Nonlinear Fluid Dynamics from Gravity-1

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Outline

- Introduction
- Spacetimes dual to boundary fluid flows
- Global structure and entropy current
- A generalization
- Discussion

References

Talk based on

- arXiv: 0712.2456, S. Bhattacharyya, V. Hubeny, S.M. , M. Rangamani
- 0803.2526, above + R.Loganayagam, G. Mandal, T. Morita and H. Reall
- 0806.0006, S. Bhattacharyya, R. Loganayagam, S.M. , S. Nampuri, S. Trivedi and S. Wadia

Immediate precursors: important work by Son, Starinets, Kovtun, Policastro, Janik and collaborators. Also 0708.1770 (S. Bhattacharyya, S. Lahiri, R. Loganayagam, S.M.)
Some follow ups and other subsequent work will be reviewed by R. Loganayagam in the next talk.

Trace dynamics at Large N

- Consider any large N gauge theory. Let $\rho_m(x) = \frac{Tr\ O_m(x)}{N}$ denote set of all single trace gauge invariant operators of the theory.
- According to general lore, in the large N limit the gauge theory path integral may be rewritten as

$$\int \prod_m \mathcal{D}\rho_m(x) \exp\left[-N^2 S(\rho_m)\right]$$

Consequently large N gauge theories are effectively classical when rewritten in terms of trace variables.

Trace Dynamics from Supergravity

- Maldacena 1997: The classical large N evolution equations for $\mathcal{N} = 4$ Yang Mills are IIB SUGRA on $AdS_5 \times S^5$. $\rho_m(x, t)$ to be read off from the boundary values of bulk fields.
- Evolution equations of 10d bulk fields elegant and local. Map to unfamiliar, nonlocal and complicated looking evolution equations for $\rho_m(x, t)$.
- Would be nice to better understand the implied four dimensional dynamics for ρ_n. This talk: study ρ_n dynamics in a universal sector in a long distance limit. Will show that the bulk equations imply local and familiar boundary dynamics of $\rho_m(x)$ in this limit. Familiar dynamics= fluid dynamics

Thermodynamics, velocity and temperature

- Consider a d dimensional large N conformal field theory. The thermodynamic energy density of this theory is given by $\rho = \alpha(d-1)N^2T^d$ for some constant α.
- Let $T^{\mu\nu}(x)$ denote the expectation value of the stress tensor in any quantum state of this theory.
- Let $u^\mu(x)$ denote the unique time or light like eigenvector field of the stress tensor i.e.

$$T_{\mu\nu}(x)u^\nu(x) = \alpha N^2 T(x)^d u^\mu(x)$$

- We will refer to $u^\mu(x)$ as the velocity field and $T(x)$ as the temperature field associated with the state. Coincides with thermodynamic notions in equilibrium.

Local Equilibriation

- Fluctuations about finite temperature states of any CFT are characterized by a length scale, $l_{mfp} \sim \frac{\eta}{\rho}$. This is the length and time scale associated with equilibriation. In the theories we study $l_{mfp} \sim 1/T$.
- Key physical assumption: all Fourier components of the stress tensor with $kl_{mfp} > 1$ decay away exponentially over time scales of order l_{mfp}. After this time the system is approximately locally equilibriated. Its dynamical variables are simply $u^\mu(x)$ and $T(x)$. These fields subsequently vary on length and time scales long compared to l_{mfp}

Fluid Dynamics

- After local equilibrium is attained, the system begins to relax towards global equilibrium. Described by fluid dynamics. Variables $u^\mu(x)$ and $T(x)$. Universal dynamical equations: $\partial_\mu T^{\mu\nu} = 0$. Need a constituitive relationship to express $T^{\mu\nu}(x)$ in terms of $u^\mu(x)$ and $T(x)$
- As fluid dynamics only works when length and time scales of variation are long compared to l_{mfp}. Consequently it only makes sense to specify the constituitive relations in a expansion in derivatives. Form of this expansion greatly constrained by symmetries: e.g. leading order

$$T^{\mu\nu} = \alpha N^2 T^d \left(d u^\mu u^\nu + \eta^{\mu\nu}\right)$$

Higher order constituitive relations

- At first order in the derivative expansion, the only additional term allowed by symmetries is a piece proportional to the shear tensor $\sigma_{\mu\nu}$.
- At next order there are four linearly independent (onshell inequivalent) possible additions to the stress tensor in flat space.
- Consequently, the constituitive relations of an arbitrary conformal fluid are completely specified by one and four dimensionless numbers at first and second order respectively. In this talk: see how gravity reduces to fluid dynamics. Results will prove ‘universal’ in a sense I now explain.

