# Brane Crystals, Chern-Simons theories, and M2-branes

Sangmin Lee
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11 Dec. 2008, Indian Strings Meeting, Pondicherry

# This talk is based on ...

- Brane Crystal model
  [SL, 0610204][SL, Sungjay Lee, Jaemo Park, 0702120]
  [Seok Kim, SL, Sungjay Lee, Jaemo Park, 0705.3540]

- Superconformal Chern-Simons ($\mathcal{N} = 4,5,6$)

  [Kazuo Hosomichi, Ki-Myeong Lee, SL, Sungjay Lee, 0805.3662/0806.4977]

- Instantons in ABJM
  [Kazuo Hosomichi, Ki-Myeong Lee, SL, Sungjay Lee, Jaemo Park, Piljin Yi, 0809.1771]

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Pre-BLG

Post-BLG

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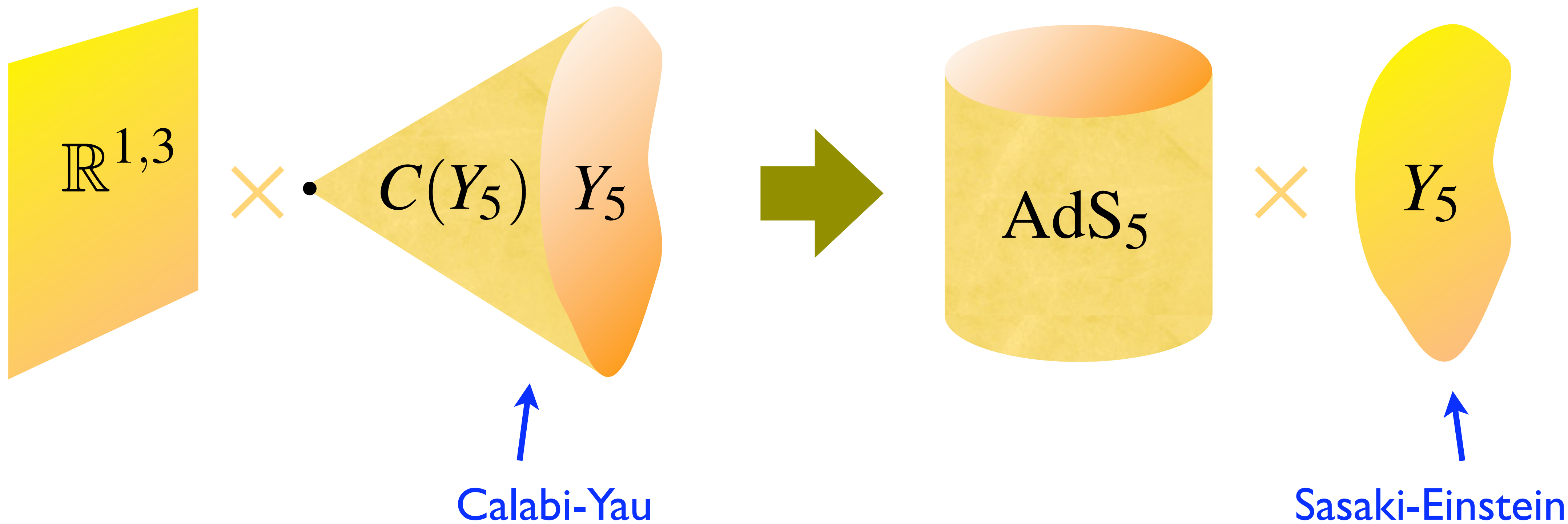
# Brane Crystals

