லோரென்ட்சொடிய 3-அக்ஷரகணித அசைதல்

Dynamiques des 3-algèbres Lorentziennes

Sunil Mukhi
Institut Tata de Recherche Fondamentale, Mumbai

Réunion Indienne des Cordes, Pondichéry
Décembre 11, 2008

- Based on:

  "D2 to D2",
  Bobby Ezhuthachan, SM and Costis Papageorgakis,
  arXiv:0806.1639 [hep-th], JHEP 0807:041, (2008).

  "Higher-derivative 3-algebras",
  Mohsen Alishahiha and SM,
  arXiv:0808.3067 [hep-th], JHEP 0810:yyy, (2008).

Outline

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- However M-theory does have two kinds of stable branes:
  - M2-branes (membranes)
  - M5-branes
- This raises the question of what is the worldvolume field theory on these branes.
- In this talk I will describe one aspect of the recent progress in finding the field theory on multiple M2-branes.

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- The field theory on N D2-branes is just maximally supersymmetric or $\mathcal{N} = 8$ Yang-Mills theory in $2 + 1$ d, which has 7 transverse scalars.
- This is a super-renormalisable theory that inherits its coupling from the string coupling g_s:

$$g_{\mathsf{YM}} = \sqrt{\frac{g_s}{l_s}}$$

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  and the problem is to find an explicit form for this limiting theory.
- The limiting theory, if interacting, must be an infrared fixed point and therefore conformal invariant.
- Also, if the brane interpretation is to make sense, the field theory should have 8 scalars with an SO(8) global symmetry, describing transverse motion.

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  - Superconformal invariance.

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- Like the Bagger-Lambert theories described in the previous talk, these theories make use of the 3-algebra structure and are Chern-Simons-like, though really of $B \wedge F$ type.
- They have maximal $\mathcal{N} = 8$ superconformal invariance, their structure constants obey the fundamental identity, and they can be generalised to arbitrary Lie algebras.
- A constraint has to be imposed that eliminates negative-norm states [Bandres-Lipstein-Schwarz, Gomis-Rodriguez-Gomez-van Raamsdonk-Verlinde].

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- We will see [Alishahiha-SM] that the 3-algebra structure survives even after the corrections are included, giving us confidence that it is fundamental to the theory.
- The usefulness of these theories to describe membranes remains an open question at present.

Outline

Abelian duality

- The Abelian DBI action for D2-branes is:

$$\mathcal{L} = -\frac{1}{\alpha'^2 g_{YM}^2}\sqrt{-\det(g_{\mu\nu} + \alpha' F_{\mu\nu})}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

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- The factor of $(g_{YM})^{-2}$ in front shows that it is a tree-level action in open string theory.
- Abelian duality is implemented by replacing the above action by the equivalent action:

$$\mathcal{L} = \frac{1}{2}\varepsilon^{\mu\nu\lambda} B_\mu F_{\nu\lambda} - \frac{1}{\alpha'^2 g_{YM}^2}\sqrt{-\det(g_{\mu\nu} + \alpha'^2 g_{YM}^4 B_\mu B_\nu)}$$

Integrating out B_μ, one recovers the original action.

- If instead we integrate out the gauge field A_μ, its equation of motion tells us that

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- Noting also that in static gauge:

$$g_{\mu\nu} = \eta_{\mu\nu} + \alpha'^2 g_{YM}^2 \partial_\mu X^i \partial_\nu X^i, \quad i = 1, 2, \cdots, 7$$

and that

$$\alpha'^2 g_{YM}^2 = \alpha'^{\frac{3}{2}} g_s = \ell_p^3$$

we end up with the action:

$$\mathcal{L} = -\frac{1}{\ell_p^3}\sqrt{-\det(\eta_{\mu\nu} + \ell_p^3\, \partial_\mu X^I \partial_\nu X^I)}, \quad I = 1, 2, \cdots, 8$$

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- It can then be interpreted as the action for a single M2-brane.
- Note that the actions related by this duality contain all orders in derivatives, though restricted to the DBI approximation of slowly varying first derivatives.

