Properties of Baryons in Holographic QCD

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Based on

→ **arXiv:0806.3122** K.Hashimoto, T.Sakai and S.S.

hep-th/0701280 H.Hata, T.Sakai, S.S. and S.Yamato

Closely related works:

arXiv:0807.0033 K.Y.Kim and I.Zahed

arXiv:0803.0180 H.Hata, M.Murata and S.Yamato

hep-th/0701276, arXiv:0705.2632, **arXiv:0710.4615, …**
D.K.Hong, M.Rho, H.U.Yee and P.Yi

Indian Strings Meeting 2008 (ISM08) @Pondicherry 12/6-13/2008

1 Introduction

Claim :

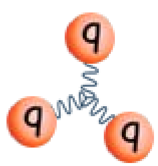
Hadrons can be described by ***string theory***

without using quarks !

QCD ***dual !*** ⟷ String theory (in a certain curved background)

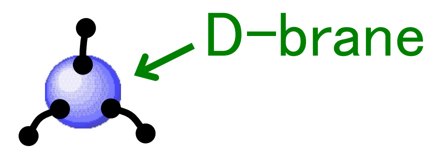
" Holographic QCD "

meson

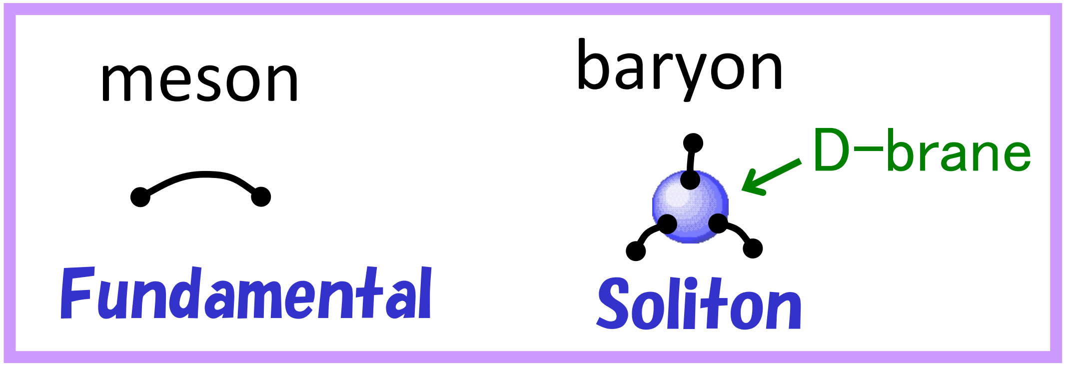


baryon

string



D−brane with N_c strings



Prototype: ***Skyrme model***

- In 1961, Skyrme proposed

  Baryons are solitons (Skyrmion) in the pion effective theory.

- In 1983, Adkins-Nappi-Witten (ANW)

  succeeded to calculate the static properties
  (charge radii, magnetic moments, axial coupling, etc.)
  by quantizing the collective modes of the Skyrmion.

  Roughly agree with the experimental data!

Q. Can we apply the idea of ANW to holographic QCD ?

[Sakai-S.S. 2004, 2005]

- Recently, based on gauge/string duality + probe approximation, we proposed that meson effective theory is given by a 5 dim $U(N_f)$ YM-CS theory in a curved space-time.

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2\right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

  - This system is equivalent to 4 dim effective theory with (infinitely) many mesons. $\pi, \rho, a_1, \rho', a_1', \cdots$
  - Masses and couplings calculated in this system roughly agree with the experimental data!

(x^1, x^2, x^3, z)

- Baryons are realized as instantons localized on the 4 dim space.

Goal: extract properties of baryons using this description

Plan

1. Introduction
2. Brief summary of the model
3. Baryons as instantons
4. Quantization
5. Currents
6. Exploration
7. Conclusion

2 Brief summary of the model [Sakai-S.S. 2004]

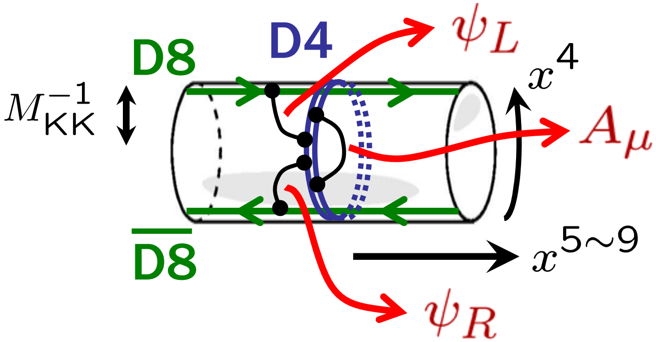
Type IIA string theory
in Witten's D4 background