# Brane Tiling model

Ramadevi's talk

## D3 on CY3

$D = 4$
$\mathcal{N} = 1$



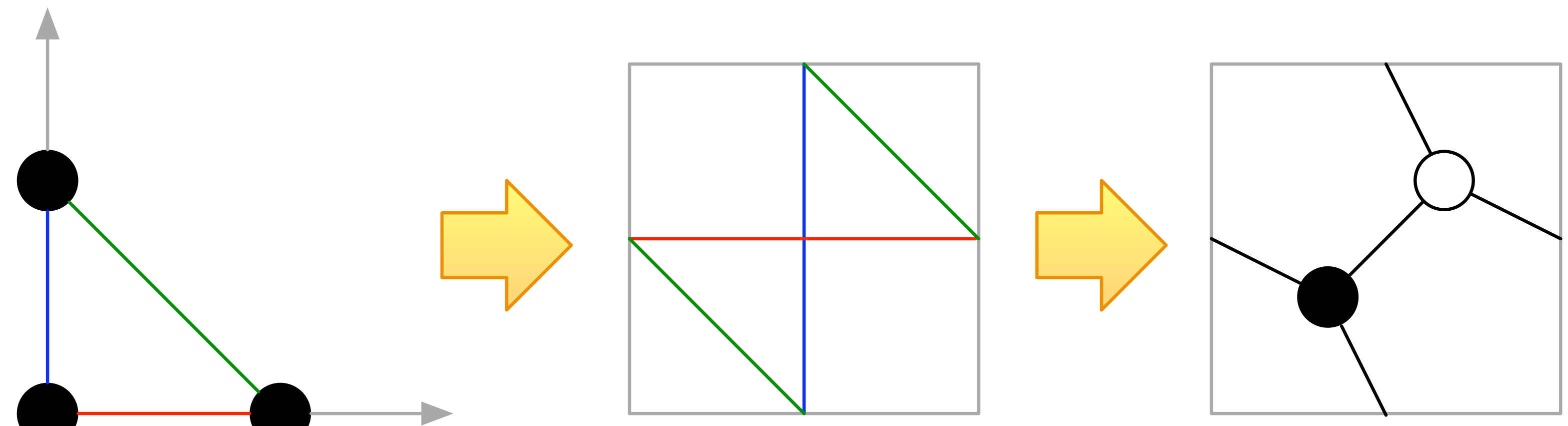

Flat $C^3$ > Orbifolds > Non-orbifolds [Klebanov-Witten] [Morrison-Plesser]

$C(T^{1,1})$ partial resolution

# Brane Tiling model

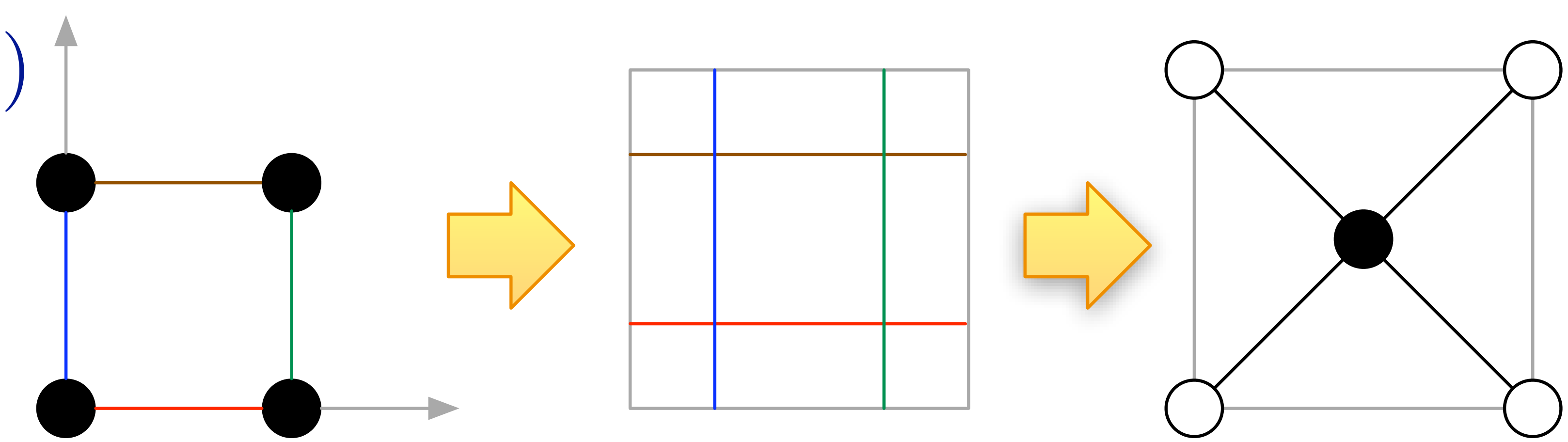
[Hanany-Benvenuti-Franco-Vegh-Wecht]
[Feng-He-Kennaway-Vafa] [Iqbal-Uranga]