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$$-\frac{1}{4g_{YM}^2}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} \quad \to \quad \frac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu\boldsymbol{F}_{\nu\lambda} - \frac{1}{2}\left(D_\mu\boldsymbol{\phi} - g_{YM}\boldsymbol{B}_\mu\right)^2$$

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  Here D_μ is the covariant derivative with respect to $\boldsymbol{A}$.
- In addition to the gauge symmetry G, the new action has a noncompact abelian gauge symmetry:

$$\delta\boldsymbol{\phi} = g_{YM}\boldsymbol{M}, \qquad \delta\boldsymbol{B}_\mu = D_\mu\boldsymbol{M}$$

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  where $\boldsymbol{M}(x)$ is an arbitrary matrix in the adjoint of G.
- To prove the duality, use this symmetry to set $\boldsymbol{\phi} = 0$. Then integrating out $\boldsymbol{B}_\mu$ gives the usual YM kinetic term for $\boldsymbol{F}_{\mu\nu}$.

- The duality-transformed $\mathcal{N} = 8$ SYM is:

$$\begin{aligned} L &= \mathrm{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\big(D_\mu\boldsymbol{\phi} - g_{\mathsf{YM}}\boldsymbol{B}_\mu\big)^2 \\ &- \tfrac{1}{2}D_\mu \boldsymbol{X}^i D^\mu \boldsymbol{X}^i - \tfrac{g_{\mathsf{YM}}^2}{4}[\boldsymbol{X}^i, \boldsymbol{X}^j]^2 + \text{fermions}\Big) \end{aligned}$$

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- Rename $\boldsymbol{\phi} \to \boldsymbol{X}^8$. Then the scalar kinetic terms are:

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  where $g_{\mathsf{YM}}^I = (0, \ldots, 0, g_{\mathsf{YM}})$.
- Next, we can allow g_{YM}^I to be an arbitrary 8-vector.

- The action is now $SO(8)$-invariant if we rotate both the fields X^I and the coupling-constant vector g^I_{YM}:

$$\begin{aligned} L &= \operatorname{tr}\Big(\tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \\ &- \tfrac{1}{12}\left(g^I_{YM}[\boldsymbol{X}^J, \boldsymbol{X}^K] + g^J_{YM}[\boldsymbol{X}^K, \boldsymbol{X}^I] + g^K_{YM}[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big) \end{aligned}$$

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- This is legitimate if and only if $X^I_+(x)$ has an equation of motion that renders it constant. Then on-shell we can recover the original theory by writing $\langle X^I_+ \rangle = g^I_{\textsf{YM}}$.

- Constancy of X_+^I is imposed by introducing a new set of abelian gauge fields and scalars: C_μ^I, X_-^I and adding the following term:

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which will remove the negative-norm states associated to C_μ^I.

- We have thus ended up with the Lorentzian 3-algebra action:

$$\begin{aligned} L = \operatorname{tr}\Big(& \tfrac{1}{2}\epsilon^{\mu\nu\lambda}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\lambda} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}_\mu \boldsymbol{X}^I \\ & - \ \tfrac{1}{12}\left(X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]\right)^2 \Big) \\ & + \ (C^{\mu\, I} - \partial^\mu X_-^I)\partial_\mu X_+^I + L_{\text{gauge-fixing}} + \mathcal{L}_{\text{fermions}} \end{aligned}$$

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- The interactions depend on the triple product:

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- However, both are spontaneously broken by giving a vev $\langle X_+^I \rangle = g_{\mathsf{YM}}^I$ and the theory reduces to $\mathcal{N} = 8$ SYM with coupling $|g_{\mathsf{YM}}|$.

- The final action has some remarkable properties.
- The interactions depend on the triple product:

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- The final theory has seven massless scalars, and they are the Goldstone bosons for the spontaneous breaking $SO(8) \to SO(7)$.
- To actually describe M2-branes we need to find a way to take $\langle X_+^I \rangle = \infty$. That has not yet been done.

Outline

Higher-order corrections for Lorentzian 3-algebras

- It is natural to ask if the non-Abelian duality that we have just performed works when α' corrections are included in the D2-brane action.

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- I will now show [Alishahiha-SM] that to lowest nontrivial order (F^4-type corrections) one can indeed dualise the non-Abelian theory into an $SO(8)$-invariant form.
- Here of course one cannot do all orders in α' because a non-Abelian analogue of DBI is still not known. (Nevertheless a claim of doing this has been made by [Iengo-Russo].)