Universal dynamics of the stress tensor

- Consider any 2 derivative theory of gravity interacting with other fields, that admits AdS_{d+1} space as a solution.
- Every such theory admits a consistent truncation to Einstein gravity with a negative cosmological constant. All fields other than the Einstein frame graviton are simply set to their background AdS_{d+1} values under this truncation.
- Dual implication: Simple universal dual dynamics for the stress tensor of all the (infinitely many) large N field theories with a 2 derivative bulk dual. Most of the rest of this talk: study this simple universal sector subsector at long wavelengths.

Einstein's Equations reduce to fluid dynamics

- In this talk we conjecture and largely demonstrate that the set of all regular long wavelength solutions to Einstein's equations with a negative cosmological constant in $d+1$ dimensions is identical to the set of solutions of the boundary Navier Stokes equations (with holographically determined values of transport coefficients) in d dimensions.
- Thus Einstein Equations (1915) $\rightarrow$ Navier Stokes equations (1822), adding to the list of connections uncovered by string theory between classic but apparently unrelated equations of physics.

Boosted Black Branes

$$R_{MN} - \frac{R}{2} g_{MN} = \frac{d(d-1)}{2} g_{MN} : \quad : M, N = 1 \ldots d+1$$

Simplest soln : AdS_{d+1} space

$$ds^2 = \frac{dr^2}{r^2} + r^2 g_{\mu\nu} dx^\mu dx^\nu ; \quad : \mu, \nu = 1 \ldots d$$

($g_{\mu\nu}$ = constant boundary metric). Another solution: black brane at temperature T and velocity u_μ

$$ds^2 = \frac{dr^2}{r^2 f(r)} + r^2 \mathcal{P}_{\mu\nu} dx^\mu dx^\nu - r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu$$

$$f(r) = 1 - \left(\frac{4\pi T}{d\ r}\right)^d ; \quad \mathcal{P}_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$u_\mu(x)$ and $T(x)$

- The boundary stress tensor for the boosted black brane is

$$T_{\mu\nu} = K T^d \left(g_{\mu\nu} + d u_\mu u_\nu\right); \quad K = \frac{1}{16\pi G_{d+1}} \left(\frac{4\pi}{d}\right)^d$$

- Note that

$$T_{\mu\nu}(x) u^\nu(x) = K' T(x)^d u^\mu(x), \quad K' = (1-d)K$$

(u^μ is the unique timelike eigenvector).

- As explained above we use this equation to define the velocity and temperature field of any locally asymptotically *AdS* solution of Einsteins equations. Simple physical interpretation.

Our Question

- Consider an arbitrary evolution $T_{\mu\nu}(x)$ on a boundary with metric $g_{\mu\nu}(x)$. Let $\Delta(x)$ denote the minimum length scale of variation of $T_{\mu\nu}(x)$ and $g_{\mu\nu}(x)$. Let $\epsilon(x) = \frac{1}{T(x)\Delta(x)}$.
- If $\epsilon(x) \ll 1$ then $T_{\mu\nu}(x)$, $g_{\mu\nu}(x)$ 'slowly varying' (vary on length scales large comp to the equilibriation length, $\frac{1}{T}$).
- Question: Given arbitrary slowly varying boundary stress tensor $T_{\mu\nu}(x)$. What are its boundary 'equations of motion', i.e. under what conditions can $T_{\mu\nu}(x)$ be obtained from a regular solutions to Einstein's equations? What is the bulk metric dual to any $T_{\mu\nu}(x)$ that satisfies these conditions?
- Address this question: perturbatively construct families of (we conjecture all) 'slowly varying' bulk spacetimes.