$\mathbb{C}^3$



$$W = \mathrm{tr}(XYZ - ZYX)$$

$C(T^{1,1})$



$$W = \mathrm{tr}(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1)$$

Faces = Gauge groups
Edges = Bi-fundamentals
Vertices = Super-potential

# Brane Crystal model

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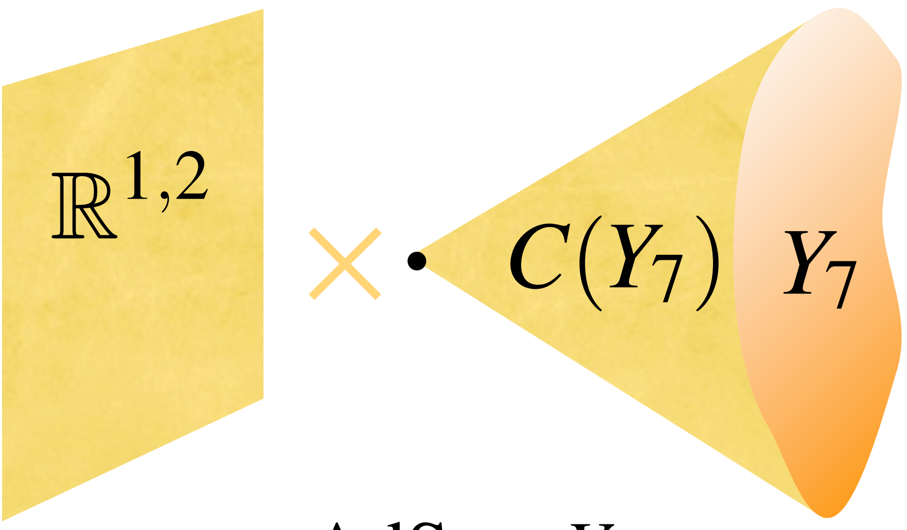
Apply the idea of brane tiling to ...

M2 on CY4

$D = 3, \mathcal{N} = 2$ SUSY




$\rightarrow \ \mathrm{AdS}_4 \times Y_7$

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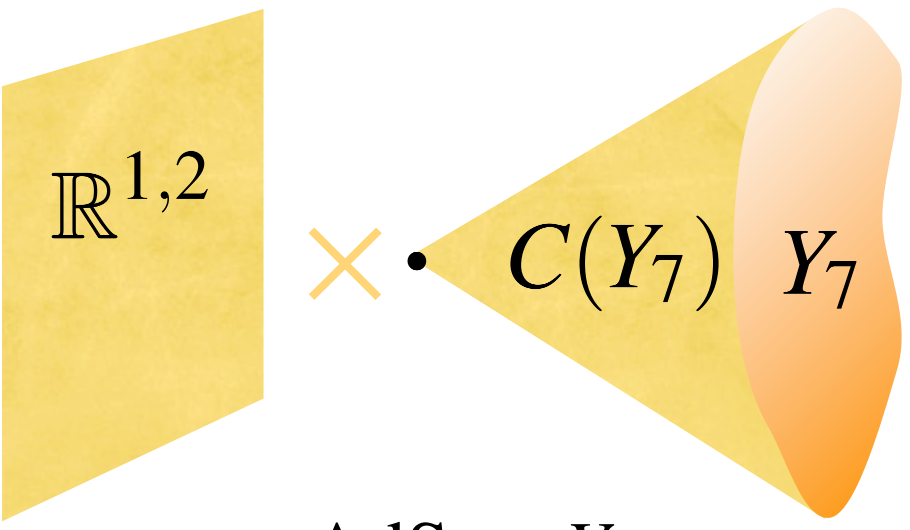


$\rightarrow \quad \mathrm{AdS}_4 \times Y_7$

[Doubt]
Even M2-branes in flat background not understood !

# Brane Tiling from T-duality

[Franco-Hanany-Kennaway-Vegh-Wecht]
[Imamura] [Feng-He-Kennaway-Vafa]

Toric $CY_3$ has $T^3$ fibration $\longrightarrow$ Take T-duality along $T^2$


*N* D3-branes | *N* D4-branes | *N* D5-branes

Shrinking $S^1$ fibers | NS5-brane

$$SL(2,\mathbb{Z}) \ : \ \tau = B_{12} + \sqrt{g_{T^2}} \rightarrow -1/\tau$$

# Brane Tiling from T-duality

Stack of *N* D3-branes

$\mathbb{R}^2$ | $T^2$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| D5 | ∘ | ∘ | ∘ | ∘ | | | ∘ | ∘ | | |
| NS5 | ∘ | ∘ | ∘ | ∘ | | $\Sigma$ | | | | |

degenerating circle fibers

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Stack of $N$ D3-branes

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<table>
<tr><th></th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th></tr>
<tr><td>D5</td><td>∘</td><td>∘</td><td>∘</td><td>∘</td><td></td><td></td><td>∘</td><td>∘</td><td></td><td></td></tr>
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degenerating circle fibers

