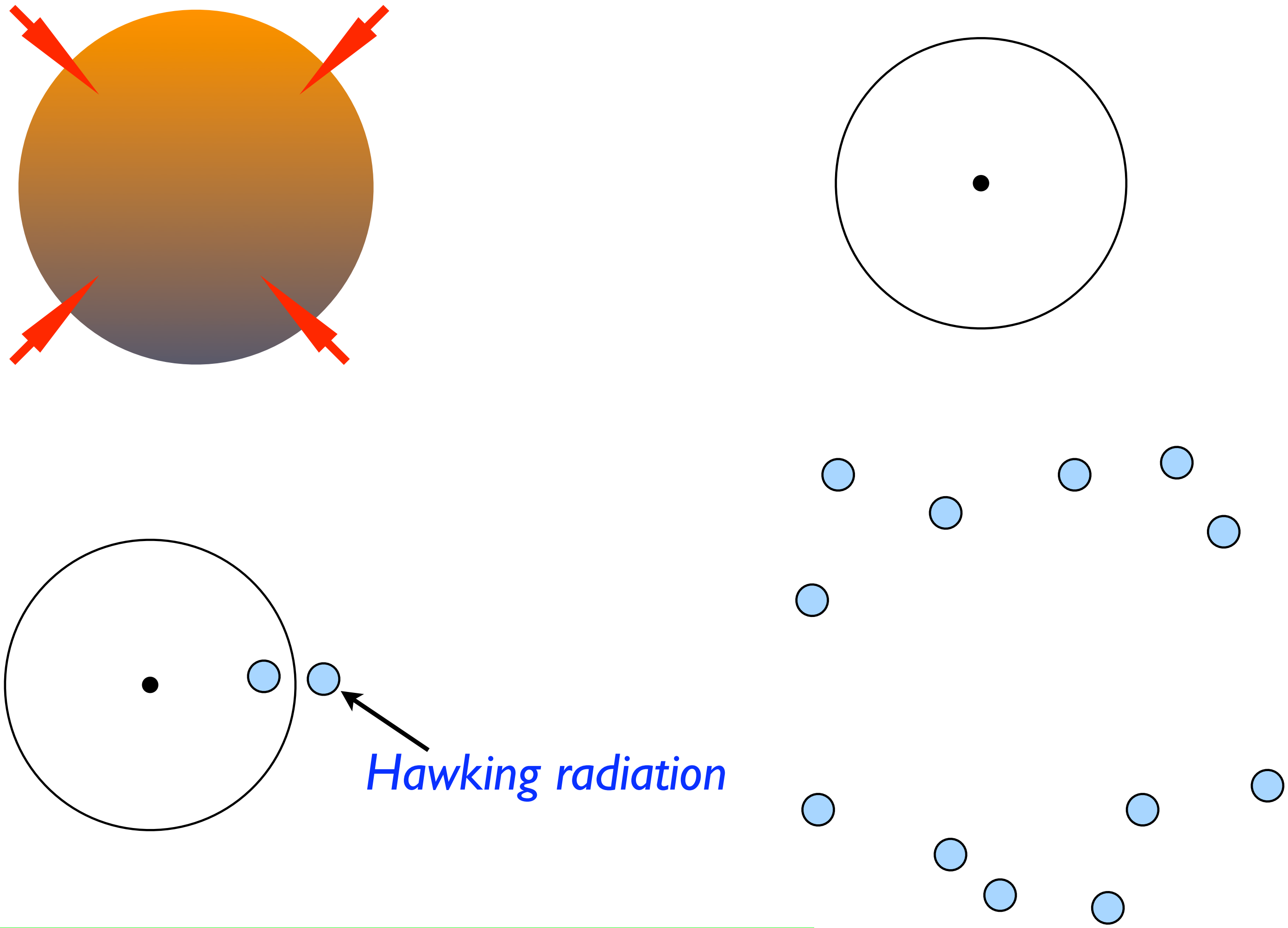
Collapse of a shell
Infalling observer

http://www.physics.ohio-state .edu/~mathur

The Information paradox

(a review can be found in SDM 2008)

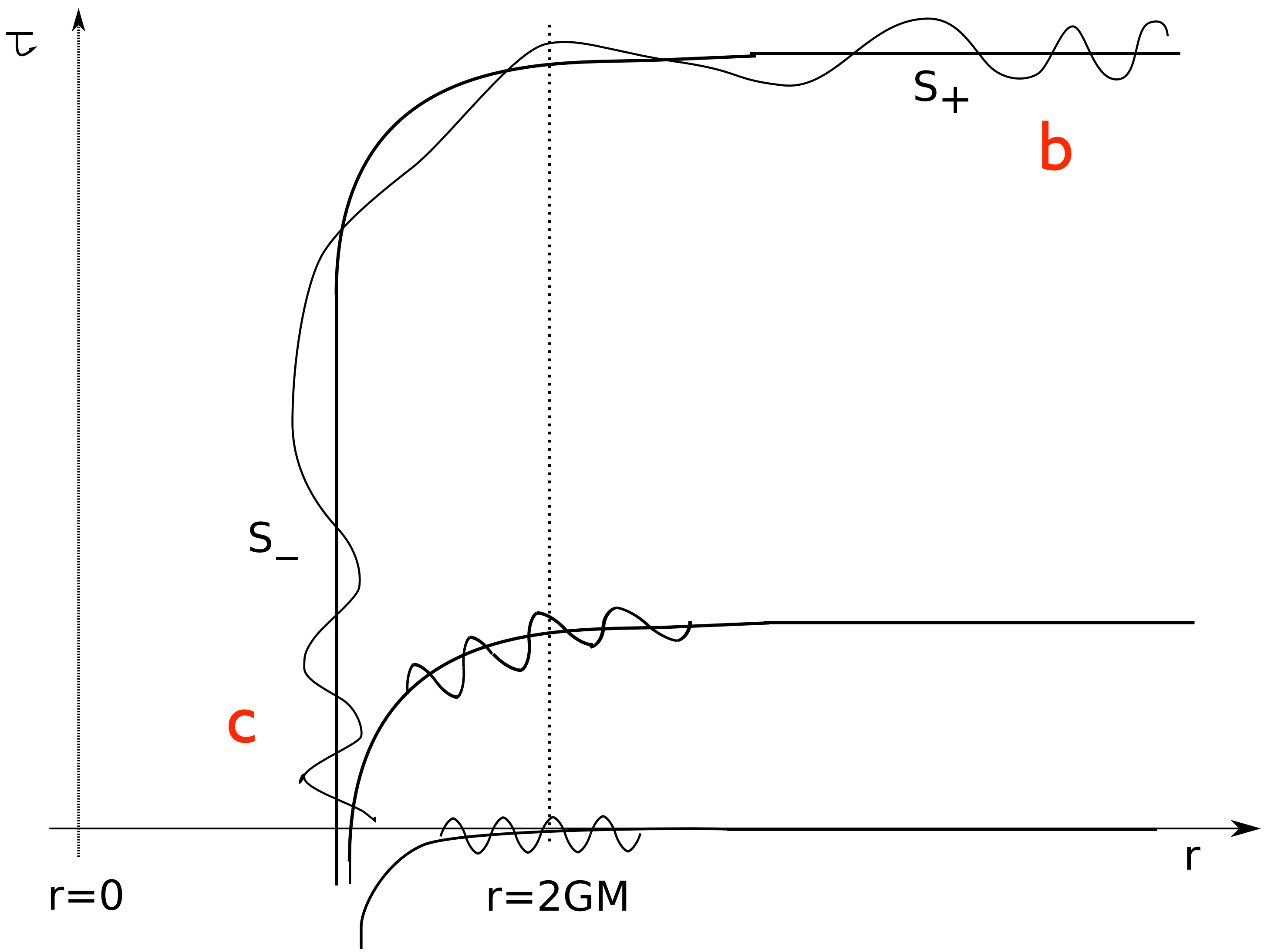
The information problem: a first pass



How can the Hawking radiation carry the information of the initial matter ?

(Hawking '74)

If a wavepacket sits across the horizon, then we will get particle creation. The mode gets cut in two parts ...