The tubewise approximation

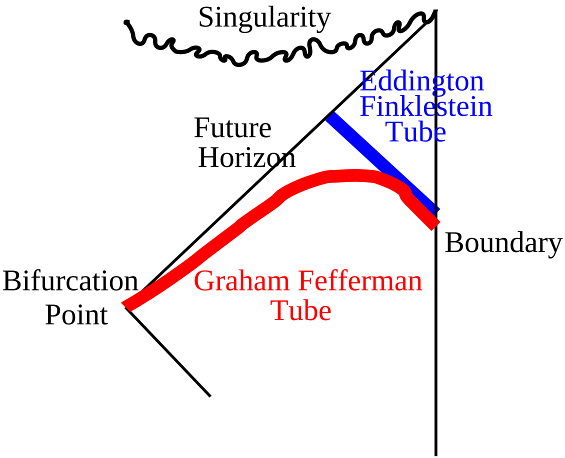
- We expect slowly varying boundary configurations to be locally thermalized. Suggests bulk solution tubewise approximated by black branes. But along which tubes?
- Naive guess: lines of constant x^μ in Schwarschild (Graham Fefferman) coordinates, i.e. metric approximately

$$ds^2 = \frac{dr^2}{r^2 f(r)} + r^2 \mathcal{P}_{\mu\nu}(x) dx^\mu(x) dx^\nu(x) - r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu$$

$$f(r) = 1 - \left(\frac{4\pi T(x)}{d\ r}\right)^d ; \quad \mathcal{P}_{\mu\nu} = g_{\mu\nu}(x) + u_\mu(x) u_\nu(x)$$

- Wrong. Metrics not regular. Bad starting point for perturbation theory. Also intuitively problem with causality.

Penrose diagram



Zero order metric

- Causality suggests the use of tubes centered around ingoing null geodesics. In particular we try

$$
\begin{aligned}
ds^2 = g^{(0)}_{MN} dx^M dx^N &= -2u_\mu(x)dx^\mu dr + r^2 \mathcal{P}_{\mu\nu}(x)dx^\mu dx^\nu \\
&- r^2 f(r, T(x))u_\mu(x)u_\nu(x)dx^\mu dx^\nu
\end{aligned}
$$

- Metric generally regular but not solution to Einstein's equations. However solves equations for constant $u^\mu, T, g_{\mu\nu}$. Consequntly appropriate starting point for a perturbative soln of equations in the parameter $\epsilon(x)$.

Perturbation Theory: Redn to ODEs

- That is we set

$$g_{MN} = g^{(0)}_{MN}(\epsilon x) + \epsilon g^{(1)}_{MN}(\epsilon x) + \epsilon^2 g^{(2)}_{MN}(\epsilon x) \ldots$$

  and attempt to solve for $g^{(n)}_{MN}$ order by order in ϵ.
- Perturbation expansion surprisingly simple to implement. Nonlinear partial differential equation $\rightarrow$ 15 ordinary differential equations, in the variable r at each order and each boundary point.

Perturbation Theory: Constraint Equations

- Gauge choice: $g_{r\mu}(x) = -u_\mu(x), \quad g_{rr} = 0$. Ten undetermined metric components $g^{(n)}_{\mu\nu}$ at each order. Naively 15 but actually 14 independent Einstein equations. Split up into 4 constraint equations and 10 dynamical equations.
- The constraint equations at n^{th} order are independent of $g^{(n)}_{\mu\nu}$: they are $\nabla^\mu T^{(n-1)}_{\mu\nu} = 0$, where $T^{(n-1)}_{\mu\nu} = 0$ is the boundary stress tensor dual to the solution upto $(n-1)^{th}$ order.