\+ N_f Probe D8-branes
(assuming $N_c \gg N_f$)

dual ⟷
at low energy

4 dim QCD with
N_f massless quarks

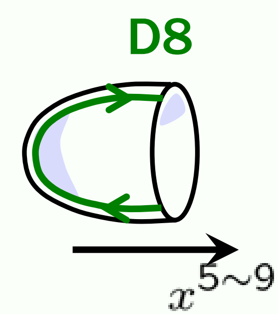
$\overset{N_c}{\text{D4}}$-$\overbrace{\text{D8-}\overline{\text{D8}}}^{N_f \text{ pairs}}$ system
on ~~SUSY~~ S^1



QCD with N_f massless quarks
(at low energy)

dual

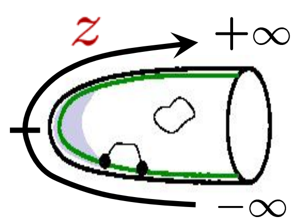
String theory
in the D4 background
\+ N_f probe D8-branes
(assuming $N_c \gg N_f$)



The effective theory on the D8-branes

N_f D8-branes extended along $(x^\mu, z) \times S^4 \subset \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$

← Low energy



9 dim $U(N_f)$ gauge theory

← Reducing S^4 (Here we only consider *SO(5)* invariant states)

5 dim $U(N_f)$ YM-CS theory

$A_\mu(x^\nu, z), A_z(x^\nu, z)$ $\mu, \nu = 0 \sim 3$

5 dim gauge field

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5-form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$$\kappa = \frac{\lambda N_c}{216\pi^3} \equiv a\lambda N_c$$

$h(z) = (1 + z^2)^{-1/3}$

$(M_{\text{KK}} = 1 \text{ unit})$

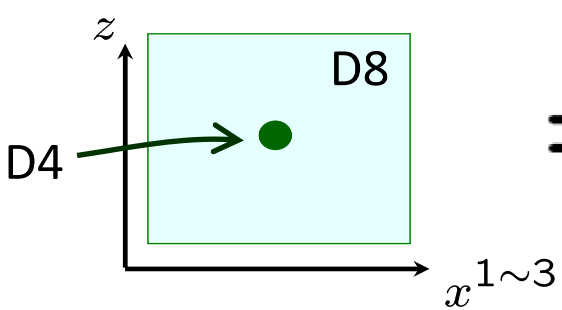
3 Baryons as instantons

- Baryon is described as D4 wrapped on S^4 [Witten, Gross-Ooguri 1998]

D4-brane D8-brane Topology of the background

$$\underset{x^0}{\mathbf{R}} \times S^4 \subset \underset{x^\mu}{\mathbf{R}^{1,3}} \times \underset{z}{\mathbf{R}} \times S^4 \subset \mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

Bahaves as a point-like particle ➡ **baryon**



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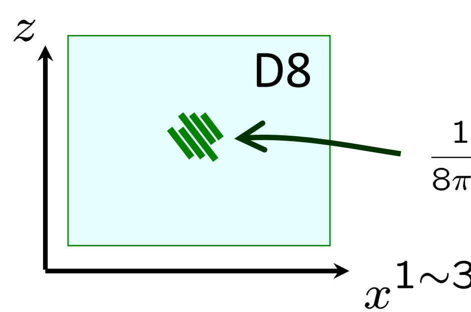


$\frac{1}{8\pi^2}\int \mathrm{Tr} F \wedge F \neq 0$

[Douglas 1995]

D4 within D8

$M = 1, 2, 3, z$

Instanton in $x^M = (\vec{x}, z) \in \mathbf{R}^4$

$$\text{Baryon number} = \text{number of D4} = \frac{1}{8\pi^2}\int_{4\text{dim}} \mathrm{tr} F \wedge F$$

(x^1, x^2, x^3, z)

Classical solution (We consentrate on the $N_f = 2$ case.)