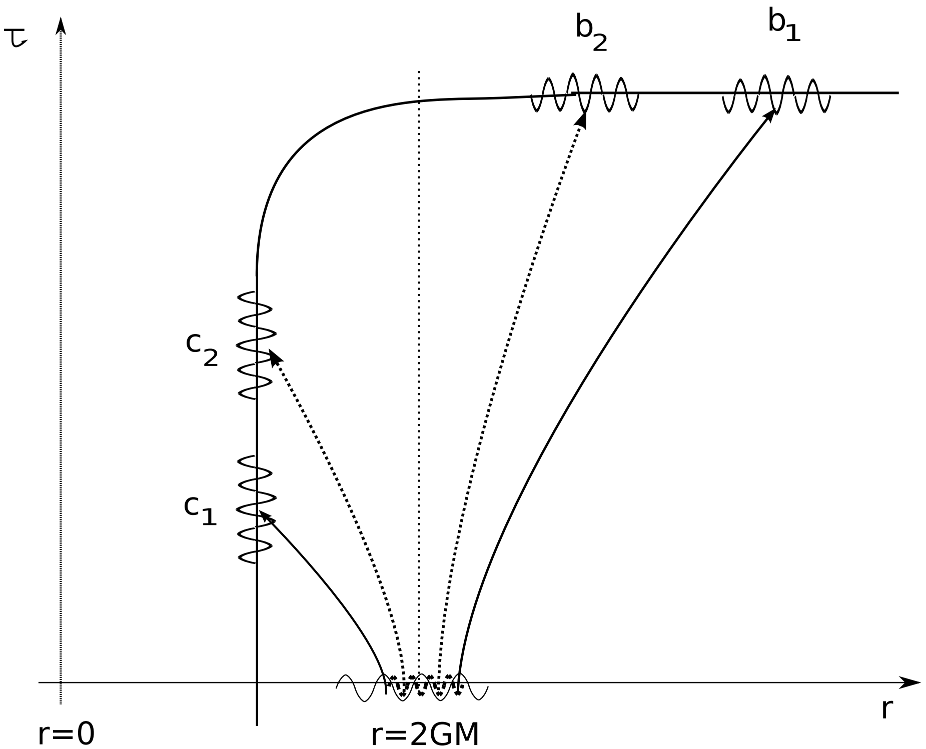
The entangled nature of the state

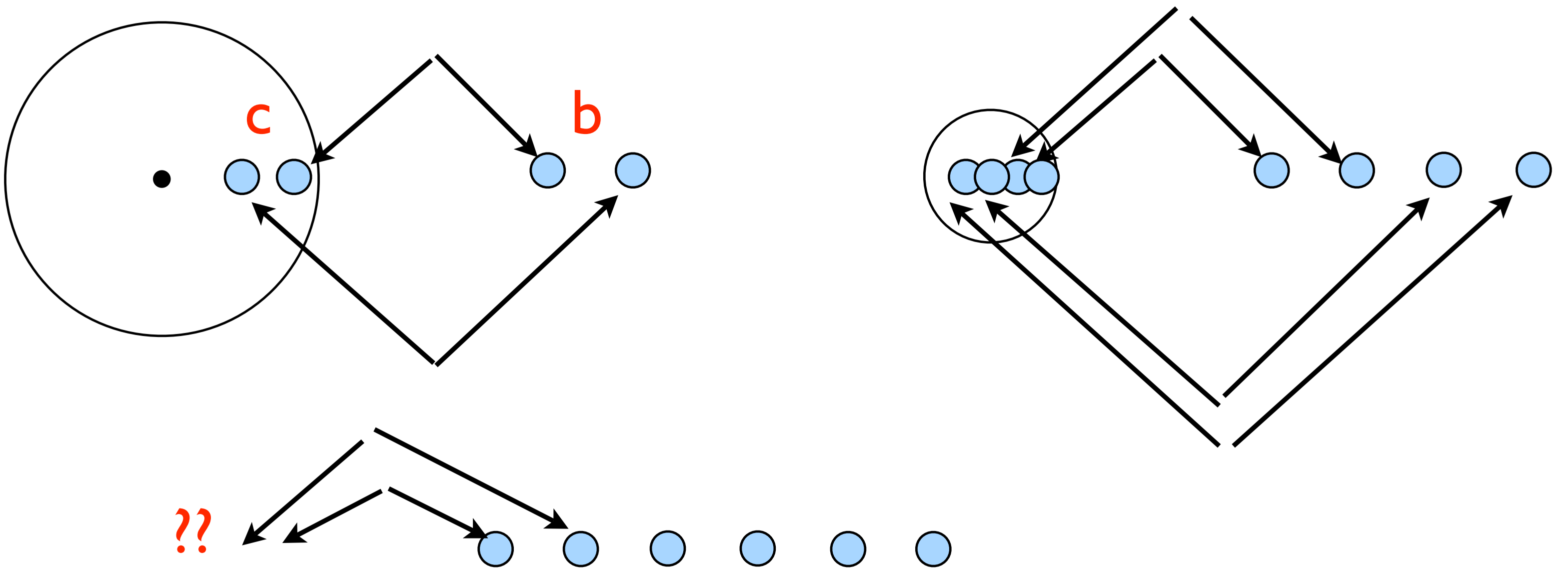
$$|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}$$

$$\begin{aligned}|\psi\rangle_1 &= C\left(|0\rangle_{b_1}\otimes|0\rangle_{c_1} + \gamma \hat{b}_1^\dagger|0\rangle_{b_1}\otimes \hat{c}_1^\dagger|0\rangle_{c_1} + \frac{\gamma^2}{2}\hat{b}_1^\dagger\hat{b}_1^\dagger|0\rangle_{b_1}\otimes\hat{c}_1^\dagger\hat{c}_1^\dagger|0\rangle_{c_1} + \ldots\right)\\ &= C\left(|0\rangle_{b_1}\otimes|0\rangle_{c_1} + \gamma|1\rangle_{b_1}\otimes|1\rangle_{c_1} + \gamma^2|2\rangle_{b_1}\otimes|2\rangle_{c_1} + \ldots\right)\end{aligned}$$

$$\boxed{|\psi\rangle_1 = Ce^{\gamma \hat{b}_1^\dagger \hat{c}_1^\dagger}|0\rangle_{b_1}|0\rangle_{c_1}}$$

$$\boxed{|\psi\rangle_2 = Ce^{\gamma \hat{b}_2^\dagger \hat{c}_2^\dagger}|0\rangle_{b_2}|0\rangle_{c_2}}$$





(a) The b quanta are entangled with the c quanta

(b) Thus there is no state as such for the b quanta alone,
but there is a state for the b and c quanta together

(c) If the black hole vanishes, then the b quanta are
left 'entangled with nothing'

(d) There is not supposed to be any such state in quantum mechanics !!

Our state is of this essential form

$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(|0\rangle_{b_1} \otimes |0\rangle_{c_1} + |1\rangle_{b_1} \otimes |1\rangle_{c_1}\right)$$

A factored state would be of the form

$$|\psi\rangle_1 = (C_0|0\rangle_{b_1} + C_1|1\rangle_{b_1} + \ldots) \otimes (D_0|0\rangle_{c_1} + D_1|1\rangle_{c_1} + \ldots)$$

The essential point is that a small change in our state will not make it a factored state :

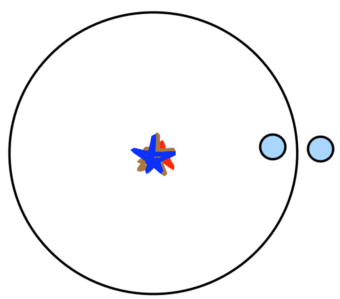
$$|\psi\rangle_1 = \frac{1}{\sqrt{2}}\left(1.1|0\rangle_{b_1} \otimes |0\rangle_{c_1} + 0.9|1\rangle_{b_1} \otimes |1\rangle_{c_1}\right)$$

is almost as entangled as the initial state we had

Thus a small change in the evolution of the wavemode will NOT solve the information problem

We need a change of ORDER UNITY in the evolution of low energy outgoing radiation modes

If we do not find such an order unity change, we will have to give up either General Relativity or Quantum Mechanics



The Hawking 'theorem':

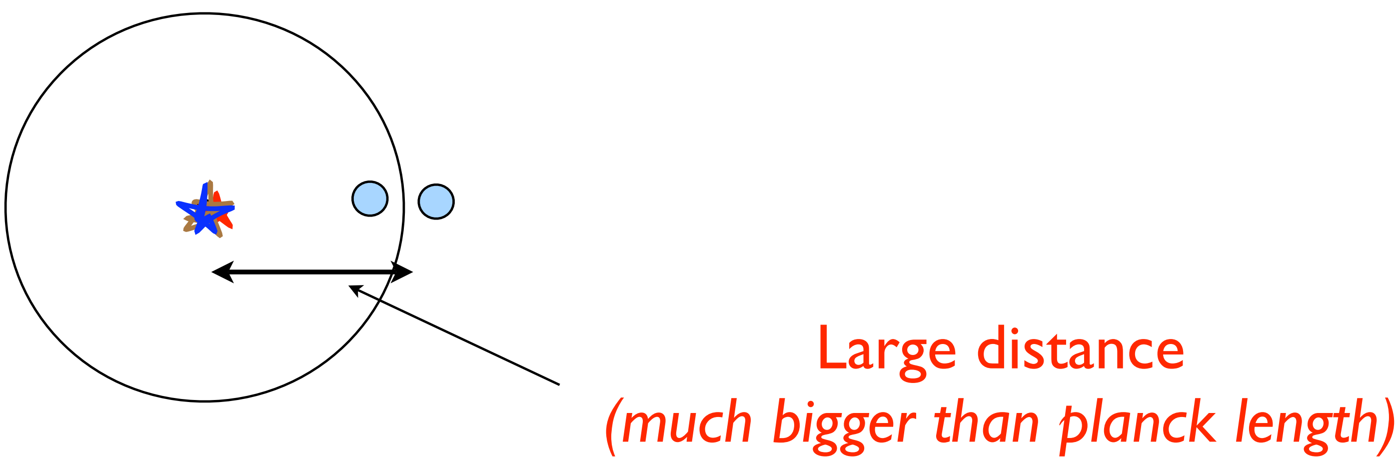
If we are given that

(a) All quantum gravity effects are confined
to within a bounded distance like planck length or string length