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- The bosonic ${\alpha'}^2$ correction to $\mathcal{N}=1$ SYM in 10d is known to be:

$$L^{(10)} = \frac{1}{8}\,\mathrm{STr}\Big[\boldsymbol{F}_{MN}\boldsymbol{F}_{RS}\boldsymbol{F}^{MR}\boldsymbol{F}^{NS} - \frac{1}{4}\boldsymbol{F}_{MN}\boldsymbol{F}^{MN}\boldsymbol{F}_{RS}\boldsymbol{F}^{RS}\Big]$$

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  where we have used the symmetrised trace STr.
- To get the corresponding correction in 3d, we merely need to dimensionally reduce this.
- Henceforth we set $\alpha' = 1$ since we know it will re-appear as ℓ_p^3 at the end.

- The result of this dimensional reduction (here $\boldsymbol{X}^{ij} = [\boldsymbol{X}^i, \boldsymbol{X}^j]$) is:

$$\begin{aligned}
L_1^{(4)} &= \frac{1}{8g_{YM}^2}\,\mathrm{STr}\Big[\boldsymbol{F}_{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\mu\rho}\boldsymbol{F}^{\nu\sigma} - \tfrac{1}{4}\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\rho\sigma}\boldsymbol{F}^{\rho\sigma}\Big] \\
L_2^{(4)} &= \tfrac{1}{2}\,\mathrm{STr}\Big[\boldsymbol{F}_{\mu\nu}\,D^\mu\boldsymbol{X}^i\,\boldsymbol{F}^{\rho\nu}\,D_\rho\boldsymbol{X}^i - \tfrac{1}{9}\boldsymbol{F}_{\mu\nu}\,\boldsymbol{F}^{\mu\nu}\,D^\rho\boldsymbol{X}^i\,D_\rho\boldsymbol{X}^i\Big] \\
&\qquad - \frac{g_{YM}^2}{16}\,\mathrm{STr}\,\boldsymbol{F}_{\mu\nu}\,\boldsymbol{F}^{\mu\nu}\,\boldsymbol{X}^{ij}\,\boldsymbol{X}^{ij} \\
L_3^{(4)} &= -\frac{g_{YM}^2}{2}\,\mathrm{STr}\,D^\mu\boldsymbol{X}^i\,D^\nu\boldsymbol{X}^j\boldsymbol{F}_{\mu\nu}\boldsymbol{X}^{ij} \\
L_4^{(4)} &= \frac{g_{YM}^2}{4}\,\mathrm{STr}\Big[D_\mu\boldsymbol{X}^i\,D_\nu\boldsymbol{X}^j\,D^\nu\boldsymbol{X}^i\,D^\mu\boldsymbol{X}^j \\
&\qquad\qquad - \tfrac{1}{2}\,D_\mu\boldsymbol{X}^i\,D^\mu\boldsymbol{X}^i\,D_\nu\boldsymbol{X}^j\,D^\nu\boldsymbol{X}^j\Big] \\
L_5^{(4)} &= \frac{g_{YM}^4}{2}\mathrm{STr}\Big[\boldsymbol{X}^{kj}\,D_\mu\boldsymbol{X}^k\,\boldsymbol{X}^{ij}\,D^\mu\boldsymbol{X}^i - \tfrac{1}{8}\boldsymbol{X}^{ij}\,\boldsymbol{X}^{ij}\,D_\mu\boldsymbol{X}^k\,D^\mu\boldsymbol{X}^k\Big] \\
L_6^{(4)} &= \frac{g_{YM}^6}{8}\mathrm{STr}\Big[\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{ik}\boldsymbol{X}^{jl} - \tfrac{1}{4}\boldsymbol{X}^{ij}\boldsymbol{X}^{ij}\boldsymbol{X}^{kl}\boldsymbol{X}^{kl}\Big]
\end{aligned}$$

- Now the aim is to rewrite the above Lagrangian by introducing new fields $\boldsymbol{B}_{\mu}, \boldsymbol{X}^{8}$ such that it becomes manifestly $SO(8)$ invariant.

- Now the aim is to rewrite the above Lagrangian by introducing new fields $\boldsymbol{B}_\mu, \boldsymbol{X}^8$ such that it becomes manifestly $SO(8)$ invariant.
- It is useful to proceed in two steps. First we simply rewrite the Lagrangian in terms of the Poincare dual field strength defined by:

$$\boldsymbol{f}_\mu = \frac{1}{2}\epsilon_{\mu\nu\lambda}\boldsymbol{F}^{\nu\lambda}$$