- The instanton solution for the Yang-Mills action

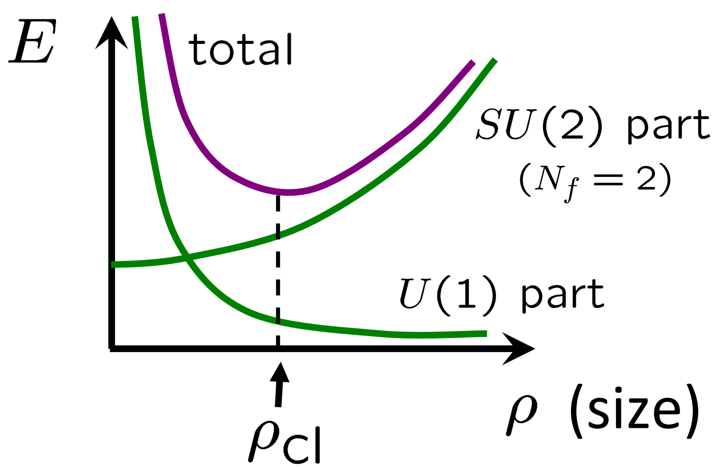
$$S_{\rm YM} = \kappa \int d^4x dz \, {\rm Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right)$$

shrinks to zero size !

- The Chern-Simons term makes it larger

$$S_{\rm CS} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A) = \frac{N_c}{16\pi^2} \int d^4x dz \, \underset{U(1)\ \text{part}}{A_0^{U(1)}} \, \underbrace{\epsilon^{ijk} {\rm Tr} F_{ij} F_{kz}}_{\text{Non-zero for instanton}} + \cdots$$

→ source of the U(1) charge



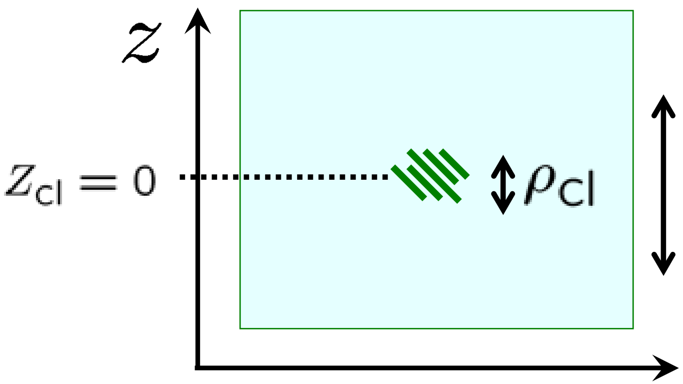
→ Stabilized at $\rho_{\rm cl}^2 = \frac{N_c}{8\pi^2 \kappa} \sqrt{\frac{6}{5}}$

[Hong-Rho-Yee-Yi 2007]
[Hata-Sakai-S.S.-Yamato 2007]

- Note that $\rho_{\rm cl} \sim \mathcal{O}(\lambda^{-1/2})$

λ : 't Hooft coupling (assumed to be large)

If λ is large enough, the 5 dim space-time can be approximated by the flat space-time. (The effect of the non-trivial z-dependence is taken into account perturbatively.)



$k(z) = 1 + z^2, \ h(z) = (1+z^2)^{-1/3}$

$k(z) \simeq h(z) \simeq 1$ (for $|z| \ll 1$)

➡ The leading order classical solution is the BPST instanton with $\rho = \rho_{\rm cl}$ and $Z = Z_{\rm cl} = 0$

$$A_M^{\rm cl} = -i \frac{\xi^2}{\xi^2 + \rho^2} g \partial_M g^{-1}$$

$M = 1, 2, 3, z$

$$g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi}$$

$$\xi = \sqrt{(\vec{x} - \vec{X})^2 + (z - Z)^2}$$

ρ : size $(\vec{X}, Z)$: position of the instanton

4 Quantization

[Hata-Sakai-S.S.-Yamato 2007]

- Consider a slowly moving (rotating) baryon configuration.
  moduli space approximation method :

  Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ $(\alpha = 1, 2, \cdots, \dim \mathcal{M})$

  time

  $\boxed{A_M(t,x) \sim A_M^{\mathrm{cl}}(x; X^\alpha(t))}$

  $S_{5\mathrm{dim}} \rightarrow$ **Quantum Mechanics for** $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z}_{\text{position}}, \rho)\} \times SU(2)/\mathbf{Z}_2 \qquad \mathbf{Z}_2 : \mathbf{a} \rightarrow -\mathbf{a}$$

size $\quad \mathbf{a} \in SU(2)$ $\leftarrow$ *SU(2)* orientation

$$\rightarrow \quad L_{\mathrm{QM}} = \frac{G_{\alpha\beta}}{2}\dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\lambda^2\rho^2} + \frac{Z^2}{3}\right) + \cdots\right)$$

<u>**Note**</u> $(\vec{X}, \mathbf{a})$: genuine moduli (the same as in the Skyrme model)

(ρ, Z) : new degrees of freedom, added since they are light compared with the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics, we obtain the baryon states

→ Generalization of Adkins-Nappi-Witten including vector mesons and ρ , Z modes

We can construct baryon states for

$$n, p, \Delta(1232), N(1440), N(1530), \cdots$$

<u>Example</u> Nucleon wave function:

$$\psi(\vec{X}, \mathbf{a}, \rho, Z) \propto e^{i\vec{p}\cdot\vec{X}} R(\rho)\psi_Z(Z)T(\mathbf{a})$$

$$\left(\begin{array}{ll} R(\rho) = \rho^{\tilde{l}} e^{-A\rho^2} & \tilde{l} = -1 + 2\sqrt{1 + N_c^2/5} \\ \psi_Z(\rho) = e^{-AZ^2} & A = \frac{8\pi^2\kappa}{\sqrt{6}} \\ T(\mathbf{a}) = a_1 + ia_2 \ \text{ for } \ |p\uparrow\rangle & \text{etc.} \end{array}\right)$$

5 Currents

- Chiral symmetry

$$U(N_f)_L \times U(N_f)_R \xrightarrow{\text{gauge}} (A_{L\mu}(x), A_{R\mu}(x))$$

- Interpreted as

$$A_{L\mu}(x) = \lim_{z\to+\infty} A_\mu(x,z) \qquad A_{R\mu}(x) = \lim_{z\to-\infty} A_\mu(x,z)$$

$$\Rightarrow \quad S_{5\,\mathrm{dim}}\Big|_{\mathcal{O}(A_L,A_R)} = -\int d^4x \left(A^a_{L\mu} J^{a\mu}_L + A^a_{R\mu} J^{a\mu}_R\right)$$

with

$$J_{L\mu} = -\kappa\left(k(z)F_{\mu z}\right)\Big|_{z=+\infty} \qquad J_{R\mu} = +\kappa\left(k(z)F_{\mu z}\right)\Big|_{z=-\infty}$$

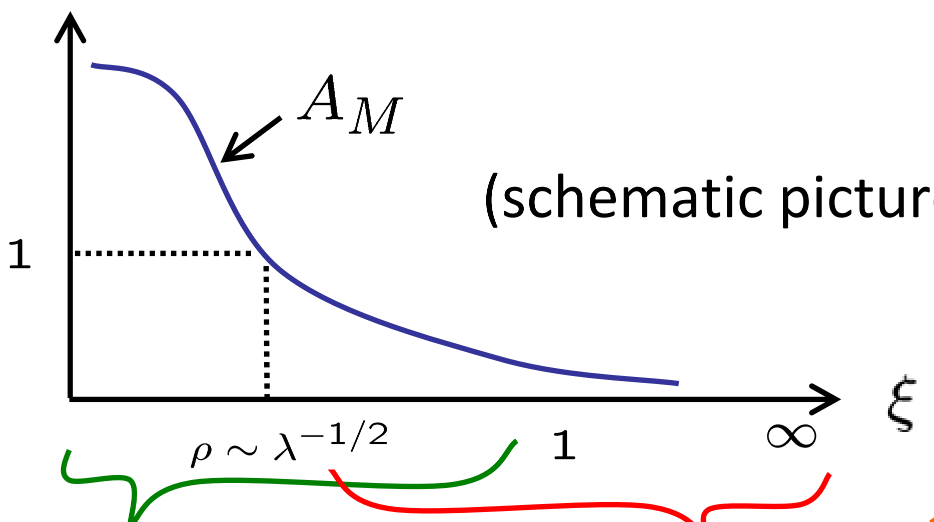
- vector and axial vector currents

$$J^\mu_V \equiv J^\mu_L + J^\mu_R = -\kappa\Big[k(z)F^{\mu z}\Big]^{z=+\infty}_{z=-\infty}$$

$$J^\mu_A \equiv J^\mu_L - J^\mu_R = -\kappa\Big[\psi_0(z)k(z)F^{\mu z}\Big]^{z=+\infty}_{z=-\infty} \quad (\psi_0(\pm\infty) = \pm 1)$$