and

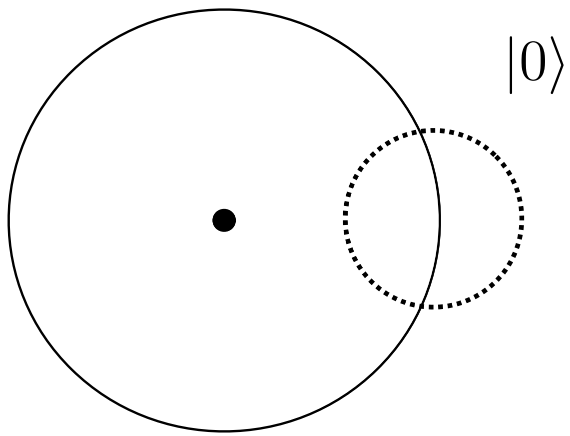
(b) The vacuum of the theory is unique

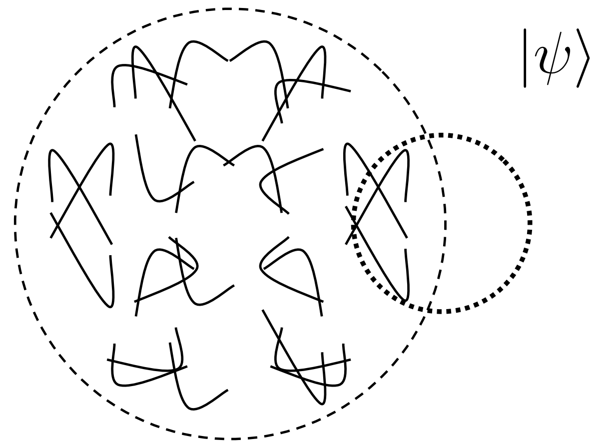
Then there WILL be information loss



Review of fuzzball results

The fuzzball picture





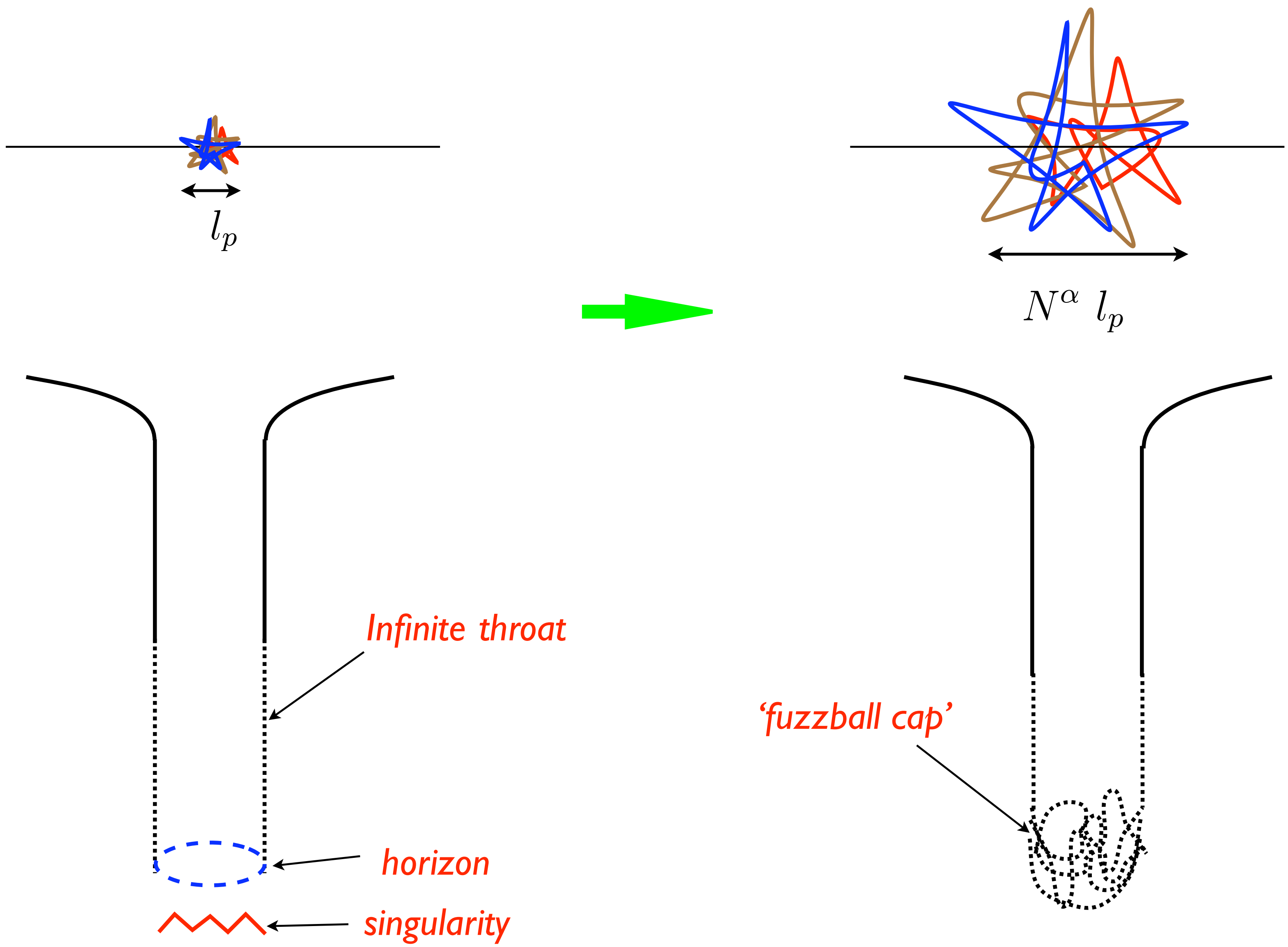
$$\langle 0|\psi\rangle \approx 0$$

In the traditional black hole, quantum gravity effects are assumed to stretch only over distances $\sim l_p$, and so the state near the horizon is the vacuum.

But a black hole is made of a large number of quanta N, so we must ask if the relevant length scales are $\sim l_p$ or $\sim N^{\alpha}\, l_p$

The paradigm for extremal holes

A supersymmetric brane state in string theory: **Mass = Charge**



Microscopic entropy expressions :

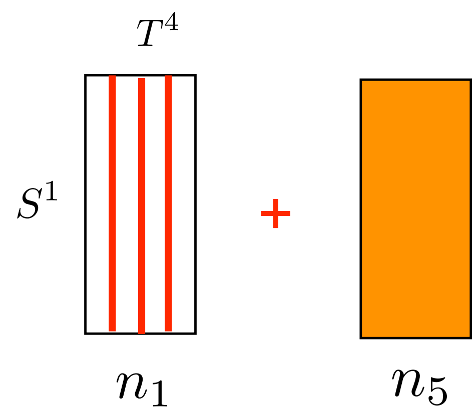
2-charges $S = 2\sqrt{2}\pi\sqrt{n_1 n_2}$

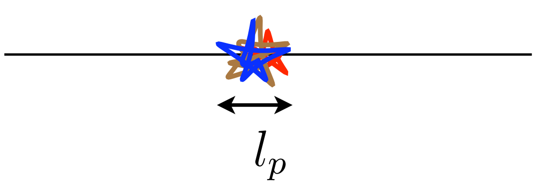
3-charges $S = 2\pi\sqrt{n_1 n_2 n_3}$

4-charges $S = 2\pi\sqrt{n_1 n_2 n_3 n_4}$

IIB $M_{9,1} \rightarrow M_{4,1} \times T^4 \times S^1$

2-charge D1D5 system





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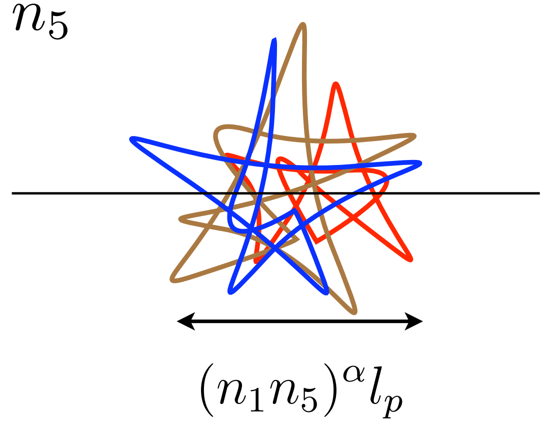

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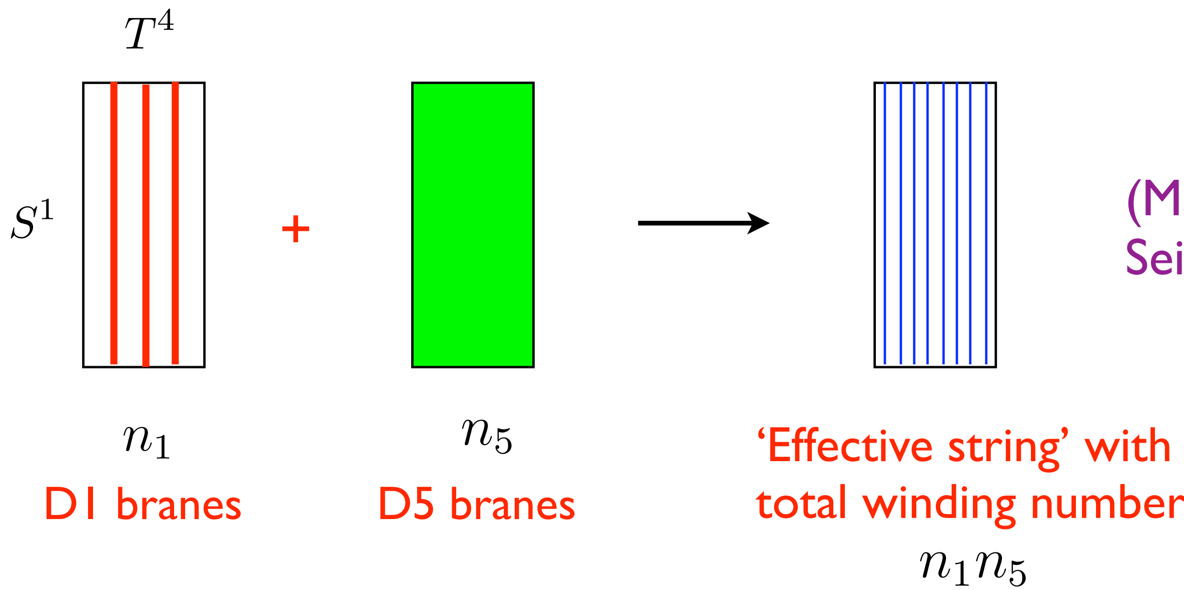
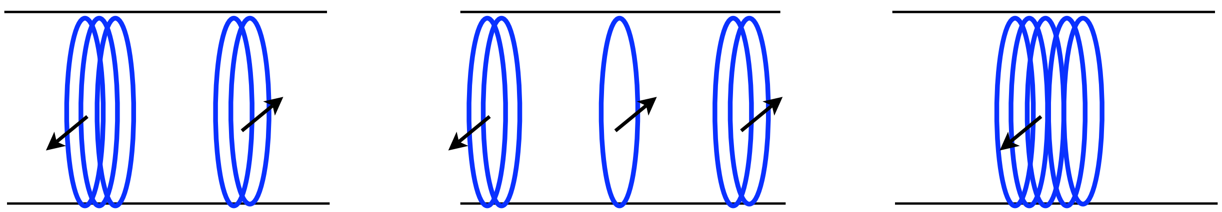