Perturbation Theory: Dynamical Equations

- The dynamical equations take the form $M\ g^{(n)} = s^{(n)}$. Here M is a 'homogeneous' differential operator in r that is the same at every order. $s^{(n)}$ is a source function that is independent of $g^{(n)}$ and is determined by the solution to $(n-1)^{th}$ order.
- It turns out to be possible to exactly solve the equation $M\ g^{(n)} = s^{(n)}$ for an arbitrary source function $s^{(n)}$. For any given source function s^n there is a family of solutions to the equation (which differ by solutions of the homogeneous equation $M\ g = 0$)

Perturbation Theory: Uniqueness of Solns

However provided that the source function is regular at the 'horizon' and dies off sufficiently fast at infinity (conditions that are true for $s^{(n)}$ generated in perturbation theory), the solution to this equation is unique subject to the following requirements:

1. That the solution is dual to the specified boundary metric $g_{\mu\nu}(x)$, velocity field $u_\mu(x)$ and the temperature $T(x)$. (condition on the large r behaviour of the solution).

2. That the solution is regular at the zeroth order horizon (condition at $r = \frac{4\pi T}{d}$)

Perturbation Theory: Navier Stokes Equations

- We may now construct the stress tensor $T_{\mu\nu}^{(n)}$ dual to our perturbative solution. $T_{\mu\nu}^{(n)}$ is uniquely determined as a function of n^{th} order in derivatives of $g_{\mu\nu}$, u_μ and T.
- Recall that the constraint equations are $\nabla^\mu T_{\mu\nu} = 0$. But this equation, together with the specification of $T_{\mu\nu}$ as a function of derivatives of $g_{\mu\nu}$, u_μ and T has a name: the equations of fluid dynamics.

Perturbation Theory: Summary

- Summary: Explicit map from the space of solutions of a distinguished set of fluid dynamical equations in d dimensions to long wavelenth solutions of Einstein's equations.
- Requirement of regularity of the horizon ensures this map is locally one to one in solution space.
- Naive Graham Fefferman counting: $\frac{d(d+1)}{2} - 1$ parameter solution. Rougly parameterized by fluctuation fields $g^n_{\mu\nu}$. However $\frac{d(d-1)}{2} - 1$ of these modes - the tensor sector - fixed by the requirement of regularity.
- Remaining solutions parameterized by d velocities and temperatures. Closed dynamical system.

Explicit Results at second order

We have explicitly implemented our perturbation theory to second order.

$$
\begin{aligned}
ds^2 &= -2u_\mu dx^\mu \left(dr + r\, A_\nu dx^\nu\right) + r^2 g_{\mu\nu} dx^\mu dx^\nu \\
&- \left[\omega_\mu{}^\lambda \omega_{\lambda\nu} + \frac{1}{d-2}\mathcal{D}_\lambda \omega^\lambda{}_{(\mu} u_{\nu)} - \frac{1}{d-2}\mathcal{D}_\lambda \sigma^\lambda{}_{(\mu} u_{\nu)} \right. \\
&\left. + \frac{\mathcal{R}}{(d-1)(d-2)} u_\mu u_\nu \right] dx^\mu dx^\nu \\
&+ \frac{1}{(br)^d}\left(r^2 - \frac{1}{2}\omega_{\alpha\beta}\omega^{\alpha\beta}\right) u_\mu u_\nu dx^\mu dx^\nu \\
&+ 2(br)^2 F(br) \left[\frac{1}{b}\sigma_{\mu\nu} + F(br)\sigma_\mu{}^\lambda \sigma_{\lambda\nu}\right] dx^\mu dx^\nu \ldots
\end{aligned}
$$

Explicit Results at second order

$$
\begin{aligned}
&-2(br)^2\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu}K_1(br) - \frac{u_\mu u_\nu}{(br)^{d-2}}\frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{(d-1)}K_2(br) \\
&+\frac{2\,L(br)}{(br)^{d-2}}\left[P_\mu^\lambda \mathcal{D}_\alpha {\sigma^\alpha}_\lambda u_\nu + P_\nu^\lambda \mathcal{D}_\alpha {\sigma_\lambda}^\alpha u_\mu\right] dx^\mu dx^\nu \\
&-2(br)^2 H_1(br)\left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + {\sigma_\mu}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1}P_{\mu\nu}\right. \\
&\qquad \left. + C_{\mu\alpha\nu\beta}u^\alpha u^\beta\right] dx^\mu dx^\nu \\
&+2(br)^2 H_2(br)\left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + {\omega_\mu}^\lambda \sigma_{\lambda\nu} - {\sigma_\mu}^\lambda \omega_{\lambda\nu}\right] dx^\mu dx^\nu
\end{aligned}
$$