How to calculate

- We need to know how $F_{\mu z}(x,z)$ behaves at $z \to \pm\infty$
  - ➡ We cannot use the solution in the flat space.
- The EOM are complicated non-linear equations.
  - ➡ difficult to solve exactly.
- We use the following trick to calculate the currents.



Instanton solution in the flat space

Non-linear terms are small

Approximated by linearized EOM

Now we can solve!

6 Exploration

Now we are ready to calculate various physical quantities

But, don't trust too much !

- λ may not be large enough.
- Higher derivative terms may contribute.
- $N_c = 3$ is not large enough.
- The model deviates from real QCD at high energy $\sim M_{\rm KK}$
- We use $M_{\rm KK} \simeq 949$ MeV (value consistent with ρ meson mass)
  But we know this is too large to fit the baryon mass differences.

- Baryon number current

$$J_B^\mu = -\frac{2}{N_c}\kappa\left[k(z)F_{U(1)}^{\mu z}\right]_{z=-\infty}^{z=+\infty}$$

U(1) part of the U(2) gauge field

$\Downarrow$

$$\boxed{J_B^0 \simeq \left[k(z)\partial_z G\right]_{z=-\infty}^{z=+\infty} \qquad J_B^i \simeq -\frac{J^j}{16\pi^2\kappa}\epsilon^{ijk}\partial_k J_B^0 + \cdots}$$

(G : Green's function $(h(z)\partial_i^2 + \partial_z k(z)\partial_z)G = \delta^3(\vec{x}-\vec{X})\delta(z-Z)$

J^j : Spin operator $J^j = -i4\pi^2\kappa\rho^2\,\mathrm{tr}(\tau^j \mathbf{a}^{-1}\dot{\mathbf{a}})$)

Note: $k(z) \sim z^2, \quad \partial_z G \sim 1/z^2$ at $z \to \pm\infty$

$\Rightarrow$ J_B^μ is non-zero, finite.

- Isoscalar mean square radius

$$\langle r^2\rangle_{I=0} = \int d^3x\, r^2\, J_B^0 \simeq (0.742\ \mathrm{fm})^2$$

↑

Numerical estimate using $M_{\mathrm{KK}} \simeq 949\ \mathrm{MeV}$
(fixed by ρ -meson mass)

$$\left(\mathrm{cf.}\ \langle r^2\rangle_{I=0}^{1/2}\Big|_{\mathrm{exp}} = 0.806\ \mathrm{fm}\,,\ \langle r^2\rangle_{I=0}^{1/2}\Big|_{\mathrm{ANW}} = 0.59\ \mathrm{fm}\right)$$

Isoscalar magnetic moment

$$J_B^i \simeq -\frac{J^j}{16\pi^2\kappa}\epsilon^{ijk}\partial_k J_B^0 + \cdots$$

$$\mu^i_{I=0} = \frac{1}{2}\epsilon^{ijk}\int d^3x\, x^j J_B^k \simeq \frac{J^i}{16\pi^2\kappa}$$

For a spin up proton state $|p\uparrow\rangle$

Isoscalar g-factor

$$\langle p\uparrow|\,\mu^i_{I=0}\,|p\uparrow\rangle = \frac{\delta^{i3}}{32\pi^2\kappa} \equiv \frac{g_{I=0}}{4M_N}\delta^{i3}$$

Nucleon mass

($M_N \simeq 940$ MeV)

$$\Rightarrow \quad g_{I=0} = \frac{M_N}{8\pi^2\kappa\, M_{\rm KK}} \simeq 1.68$$

$M_{\rm KK} \simeq 949$ MeV, (fixed by m_ρ) $\kappa \simeq 0.00745$ (fixed by f_π)

$$\left(\text{cf. } g_{I=0}\big|_{\rm exp} \simeq 1.76,\ \ g_{I=0}\big|_{\rm ANW} = 1.11\right)$$