Holomorphic; locally $\mathbb{R} \times S^1$

Thickened $(p,q)$-web

$$\sum_{(a,b)} c_{(a,b)} u^a v^b = 0$$

vertices of toric diagram

$$\left(u = e^{x^4 + ix^6}, v = e^{x^5 + ix^7} \in \mathbb{C}^*\right)$$

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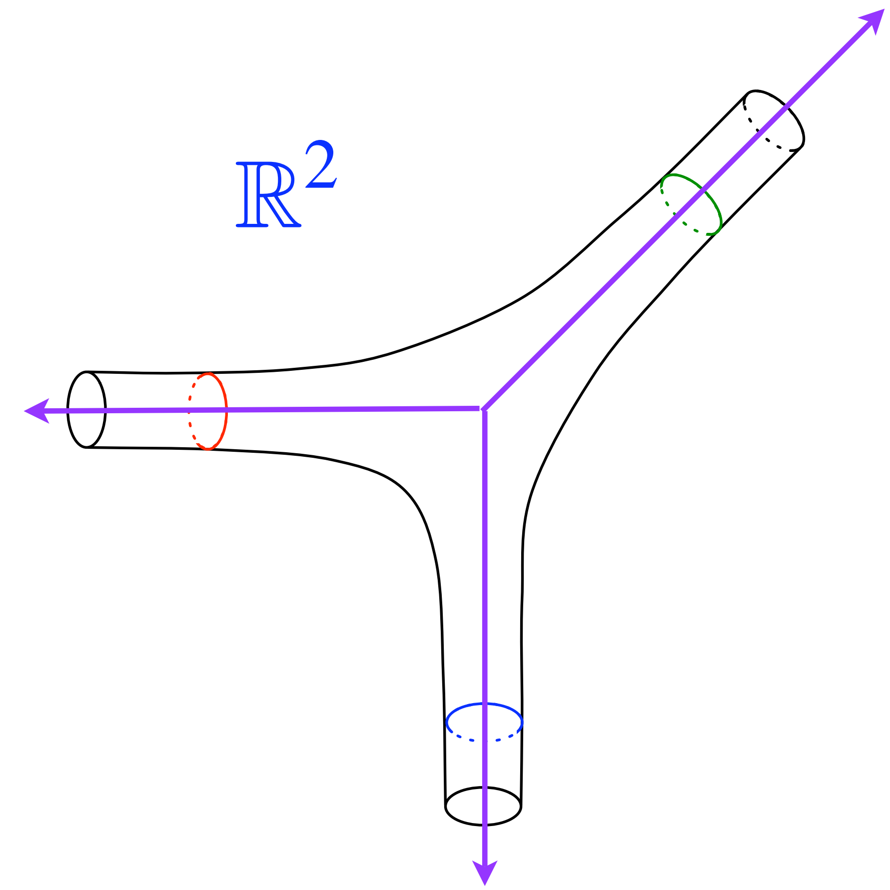
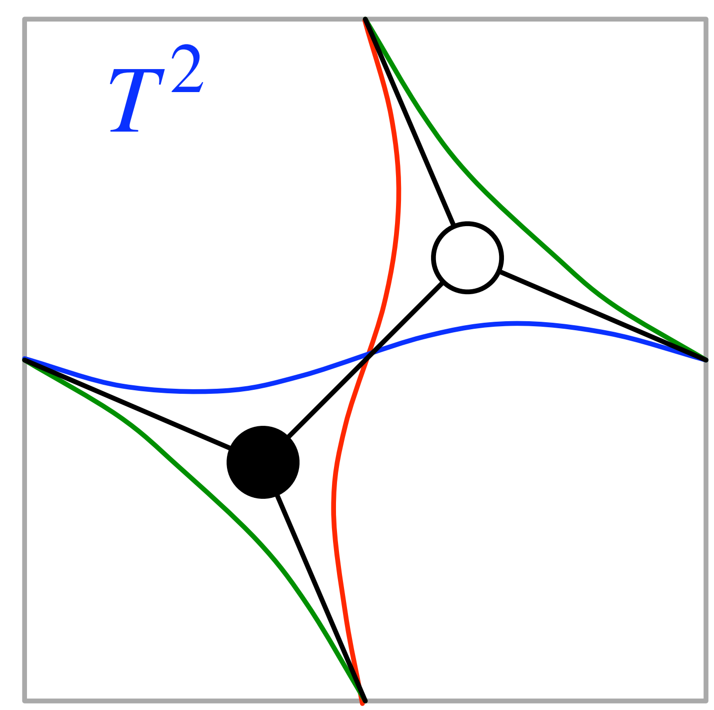
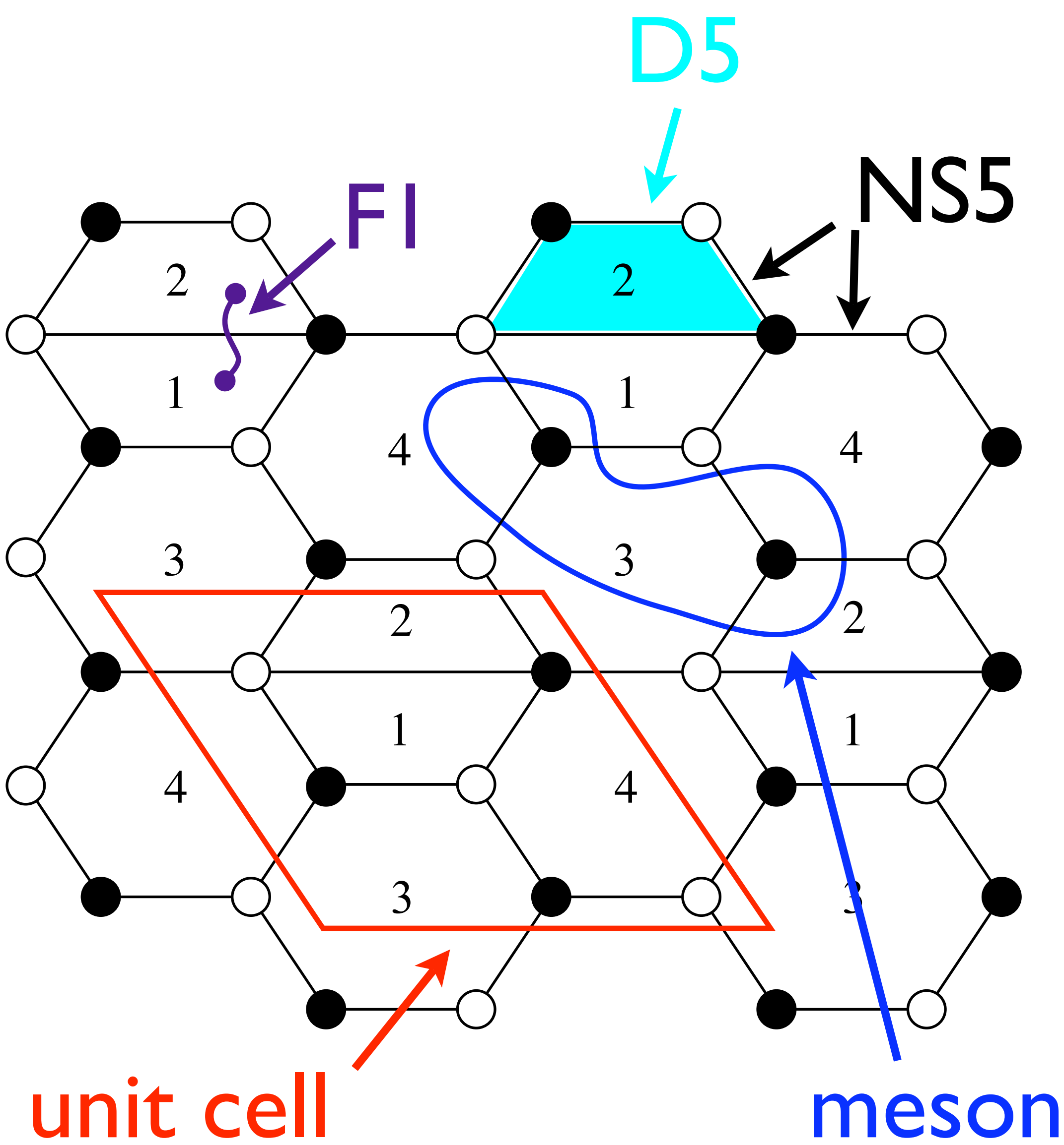