Explicit results at second order

Where

$$F(br) \equiv \int_{br}^{\infty} \frac{y^{d-1}-1}{y(y^d-1)} dy \ ; \ L(br) \equiv \int_{br}^{\infty} \xi^{d-1} d\xi \int_{\xi}^{\infty} dy \ \frac{y-1}{y^3(y^d-1)}$$

$$H_2(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi(\xi^d-1)} \int_{1}^{\xi} y^{d-3} dy \left[1+(d-1)yF(y)+2y^2F'(y)\right]$$

$$K_1(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \int_{\xi}^{\infty} dy \ y^2 F'(y)^2 \ ; \ H_1(br) \equiv \int_{br}^{\infty} \frac{y^{d-2}-1}{y(y^d-1)} dy$$

$$K_2(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \Big[1-\xi(\xi-1)F'(\xi)-2(d-1)\xi^{d-1} + \Big(2(d-1)\xi^d-(d-2)\Big) \int_{\xi}^{\infty} dy \ y^2 F'(y)^2\Big]$$

Second order boundary stress tensor

The dual stress tensor corresponding to this metric is given by $(4\pi T = b^{-1}d)$

$$
\begin{aligned}
T_{\mu\nu} &= p\left(g_{\mu\nu} + d u_\mu u_\nu\right) \\
&\quad - 2\eta\left[\sigma_{\mu\nu} - \tau_\pi u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} - \tau_\omega\left(\sigma_\mu{}^\lambda \omega_{\lambda\nu} - \omega_\mu{}^\lambda \sigma_{\lambda\nu}\right)\right] \\
&\quad + \xi_\sigma\left[\sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta}\sigma^{\alpha\beta}}{d-1} P_{\mu\nu}\right] + \xi_C C_{\mu\alpha\nu\beta} u^\alpha u^\beta
\end{aligned}
$$

$$
p = \frac{1}{16\pi G_{d+1} b^d} \quad ; \quad \eta = \frac{s}{4\pi} = \frac{1}{16\pi G_{d+1} b^{d-1}}
$$

$$
\tau_\pi = (1 - H_1(1))b \quad ; \quad \tau_\omega = H_1(1)b \quad ; \quad \xi_\sigma = \xi_C = 2\eta b
$$

Properties of soln: Stress tensor

- Schematic form of 2nd order stress tensor:

$$T_{\mu\nu} = aT^d(g_{\mu\nu} + du_\mu u_\nu) + bT^{d-1}\sigma_{\mu\nu} + T^{d-2}\sum_{i=1}^{5} c_i S^i_{\mu\nu}$$

- a is a thermodynamic parameter. b is related to the viscosity: we find $\eta/s = 1/(4\pi)$. c_i coefficients of the five traceless symmetric Weyl covariant two derivative tensors are second order transport coefficients. Values disagree with the predictions of the Israel Stewart formalism.
- Recall that results universal. Should yield correct order of magnitude estimate of transport coefficients in any strongly coupled CFT.

Properties of soln: Weyl covariance

- Weyl covariance: result is written in terms of covariant derivative built out of the 'gauge' field

$$\mathcal{A}_\nu \equiv u^\lambda \nabla_\lambda u_\nu - \frac{\nabla_\lambda u^\lambda}{d-1} u_\nu$$

R. Loganayagam . arXiv:0801.3701 [hep-th]

Can use the fact that $\mathcal{A}_\nu$ transforms like a gauge field under Weyl transformation to define a Weyl covariant derivative $\mathcal{D}$ that acts on a weight w tensor $Q^{\mu\ldots}_{\nu\ldots}$ as

$$\begin{aligned}\mathcal{D}_\lambda\, Q^{\mu\ldots}_{\nu\ldots} \equiv \nabla_\lambda\, Q^{\mu\ldots}_{\nu\ldots} &+ w\ \mathcal{A}_\lambda Q^{\mu\ldots}_{\nu\ldots} \\ &+ \left[g_{\lambda\alpha}\mathcal{A}^\mu - \delta^\mu_\lambda \mathcal{A}_\alpha - \delta^\mu_\alpha \mathcal{A}_\lambda\right] Q^{\alpha\ldots}_{\nu\ldots} + \ldots \\ &- \left[g_{\lambda\nu}\mathcal{A}^\alpha - \delta^\alpha_\lambda \mathcal{A}_\nu - \delta^\alpha_\nu \mathcal{A}_\lambda\right] Q^{\mu\ldots}_{\alpha\ldots} - \ldots\end{aligned}$$