Summary of the results

	our result	exp.	ANW
$\langle r^2\rangle^{1/2}_{I=0}$	0.742 fm	0.806 fm	0.59 fm
$\langle r^2\rangle^{1/2}_{I=1}$	0.742 fm	0.939 fm	∞ ※
$\langle r^2\rangle^{1/2}_{A}$	0.537 fm	0.674 fm	—
$g_{I=0}$	1.68	1.76	1.11
$g_{I=1}$	7.03	9.41	6.38
g_A	0.734	1.27	0.61

※ pion loop contribution is log divergent in the chiral limit.
Our calculation corresponds to the tree level in ChPT.

- We can also evaluate these for excited baryons such as $\Delta(1232), N(1440), N(1535), \cdots$

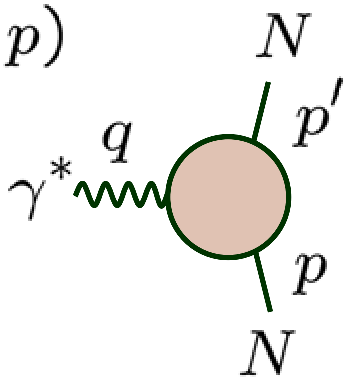
Form factors

Dirac form factor   Pauli form factor

$$\langle N,\vec{p}'|J^{\mu}_{\rm em}(0)|N,\vec{p}\rangle = \overline{u}(p',s')\left[\gamma^{\mu}F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_{\nu}F_2(q^2)\right]u(p,s)$$

$$(q = p' - p)$$

Breit frame: $\vec{p}' = -\vec{p} = \vec{q}/2$



$$\langle N,\vec{q}/2|J^{0}_{\rm em}(0)|N,-\vec{q}/2\rangle = G_E(\vec{q}^2)\,\chi^{\dagger}_{s'}\chi_s$$

$$\langle N,\vec{q}/2|J^{i}_{\rm em}(0)|N,-\vec{q}/2\rangle = \frac{i}{2m_N}G_M(\vec{q}^2)\,\chi^{\dagger}_{s'}(\vec{q}\times\vec{\sigma})\chi_s$$

Sachs form factor

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_N^2}F_2(q^2)$$ Electric form factor

$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$ Magnetic form factor

Dipole behavior

Experimental data suggest

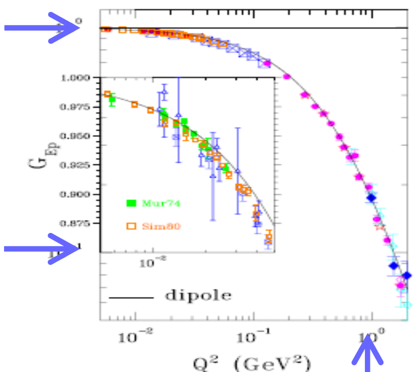
dipole $(\Lambda \simeq 0.71\ \mathrm{GeV}^2)$

$$G_E^p(Q^2) \simeq \frac{1}{\mu_p} G_M^p(Q^2) \simeq \frac{1}{\mu_n} G_M^n(Q^2) \simeq \frac{1}{(1+\frac{Q^2}{\Lambda^2})^2} \qquad G_E^n(Q^2) \simeq 0$$