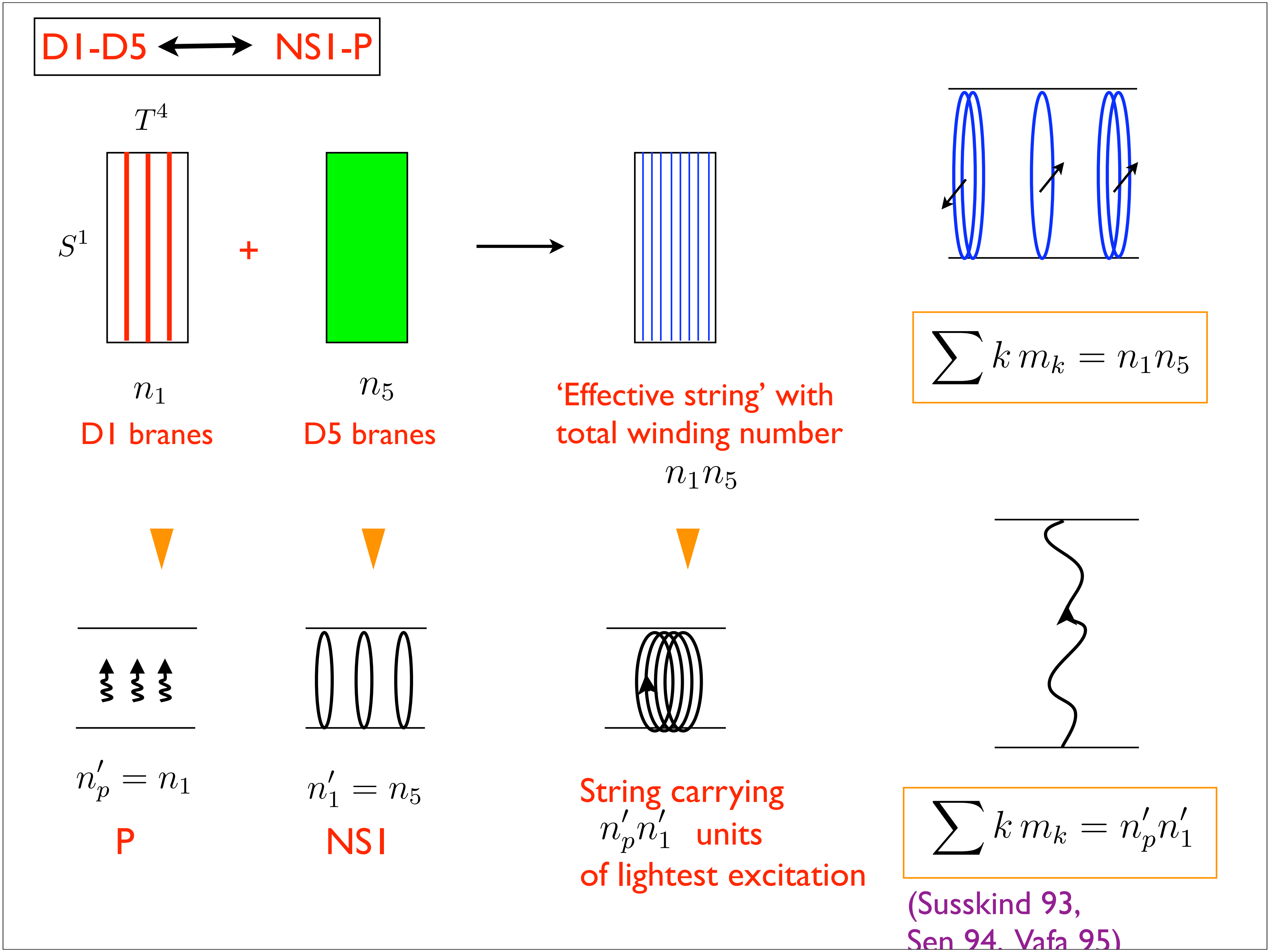
(Maldacena and Susskind 96, Seiberg and Witten 98)

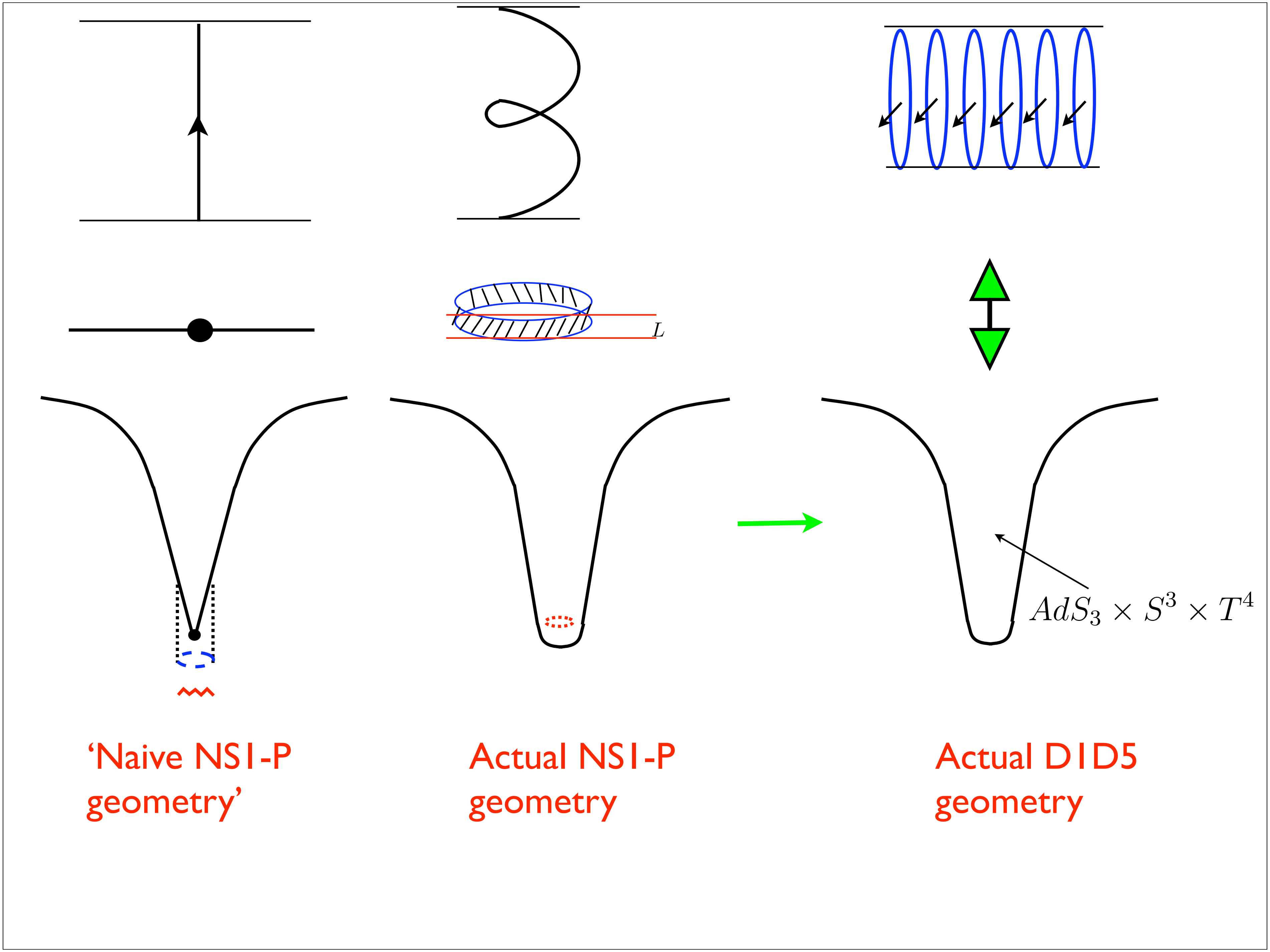


Entropy arises from different ways of partitioning the effective string into loops

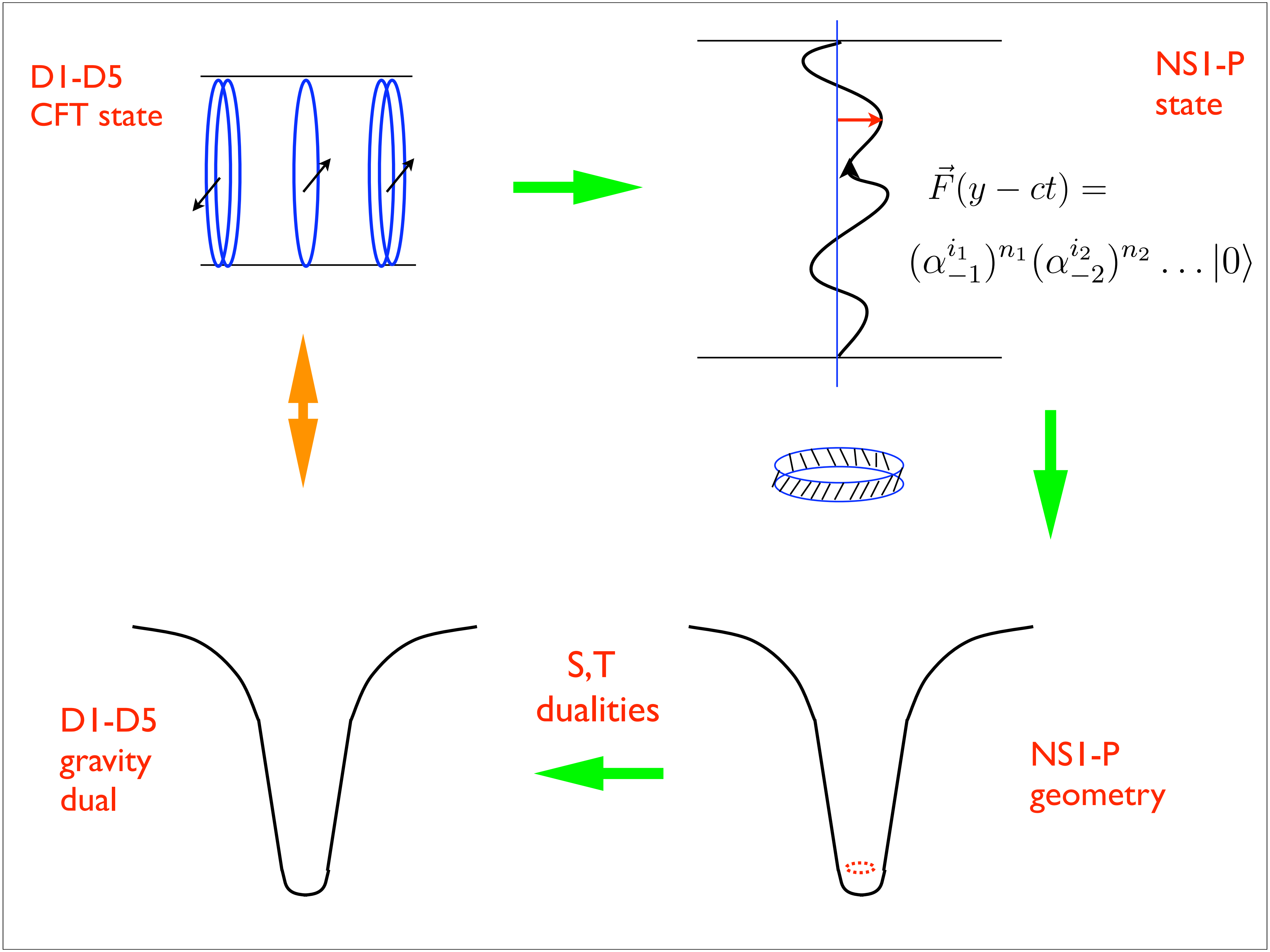
$$\boxed{\sum k\, m_k = n_1 n_5}$$

$$\boxed{S = 2\sqrt{2}\pi\sqrt{n_1 n_5}}$$


D1-D5 ⟷ NS1-P
T^4
S^1
+
n_1
D1 branes
n_5
D5 branes
'Effective string' with
total winding number
$n_1 n_5$
$\sum k\, m_k = n_1 n_5$
$n'_p = n_1$
P
$n'_1 = n_5$
NS1
String carrying
$n'_p n'_1$ units
of lightest excitation
$\sum k\, m_k = n'_p n'_1$
(Susskind 93,
Sen 94, Vafa 95)