Event Horizons

- Our solutions are singular at $r = 0$. Quite remarkably it is possible to demonstrate that under certain conditions these solutions have event horizons. The event horizon manifold $r = r(x)$, may explicitly be determined order by order in the derivative expansion. This horizon shields the $r = 0$ singularity from the boundary.
- Need some knowledge of the long time behaviour of the solution. Sufficient, though far from necessary, to assume fluid flows that reduce to constant temperature and velocity at late times. Not very strong assumption. Probably true of all finite fluctuations about uniform motion in $d \geq 2$.

Event Horizon in the derivative expansion

- The event horizon of the dual bulk geometry is the unique null manifold that reduces to the event horizon $r = \frac{4\pi T}{d} = \frac{1}{b}$ of the dual uniform black brane at late times.
- It turns out to be simple to construct this event horizon manifold in the derivative expansion: explicitly

$$r_H = \frac{1}{b} + b\left(\lambda_1 \sigma_{\alpha\beta}\sigma^{\alpha\beta} + \lambda_2 \omega_{\alpha\beta}\omega^{\alpha\beta} + \lambda_3 \mathcal{R}\right) + \ldots$$

$$\lambda_1 = \frac{2(d^2+d-4)}{d^2(d-1)(d-2)} - \frac{K_2(1)}{d(d-1)}$$

$$\lambda_2 = -\frac{d+2}{2d(d-2)} \qquad \text{and} \qquad \lambda_3 = -\frac{1}{d(d-1)(d-2)}$$

- We can put our control over the event horizon to practical use. Recall that a d dimensional event horizon is generated by a $d-1$ dimensional family of null geodesics. Let α^i $i = 1 \ldots (d-1)$ label these geodesics. Let λ be any future directed coordiante along the geodesics.
- The line element on the event horizon takes the form

$$ds^2 = g_{ij}^{eh} d\alpha^i d\alpha^j$$

Define the area $d-1$ form as $a = \sqrt{g^{eh}} d\alpha^1 \wedge d\alpha^2 \ldots d\alpha^{d-1}$.
- Now $da = \phi d\lambda \wedge d\alpha^1 \ldots d\alpha^{d-1}$ The classic area increase theorem of black hole physics implies the assertion that $\phi \geq 0$.

- Consider the pullback of a to the boundary using the map generated by the radial ingoing null geodesics described above. The boundary hodge dual of pullback of this $d-1$ form is a current whose divergence may be shown to be non negative.
- Consequently fluid dynamics dual to gravity is equipped with a local current whose divergence is always non negative, and which agrees with the thermodynamic entropy current in equilibrium. This ‘entropy current’ is a local ‘Boltzman H’ function whose non negative divergence rigorously estabilishes the locally irreversable nature of the dual fluid flows.

Entropy Current at second order

Explicitly this entropy current is given to second order by

$$4\,G_{d+1}\,b^{d-1}\,J_S^{\mu} = \left[1 + b^2\left(A_1\,\sigma^{\alpha\beta}\,\sigma_{\alpha\beta} + A_2\,\omega^{\alpha\beta}\,\omega_{\alpha\beta} + A_3\,\mathcal{R}\right)\right]u^{\mu} + b^2\left[B_1\,\mathcal{D}_{\lambda}\,\sigma^{\mu\lambda} + B_2\,\mathcal{D}_{\lambda}\,\omega^{\mu\lambda}\right]$$

where

$$A_1 = \frac{2}{d^2}(d+2) - \frac{K_1(1)d + K_2(1)}{d}\,, \qquad A_2 = -\frac{1}{2d}\,, \qquad B_2 = \frac{1}{d-2}$$

$$B_1 = -2A_3 = \frac{2}{d(d-2)}$$

Forcing and Charges

- One may attempt to generalize our construction to a bulk Lagrangian with additional fields. Gain: more solutions, wider dynamical behaviour. Price: reduced universality
- Additional fields of two sorts. Bulk gauge fields that correspond to conserved boundary charge. Plus all others.
- Gauge fields enlarge the set of fluid dynamical variables to include charge densities. Other fields yield new solns only when non normalizable part is turned on. Operator coupling leads to forcing function for fluid dynamics