- Replacing $\boldsymbol{F}_{\mu\nu}$ in terms of $\boldsymbol{f}_{\mu}$ everywhere in the preceding Lagrangian, we end up with:

$$
\begin{aligned}
&L^{(2)} + L_1^{(4)} + L_2^{(4)} + L_3^{(4)} = \\
&\mathrm{STr}\Big[\tfrac{1}{2g_{YM}^2} \boldsymbol{f}_{\mu} \boldsymbol{f}^{\mu} + \tfrac{1}{8g_{YM}^2} \boldsymbol{f}_{\mu} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\nu} \boldsymbol{f}^{\nu} \\
&+\tfrac{1}{2}\Big(\boldsymbol{f}^{\mu} \boldsymbol{f}_{\nu} D^{\nu} \boldsymbol{X}^i D_{\mu} \boldsymbol{X}^i - \tfrac{1}{2} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\mu} D_{\nu} \boldsymbol{X}^i D^{\nu} \boldsymbol{X}^i \Big) \\
&+ \tfrac{g_{YM}^2}{8} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\mu} \boldsymbol{X}^{ij} \boldsymbol{X}^{ij} + \tfrac{g_{YM}^2}{2} \epsilon_{\rho\mu\nu} \boldsymbol{f}^{\rho} D^{\mu} \boldsymbol{X}^i D^{\nu} \boldsymbol{X}^j \boldsymbol{X}^{ij} \Big]
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- Replacing $\boldsymbol{F}_{\mu\nu}$ in terms of $\boldsymbol{f}_{\mu}$ everywhere in the preceding Lagrangian, we end up with:

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& L^{(2)} + L_1^{(4)} + L_2^{(4)} + L_3^{(4)} = \\
& \mathrm{STr}\Big[\frac{1}{2g_{\mathsf{YM}}^2} \boldsymbol{f}_{\mu} \boldsymbol{f}^{\mu} + \frac{1}{8g_{\mathsf{YM}}^2} \boldsymbol{f}_{\mu} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\nu} \boldsymbol{f}^{\nu} \\
& + \tfrac{1}{2}\Big(\boldsymbol{f}^{\mu} \boldsymbol{f}_{\nu} D^{\nu} \boldsymbol{X}^i D_{\mu} \boldsymbol{X}^i - \tfrac{1}{2} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\mu} D_{\nu} \boldsymbol{X}^i D^{\nu} \boldsymbol{X}^i \Big) \\
& + \frac{g_{\mathsf{YM}}^2}{8} \boldsymbol{f}^{\mu} \boldsymbol{f}_{\mu} \boldsymbol{X}^{ij} \boldsymbol{X}^{ij} + \frac{g_{\mathsf{YM}}^2}{2} \epsilon_{\rho\mu\nu} \boldsymbol{f}^{\rho} D^{\mu} \boldsymbol{X}^i D^{\nu} \boldsymbol{X}^j \boldsymbol{X}^{ij} \Big]
\end{aligned}
$$

- Here we have written only the terms involving $\boldsymbol{f}$, as the remaining ones $L_4^{(4)}, L_5^{(4)}, L_6^{(4)}$ are obviously unaffected by our substitution.

- Now we can perform a non-Abelian duality as before.

- Now we can perform a non-Abelian duality as before.
- Introducing the independent (matrix-valued) gauge field $\boldsymbol{B}_\mu$, we get an action with only Chern-Simons-like and no Yang-Mills term:

$$
\begin{aligned}
L^{(2)} + L_1^{(4)} + L_2^{(4)} + L_3^{(4)} = \mathrm{STr}\Big[& \boldsymbol{f}_\mu \boldsymbol{B}^\mu - \frac{g_{\mathsf{YM}}^2}{2} \boldsymbol{B}_\mu \boldsymbol{B}^\mu \\
& + \frac{g_{\mathsf{YM}}^6}{8} \left(\boldsymbol{B}_\mu \boldsymbol{B}^\mu \boldsymbol{B}_\nu \boldsymbol{B}^\nu + \boldsymbol{B}_\mu \boldsymbol{B}^\nu \boldsymbol{X}^{ij} \boldsymbol{X}^{ij} \right) \\
& + \frac{g_{\mathsf{YM}}^4}{2} \Big(\boldsymbol{B}^\mu \boldsymbol{B}_\nu \, D^\nu \boldsymbol{X}^i \, D_\mu \boldsymbol{X}^i - \frac{1}{2} \boldsymbol{B}^\mu \boldsymbol{B}_\mu \, D_\nu \boldsymbol{X}^i \, D^\nu \boldsymbol{X}^i \\
& + \epsilon_{\rho\mu\nu} \boldsymbol{B}^\rho \, D^\mu \boldsymbol{X}^i \, D^\nu \boldsymbol{X}^j \, \boldsymbol{X}^{ij} \Big) \Big]
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$$

- To show that this substitution is correct, simply integrate out the field B using its equation of motion.