L
$AdS_3 \times S^3 \times T^4$
'Naive NS1-P geometry'
Actual NS1-P geometry
Actual D1D5 geometry



D1-D5
CFT state
NS1-P
state
$\vec{F}(y-ct) =$
$(\alpha^{i_1}_{-1})^{n_1}(\alpha^{i_2}_{-2})^{n_2}\ldots|0\rangle$
S,T
dualities
D1-D5
gravity
dual
NS1-P
geometry

Geometry for D1-D5

$$ds^2 = \sqrt{\frac{H}{1+K}}[-(dt - A_i dx^i)^2 + (dy + B_i dx^i)^2] + \sqrt{\frac{1+K}{H}} dx_i dx_i + \sqrt{H(1+K)} dz_a dz_a$$

$$H^{-1} = 1 + \frac{Q}{L_T}\int_0^{L_T} \frac{dv}{|\vec{x} - \vec{F}(v)|^2}$$

$$K = \frac{Q}{L_T}\int_0^{L_T} \frac{dv(\dot{F}(v))^2}{|\vec{x} - \vec{F}(v)|^2}$$

$$A_i = -\frac{Q}{L_T}\int_0^{L_T} \frac{dv\ \dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|^2}$$

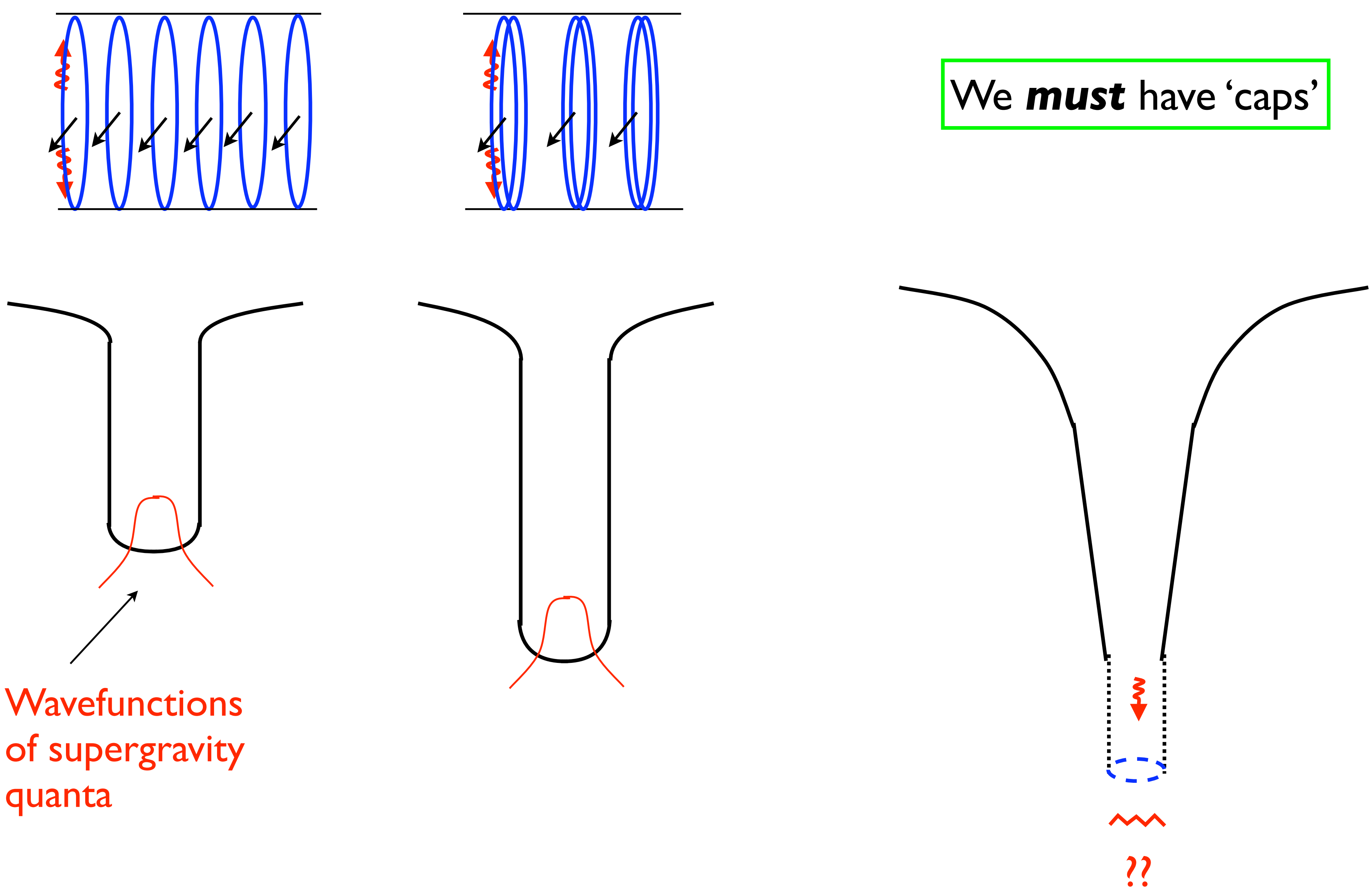
$$dB = - *_4 dA$$

(Lunin+SDM '01,

Lunin+Maldacena+Maoz 02

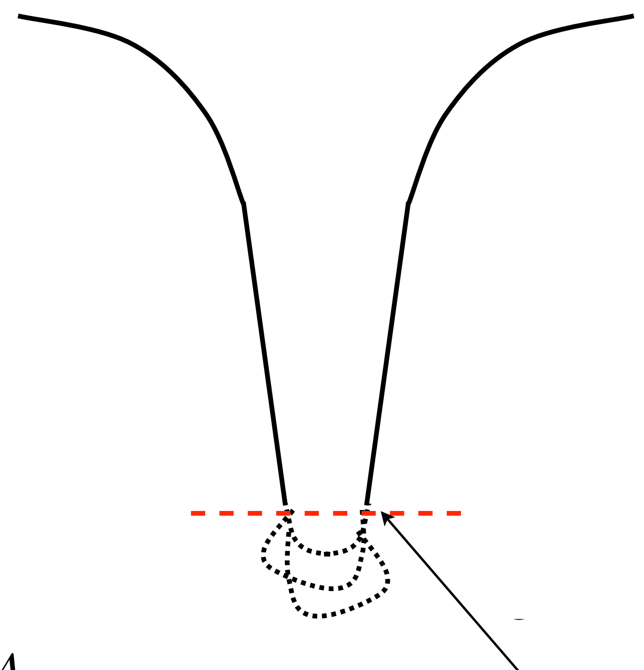
Taylor 05, Skenderis+Taylor 06)

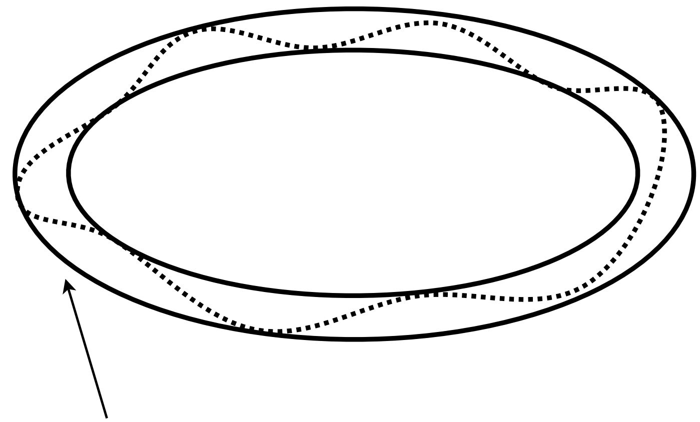
Energy gaps exactly agree between the CFT and the gravity solution...



Scale of the 'fuzzball'

Consider the typical state, and draw a boundary where it departs from the naive metric by order unity



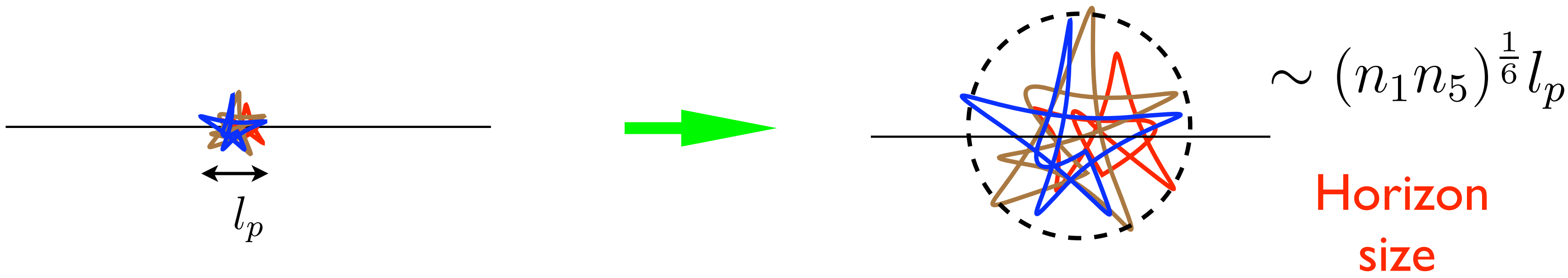


$$\frac{A}{4G} \sim \sqrt{n_1 n_5 - J} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_5 - J}$$

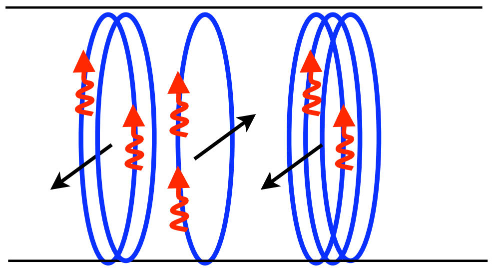
$$\frac{A}{4G} \sim \sqrt{n_1 n_5} \sim S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_5}$$

(Lunin+SDM '02)

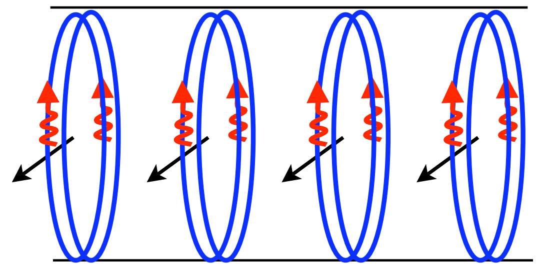
2-charge extremal D1D5 :



3-charge extremal D1D5 P ?