Dilaton Forcing

- Example of 2nd kind: $d = 4$ Einstein Dilaton System.
- Long wavelength solution of the Einstein dilaton system with a given specified slowly varying boundary dilaton field may be obtained by perturbation theory analogeous to above. Have been explicitly constructed to second order.
- Solutions are in one to one correspondence with the forced Navier Stokes equations

$$\nabla_\mu T^{\mu\nu} = -\frac{(\pi T)^3}{16\pi G_5} \nabla^\nu \phi (u.\partial)\phi + \dots$$

Simple Solutions

- A simple class of solutions to these equations are given by the dilaton chosen as a slowly varying function of time. If the fluid is initially at rest, it stays at rest but slowly heats up according to

$$\frac{dT}{dt} = \frac{(\dot{\phi})^2}{12\pi}.$$

The dual bulk solution has a dilaton pulses falling into the black hole, and at leading order is the Vaidya solution. Note that varying - whether increasing or decreasing - the dilaton heats up the gauge theory. Consistent with entropy. Speculations about the continuation to weak coupling.

Kruskal Coordinates?

- Eddington Finklestein coordinates proved very useful for our analysis. One imporant reason: future horizon regular in EF coordinates. Second equally important feature: ∂_μ were killing directions of the black brane, in these coordinates. This was crucial for obtaining ODEs - i.e. for the locality of our solutions.
- More generally, our procedure will work once we identify any foliation of the black brane metric into tubes such that ∂_a are killing, for the labels a of the tubes. Note too many options. While have not thought this through very carefully, think we have roughly the unique solution

Kruskal Coordinates cont

- For an example of something that does not work, consider the black brane written in Kruskal coordinates. If we now try to work along lines of constant U or constant V we dont get ODEs, so we dont get tube wise locality
- If one wanted to carry out the analouge of our programme in Kruskal coordinates, one would have to solve 2 variable PDEs, and would find locality on sheets not tubes. This sounds very interesting - though technically difficult. Would love to be able to control such solutions and interpret them. But have had no success so far.

Cosmic Censorship $\leftrightarrow$ singularities in equilibriation

- We have studied gravity dual of a locally equilibriated theory. What is the dual of the process of local equilibriation? Clearly the collapse to form a horizon.
- Longstanding interesting question about this process: are naked singularities permitted? Generic? Important implications for observational quantum gravity. Appears to map to questions of singularities in the process of local equilibriation in large N theories. Hint of another interesting connection. Seems like one should study.

Finite N

- Interesting question in both principle and practice: how does this story generalize to finite N. In the bulk one is instructed to quantize gravity. While this is a formidable task, it is natural to ask whether it makes any sense to simply quantize the solutions dual to fluid dynamics.
- Technically, one could compute the Witten-Crnkovic symplectic form on these solutions. If it is finite, well defined and non degenerate, it would be natural to use it to quantize the phase space of fluid dynamics. Such a quantization would give a 'wave function' on the space of temperatures and velocities (perhaps likely actual story more complicated). Dual to finite Avagadro no fluctuations?

Statistical nature of black hole spacetimes

- Clearly the spacetimes we study describe the coarse grained average properties of some more fundamental degrees of freedom. The relation between these spacetimes and one set of fundamental dofs is dual the relationship between fluid dynamics and gluon positions
- What is the relationship between the quantization of the true dofs and the quantization of gravity? Perhaps the fact that eternal black holes are dual to spacetimes with two boundaries is important here.

Other issues

- Turbulence in gravity
- Generalization to confining theories. Boundary dofs.
- Time reversal invariance
- ...