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- This brings the above Lagrangian back to the original form upto corrections of $\mathcal{O}(F^6)$.

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- This brings the above Lagrangian back to the original form upto corrections of $\mathcal{O}(F^6)$.
- Next, introduce the field $\boldsymbol{X}^8$ and replace $\boldsymbol{B}_\mu$, everywhere it occurs, by $-\frac{1}{g_{YM}}(D_\mu \boldsymbol{X}^8 - g_{YM}\boldsymbol{B}_\mu)$.

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- Using the shift symmetry one can gauge to $\boldsymbol{X}^8 = 0$ and we get back the previous action. However keeping $\boldsymbol{X}^8$ will enable us to restore $SO(8)$ invariance.

- Now following the previous procedure of making g_{YM} dynamical and re-defining covariant derivatives, we finally end up with an $SO(8)$ invariant action. (We now set $\ell_p = 1$.)

$$
\begin{aligned}
L \;=\; \mathrm{STr}\Big[& \tfrac{1}{2}\epsilon^{\mu\nu\rho}\boldsymbol{B}_\mu \boldsymbol{F}_{\nu\rho} - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I - \tfrac{1}{12}\boldsymbol{X}^{IJK}\boldsymbol{X}^{IJK} \\
& + \tfrac{1}{4}\Big(\hat{D}_\mu \boldsymbol{X}^I \hat{D}_\nu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^J - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \hat{D}_\nu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^J\Big) \\
& - \tfrac{1}{6}\,\epsilon_{\rho\mu\nu}\,\boldsymbol{X}^{IJK}\hat{D}^\rho \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^K \\
& + \tfrac{1}{8}\Big(\boldsymbol{X}^{LKJ}\boldsymbol{X}^{LIJ}\hat{D}_\mu \boldsymbol{X}^K \hat{D}^\mu \boldsymbol{X}^I - \tfrac{1}{3}\boldsymbol{X}^{LIJ}\boldsymbol{X}^{LIJ}\hat{D}_\mu \boldsymbol{X}^K \hat{D}^\mu \boldsymbol{X}^K\Big) \\
& - \tfrac{1}{72}\Big(\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NKL}\boldsymbol{X}^{MIK}\boldsymbol{X}^{MJL} - \tfrac{1}{4}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{NIJ}\boldsymbol{X}^{MKL}\boldsymbol{X}^{MKL}\Big)\Big]
\end{aligned}
$$

where:

$$
\begin{aligned}
\hat{D}_\mu \boldsymbol{X}^I &= \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I \\
\boldsymbol{X}^{IJK} &= X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]
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$$+ \tfrac{1}{4}\Big(\hat{D}_\mu \boldsymbol{X}^I \hat{D}_\nu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^J - \tfrac{1}{2}\hat{D}_\mu \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^I \hat{D}_\nu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^J\Big)$$
$$- \tfrac{1}{6}\epsilon_{\rho\mu\nu}\boldsymbol{X}^{IJK}\hat{D}^\rho \boldsymbol{X}^I \hat{D}^\mu \boldsymbol{X}^J \hat{D}^\nu \boldsymbol{X}^K$$
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where:

$$\hat{D}_\mu \boldsymbol{X}^I = \partial_\mu \boldsymbol{X}^I - [\boldsymbol{A}_\mu, \boldsymbol{X}^I] - \boldsymbol{B}_\mu X_+^I$$
$$\boldsymbol{X}^{IJK} = X_+^I[\boldsymbol{X}^J, \boldsymbol{X}^K] + X_+^J[\boldsymbol{X}^K, \boldsymbol{X}^I] + X_+^K[\boldsymbol{X}^I, \boldsymbol{X}^J]$$

- Parity is preserved if we choose $\boldsymbol{B}, X_+$ to be odd.

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  and it is remarkable that all of them depend only on the 3-algebra quantities $\hat{D}_\mu \boldsymbol{X}^I, \boldsymbol{X}^{IJK}$.
- Notice that both $\hat{D}_\mu \boldsymbol{X}^I$ and $\boldsymbol{X}^{IJK}$ have canonical dimension $\frac{3}{2}$ as required for the action to have a fixed power of ℓ_p^3 in front.