Generic D1D5P CFT state

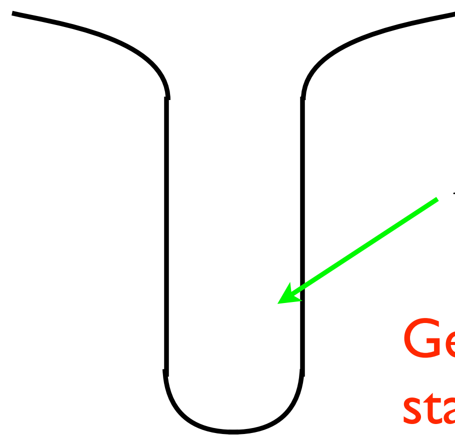


Simple states: all components the same, excitations fermionic, spin aligned

$$|k\rangle^{total} = (J^{-,total}_{-(2k-2)})^{n_1 n_5} (J^{-,total}_{-(2k-4)})^{n_1 n_5} \dots (J^{-,total}_{-2})^{n_1 n_5} \; |1\rangle^{total}$$

Can make geometries for these simple states :

U(1) X U(1) symmetry



$AdS_3 \times S^3 \times T^4$

Geometry for simple state (winding =1)

$$
\begin{aligned}
ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + \frac{Q_p}{hf}(dt-dy)^2 + hf\left(\frac{dr_N^2}{r_N^2 + a^2\eta} + d\theta^2\right) \\
&+ h\left(r_N^2 - na^2\eta + \frac{(2n+1)a^2\eta Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 \\
&+ h\left(r_N^2 + (n+1)a^2\eta - \frac{(2n+1)a^2\eta Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2 \\
&+ \frac{a^2\eta^2 Q_p}{hf}\left(\cos^2\theta d\psi + \sin^2\theta d\phi\right)^2 \\
&+ \frac{2a\sqrt{Q_1 Q_5}}{hf}\left[n\cos^2\theta d\psi - (n+1)\sin^2\theta d\phi\right](dt-dy) \\
&- \frac{2a\eta\sqrt{Q_1 Q_5}}{hf}\left[\cos^2\theta d\psi + \sin^2\theta d\phi\right]dy + \sqrt{\frac{H_1}{H_5}}\sum_{i=1}^{4} dz_i^2
\end{aligned}
$$

$$
\begin{aligned}
f &= r_N^2 - a^2\eta\, n\sin^2\theta + a^2\eta\,(n+1)\cos^2\theta \\
h &= \sqrt{H_1 H_5},\;\; H_1 = 1 + \frac{Q_1}{f},\;\; H_5 = 1 + \frac{Q_5}{f}
\end{aligned}
$$

$$
\eta \equiv \frac{Q_1 Q_5}{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}
$$

(Giusto SDM Saxena 04)

2-charges, *4+1 dimensions, noncompact excitations:* Lunin+SDM '01

2-charges, *4+1 d, torus excitations:* Lunin+Maldacena+Maoz '02, Skenderis +Taylor 07

2-charges, *4+1 d, fermionic excitations:* Taylor '05

3-charges, *4+1 d, one charge 'test quantum' wavefunction;* SDM+Saxena+Srivastava '03

3-charge, *4+1 d, U(1) X U(1) axial symmetry:* Giusto+SDM+Saxena '04, Lunin '04

3-charge, *4+1 d, U(1) axial symmetry:* Bena+Kraus '05, Berglund+Gimon+Levi '05

3 charges, *3+1 d, U(1) axial symmetry:* Bena+Kraus '05

4-charges, *3+1 d, U(1)XU(1) symmetry:* Saxena+Giusto+Potvin+Peet '05

4 charges, *3+1 d, U(1) symmetry:* Balasubramanian+Gimon+Levi '06

Non-extremal geometries, 3 charges, *4+1 d, U(1)XU(1) axial symmetry:*
Jejjala+Madden+Ross+Titchener 05

Non-extremal geometries, 4 charges, *3+1 d, U(1)XU(1) axial symmetry:*
Giusto+Ross+Saxena 07

2-charges, *4+1 d, K3 compactification:* Skenderis+Taylor 07

2-charges, *1-point functions:* Skenderis+Taylor 06

General structure of extremal solutions: hyperkahler base + 2-d fiber
(Gauntlett+Gutowski+Hull+Pakis+Reall 02, Gutowski+Martelli+Reall 03)

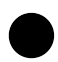
Decomposing known microstate solutions into base + fiber:
hyperkahler $\longrightarrow$ psedo-hyperkahler
(Giusto+SDM 04)

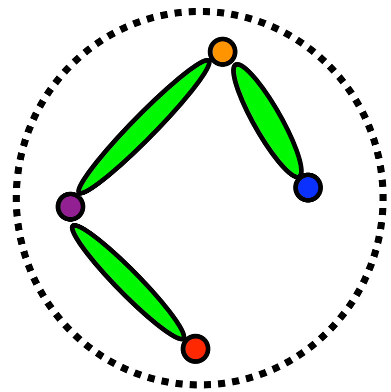
Structure of general 3-charge and 4-charge geometries :

Bound states of branes is on Higgs branch. Dipole charges form, are held apart by fluxes ...

(Bena+Warner 05)

$g \rightarrow 0$





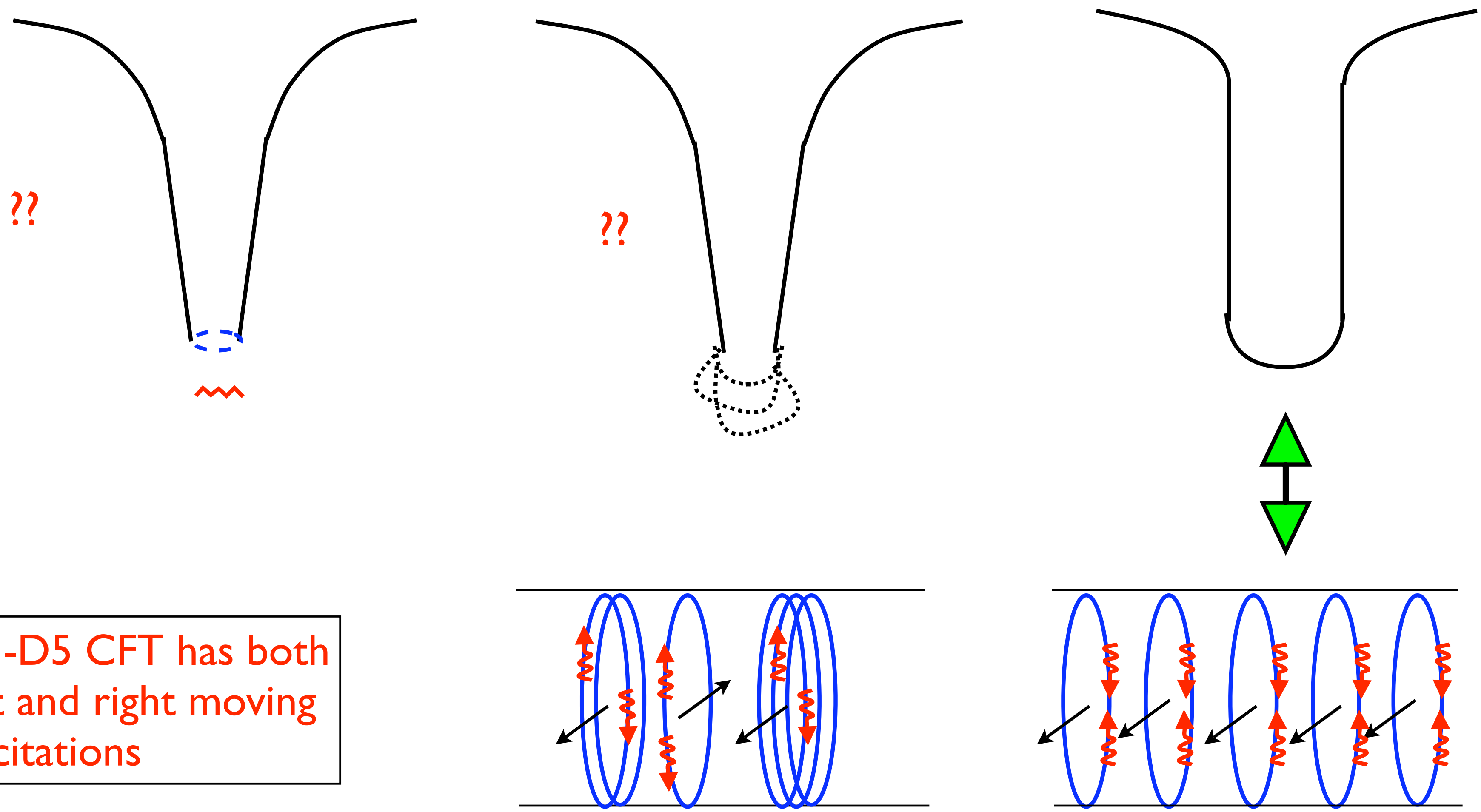
g nonzero

If we reduce to 3+1 dimensions, get metrics for 'branes at angles' (Denef '02, Balasubramanian+Gimon+Levi 05)

Recent work (Bena+Bobev+Ruef+Warner 08) ... supertubes in the `throat' might give correct order for number of states ...