G_E^p

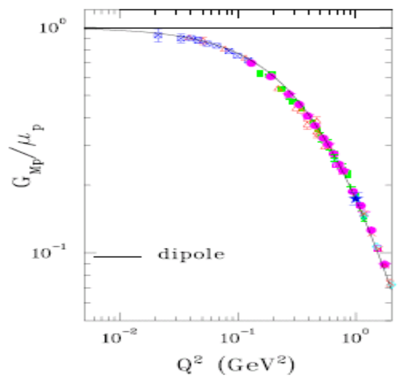
1

0.1

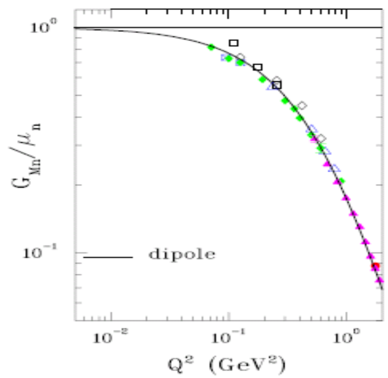


1 GeV^2

G_M^p/μ_p



G_M^n/μ_n

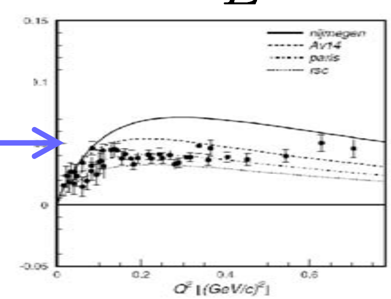


—— : dipole

dots : data

G_E^n

0.05

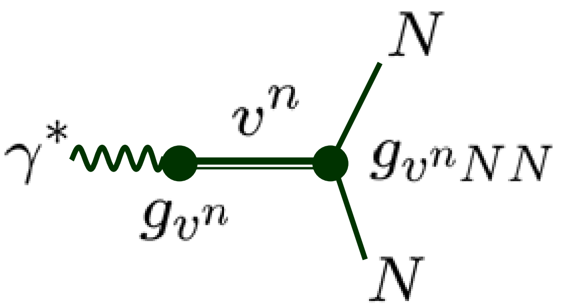


Our result

$$G_E^p(Q^2) = \frac{1}{\mu_p} G_M^p(Q^2) = \frac{1}{\mu_n} G_M^n(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2} \qquad G_E^n(Q^2) = 0$$

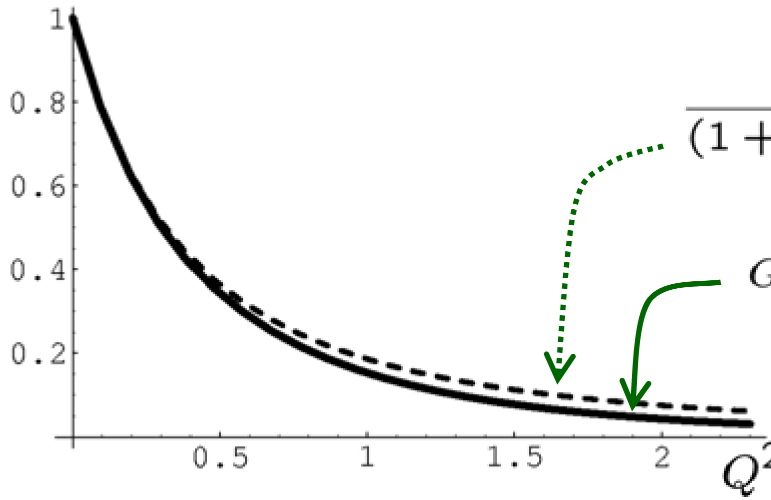
with $$g_{v^n} = -2\kappa(k(z)\partial_z \psi_{2n-1})\Big|_{z=+\infty}$$

$$g_{v^n NN} = \langle \psi_{2n-1}(Z) \rangle$$



Vector meson dominance

Can this be compatible with dipole?



Q^2 GeV^2

$$\frac{1}{(1+Q^2/\Lambda^2)^2} \quad \text{dipole}$$

$$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2} \quad \text{our result}$$

Taylor expansion

(※ here we use the approximation $g_{v^n NN} = \langle \psi_{2n-1}(Z) \rangle \simeq \psi_{2n-1}(0)$)

$$G_E^p(Q^2) \simeq 1 - 2.38Q^2 + 4.02(Q^2)^2 - 6.20(Q^2)^3 + 9.35(Q^2)^4 - 14.0(Q^2)^5 + \cdots$$

$$\frac{1}{(1+Q^2/\Lambda^2)^2} \simeq 1 - 2.38Q^2 + 4.24(Q^2)^2 - 6.71(Q^2)^3 + 9.97(Q^2)^4 - 14.2(Q^2)^5 + \cdots$$

$(M_{\mathrm{KK}} = 1 \text{ unit})$

with $\Lambda^2 = 0.758$ GeV^2

5 Conclusion

- We proposed a new method to analyze static properties of baryons.
- Our model automatically includes the contributions from various massive vector and axial-vector mesons.
- Compared with the similar analysis in the Skyrme model (ANW), the agreement with the experimental values are improved in most of the cases.
- But, we should keep in mind that our analysis is very crude and there are a lot of ambiguities remain unsolved.