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- The leading term can be written:

$$\frac{i}{2}\mathrm{tr}\left(\bar{\boldsymbol{\lambda}}\Gamma^{\mu}D_{\mu}\boldsymbol{\lambda} + \frac{1}{2}\bar{\boldsymbol{\lambda}}\Gamma^{IJ}\boldsymbol{\lambda}^{IJ}\right)$$

Here,

$$\boldsymbol{\lambda}^{IJ} \equiv X_{+}^{I}[\boldsymbol{X}^{J}, \boldsymbol{\lambda}] - X_{+}^{J}[\boldsymbol{X}^{I}, \boldsymbol{\lambda}]$$

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Here,

$$\boldsymbol{\lambda}^{IJ}\equiv X_{+}^{I}[\boldsymbol{X}^{J},\boldsymbol{\lambda}]-X_{+}^{J}[\boldsymbol{X}^{I},\boldsymbol{\lambda}]$$

- The ℓ_p^3 corrections involve the building blocks $D_{\mu}\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}^{IJ}$ in addition to $\hat{D}_{\mu}\boldsymbol{X}^{I}$ and $\boldsymbol{X}^{IJK}$.

- As an example let us do the 4-fermion terms. On the D2-brane side we find (by dimensional reduction):

$$-\tfrac{1}{16}\mathrm{Str}\Big(\bar{\boldsymbol{\lambda}}\Gamma^{\mu}D^{\nu}\boldsymbol{\lambda}\,\bar{\boldsymbol{\lambda}}\Gamma_{\mu}D_{\nu}\boldsymbol{\lambda} + g_{YM}\bar{\boldsymbol{\lambda}}\Gamma^{i}D^{\nu}\boldsymbol{\lambda}\,\bar{\boldsymbol{\lambda}}\Gamma_{\nu}[\boldsymbol{X}^{i},\boldsymbol{\lambda}]$$
$$+ g_{YM}\bar{\boldsymbol{\lambda}}\Gamma^{\mu}[\boldsymbol{X}^{i},\boldsymbol{\lambda}]\,\bar{\boldsymbol{\lambda}}\Gamma^{i}D_{\mu}\boldsymbol{\lambda} + g_{YM}^{2}\bar{\boldsymbol{\lambda}}\Gamma^{i}[\boldsymbol{X}^{j},\boldsymbol{\lambda}]\,\bar{\boldsymbol{\lambda}}\Gamma^{j}[\boldsymbol{X}^{i},\boldsymbol{\lambda}]\Big)$$

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- Using our notation the above action can be recast in the following $SO(8)$ invariant form:

$$-\frac{1}{16}\mathrm{Str}\Big(\bar{\boldsymbol{\lambda}}\Gamma^\mu D^\nu\boldsymbol{\lambda}\,\bar{\boldsymbol{\lambda}}\Gamma_\mu D_\nu\boldsymbol{\lambda} + \frac{1}{2}\bar{\boldsymbol{\lambda}}\Gamma^{IJ}D^\nu\boldsymbol{\lambda}\,\bar{\boldsymbol{\lambda}}\Gamma_\nu\boldsymbol{\lambda}^{IJ} + \frac{1}{2}\bar{\boldsymbol{\lambda}}\Gamma^\mu\boldsymbol{\lambda}^{IJ}\,\bar{\boldsymbol{\lambda}}\Gamma^{IJ}D_\mu\boldsymbol{\lambda} + \frac{1}{4}\bar{\boldsymbol{\lambda}}\Gamma^{IJ}\boldsymbol{\lambda}^{KL}\,\bar{\boldsymbol{\lambda}}\Gamma^{KL}\boldsymbol{\lambda}^{IJ}\Big)$$

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- The fermionic terms confirm that the corrections obey the Lorentzian 3-algebra structure.
- Moreover they make it possible to test supersymmetry to this order.
- It is likely that the supersymmetry transformations are simply obtained by non-Abelian duality from those for the α'-corrected SYM theory.
- We view the above results as evidence that the 3-algebra structure is fundamental to the theory, and conjecture that the correction we have found is universal (to all 3-algebras).

Outline

Summary and Conclusions

- We described a non-Abelian duality that converts the $D2$-brane action into a superconformal, $SO(8)$ invariant one. The result is a Lorentzian 3-algebra.

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- We have extended this duality to incorporate the leading α' corrections. There is no obstacle of principle to including other α' corrections.
- Our result provides evidence that the 3-algebra structure appears not only in leading order, but also in the higher-derivative terms.

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