The Non-Extremal Hole :

(Jejalla, Madden, Ross Titchener '05)



D1-D5 CFT has both left and right moving excitations

Gravity dual again has no horizon or singularity

$$
\begin{aligned}
\mathrm{d}s^2 \;=\;& -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y-c_p\mathrm{d}t)^2\\
&+\sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2)-Mr^2}+\mathrm{d}\theta^2\right)\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}-(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}+(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\mathrm{d}\phi^2\\
&+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi+a_2\sin^2\theta\mathrm{d}\phi)^2\\
&+\frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p-a_2s_1s_5s_p)\mathrm{d}t+(a_2s_1s_5c_p-a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi\\
&+\frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p-a_1s_1s_5s_p)\mathrm{d}t+(a_1s_1s_5c_p-a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi\\
&+\sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned}
$$

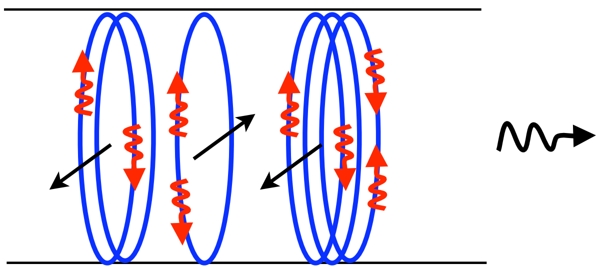
$$
\begin{aligned}
Q_1 &= \frac{g\alpha'^3}{V}n_1\\
Q_5 &= g\alpha' n_5\\
Q_p &= \frac{g^2\alpha'^4}{VR^2}n_p
\end{aligned}
$$

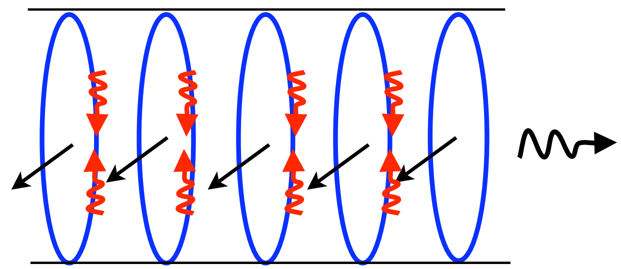
(Jejalla, Madden, Ross Titchener '05)

$$
\tilde{H}_i = f + M\sinh^2\delta_i, \quad f = r^2 + a_1^2\sin^2\theta + a_2^2\cos^2\theta
$$

$$
Q_1 = M\sinh\delta_1\cosh\delta_1, \quad Q_5 = M\sinh\delta_5\cosh\delta_5, \quad Q_p = M\sinh\delta_p\cosh\delta_p
$$

As in any statistical system, each microstate radiates a little differently





$$\Gamma_{CFT} = V \ \rho_L \ \rho_R$$

Emission vertex

Occupation numbers of left, right excitations

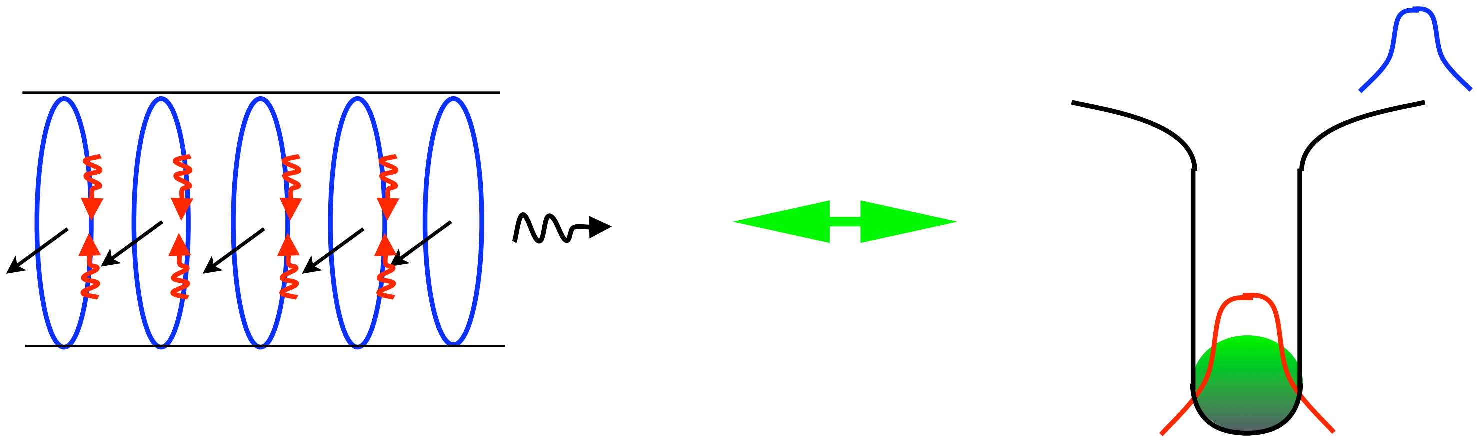
Bose, Fermi distributions for generic state

$$\Gamma_{CFT} = V \ \bar{\rho}_L \ \bar{\rho}_R$$

Occupation numbers for this particular microstate

Emission from the special microstate is peaked at definite frequencies and grows exponentially, like a laser .....

One finds :



$$\omega_R^{CFT} = \omega_R^{gravity}$$
$$\omega_I^{CFT} = \omega_I^{gravity}$$

Emission happens, not from a horizon, but from an ergoeregion

(Cardoso, Dias, Jordan, Hovdebo, Myers, '06, Chowdhury+SDM 07, 08)

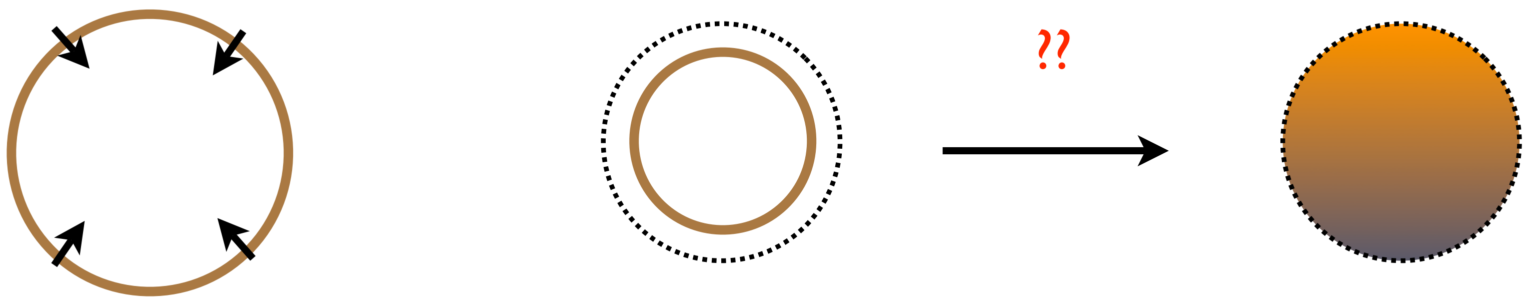
Thus for a set of (nongeneric) microstates we can explicitly see 'information carrying radiation' which is the 'Hawking radiation' for these microstates

Dynamical questions:

(A) Collapse of a shell

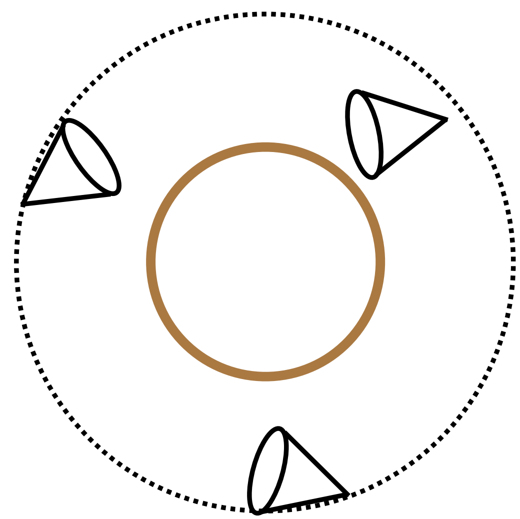
Suppose we make a black hole by collapsing a shell of matter

How can this shell change into a fuzzball ?



Light cones point inwards

How does data get out to horizon ?



Two simple estimates :

(A) Perhaps the interior of a black hole is very quantum ...

Amplitude to tunnel from any state in horizon region to any other state

$$\boxed{e^{-S} \sim e^{-GM^2}}$$

$$S = \frac{1}{16\pi G}\int R d^4x$$

$$R \sim \frac{1}{L^2} \sim \frac{1}{(GM)^2}$$

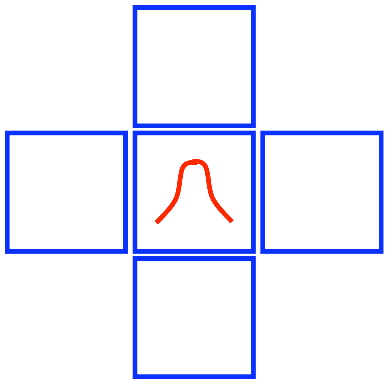
$$d^4x \sim (GM)^2$$

$$S \sim GM^2$$

Number of states that we can tunnel to

$$\boxed{e^{S_{bek}} \sim e^{GM^2}}$$

Toy model

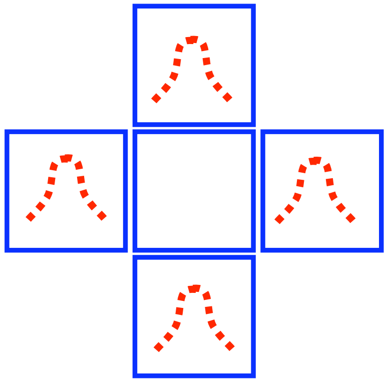


Put a quantum in a potential well

Tunneling probability is small

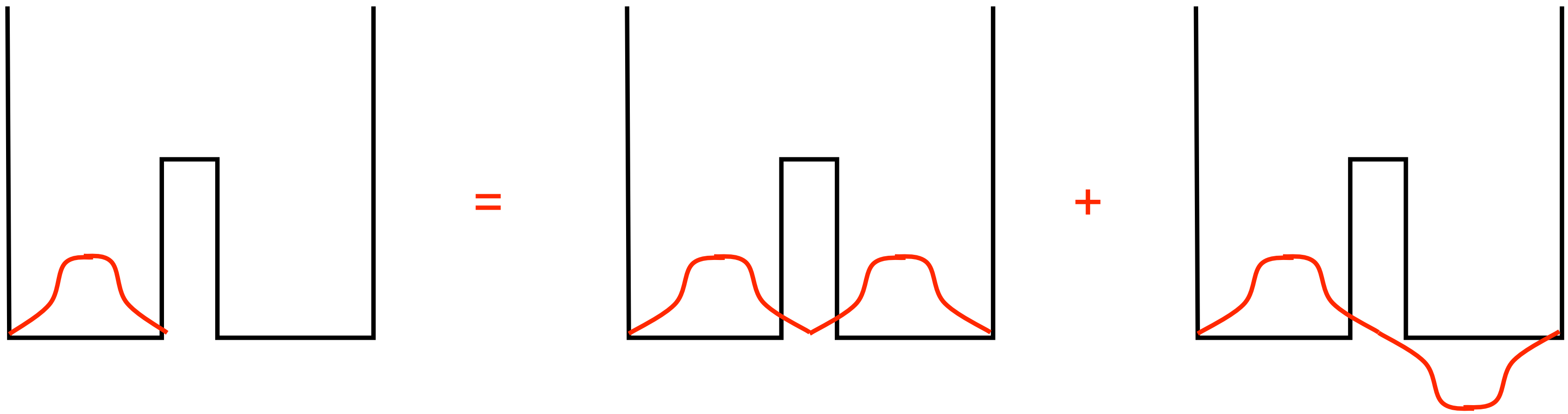
But there are many neighboring wells

In a time of order unity, the quantum spreads to a linear combination of states in all potential wells

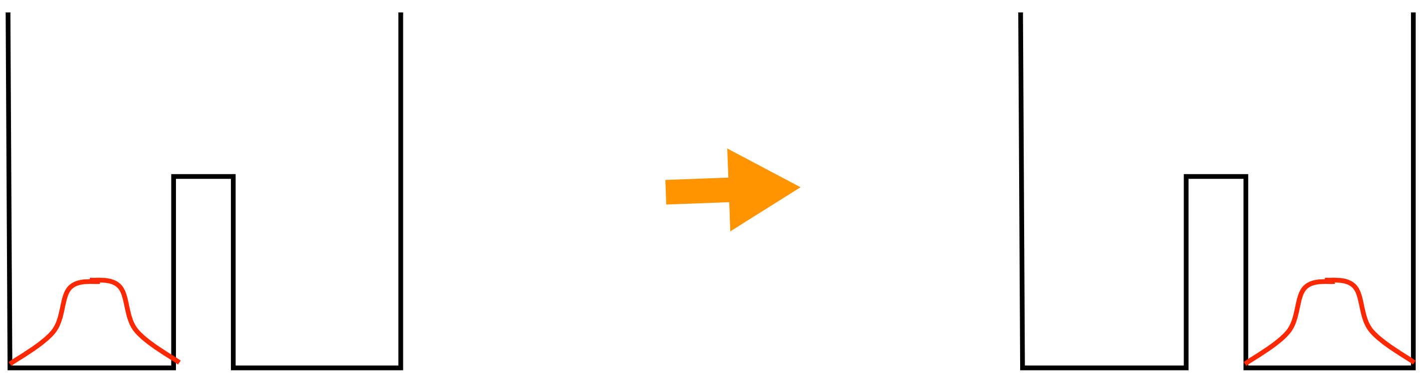


(SDM 08)

Tunneling is just 'de-phasing' of eigenstates :

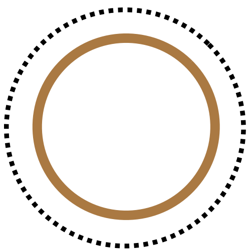


$$|\psi\rangle = \frac{1}{2}|\psi_S\rangle + \frac{1}{2}|\psi_A\rangle \;\rightarrow\; \frac{1}{2}e^{-iE_S t}|\psi_S\rangle + \frac{1}{2}e^{-iE_A t}|\psi_A\rangle$$

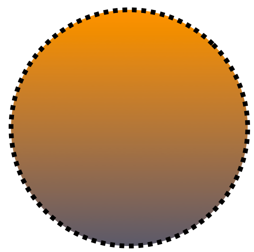


(B) How long does it take for the shell to become a general linear combination of fuzzballs ?

If it takes more than Hawking evaporation time, fuzzballs dont help !



$$|\psi\rangle = \sum_k c_k |E_k\rangle$$



$$\Delta P \gg \frac{1}{R}$$

$$\Delta E \sim \frac{P\Delta P}{M} \gg \frac{(\Delta P)^2}{M} \gg \frac{1}{MR^2}$$

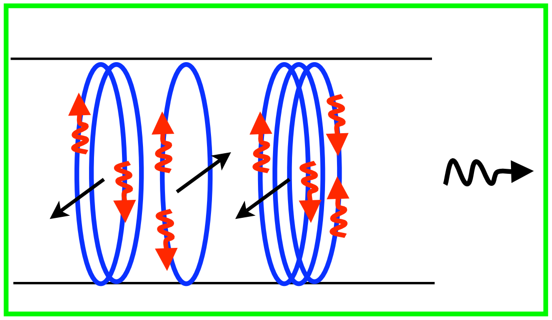
$$t_{dephase} \sim \frac{1}{\Delta E} \ll MR^2$$

Note that $\quad t_{evap} \sim MR^2$

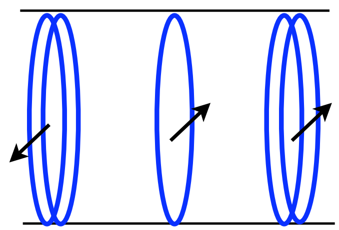
So $\quad \boxed{t_{dephase} \ll t_{evap}}$

So the state becomes a linear combination of fuzzballs much before the hole evaporates

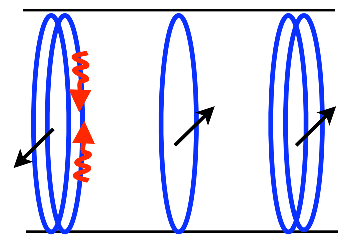
All microstates of black holes made so far are found to be ‘fuzzballs’



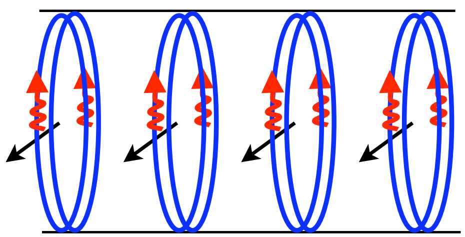
General CFT state for nonextremal D1D5



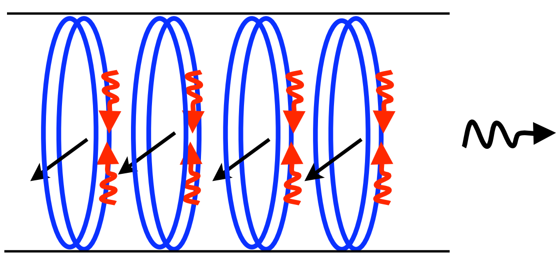
2-charge extremal

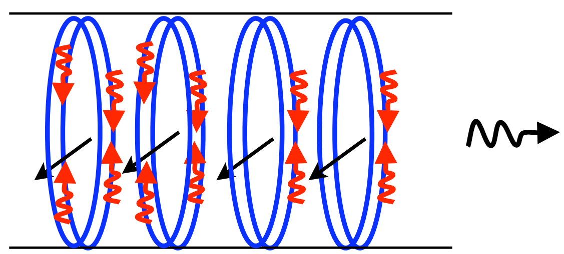


2-charge extremal + excitation



3-charge extremal: Large classes also known with CFT state not yet identified

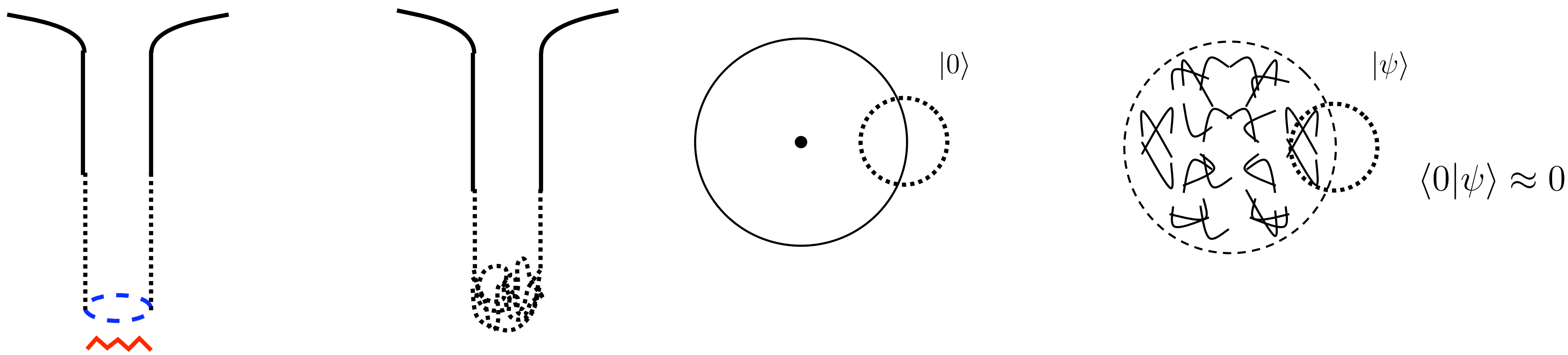




Nonextremal: Some families known, radiation agrees

Lesson: *Quantum gravity effects extend distances much longer than planck length if many quanta are involved*

$$\boxed{l_p \longrightarrow N^{\alpha}\ l_p}$$



Many pieces of evidence: 2-charge extremal, 3-charge extremal, Energy gaps, Radiation from non-extremal states

Can use this fuzzball structure to analyze 'Dynamics' ....

Large non-locality is providing interesting possibilities for early Universe dynamics ....

http://www.physics.ohio-state .edu/~mathur