# Summary : Brane Tiling



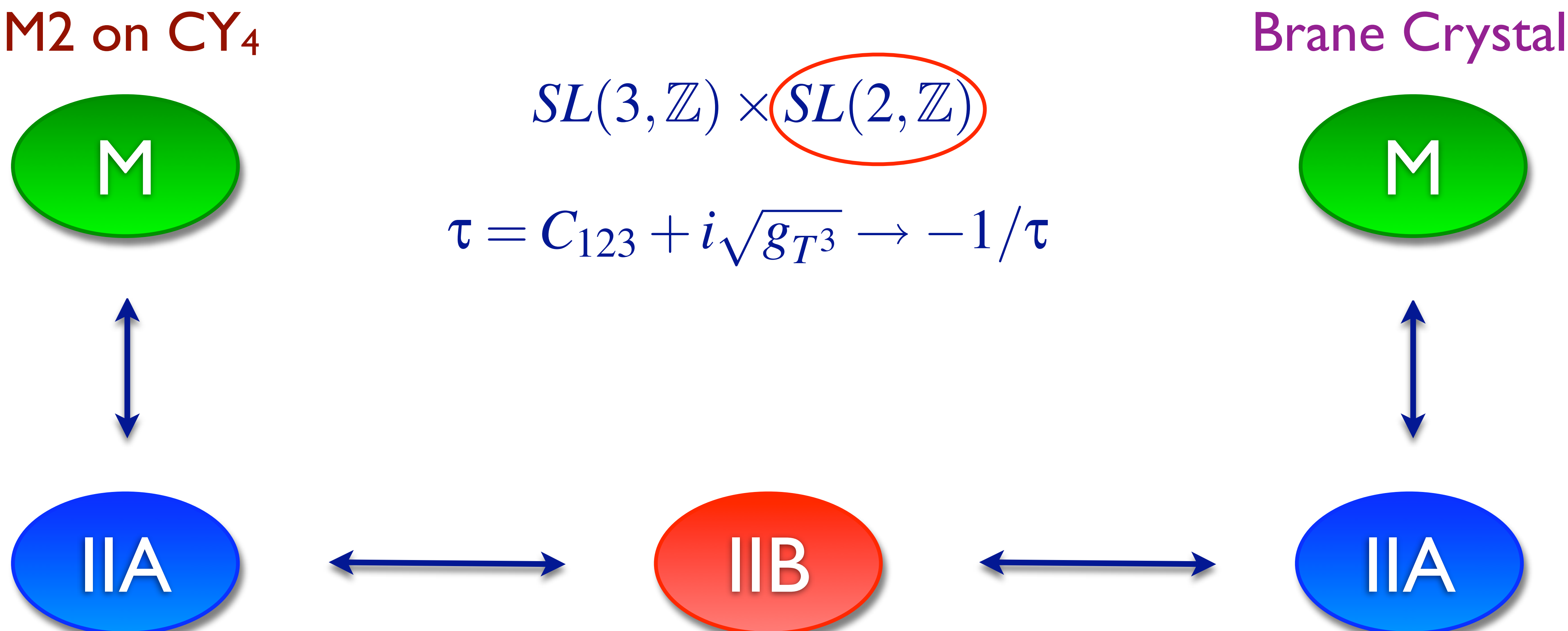

Faces = Gauge groups

Edges = Bi-fundamentals

Vertices = Super-potentials

# Brane Crystals from T-duality

Toric CY$_4$ has $T^4$ fibration $\longrightarrow$ Take T-duality along $T^3$




$$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$$

$$\tau = C_{123} + i\sqrt{g_{T^3}} \to -1/\tau$$

# T-duality and brane configuration

Stack of $N$ M2-branes

$\mathbb{R}^3$ | $T^3$

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
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degenerating circle fibers

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degenerating circle fibers

Special Lagrangian ($\mathcal{N}$=2 SUSY)

$$J = dx^1 \wedge dy^1 + dx^2 \wedge dy^2 + dx^3 \wedge dy^3$$

$$\Omega = (dx^1 + idy^1) \wedge (dx^2 + idy^2) \wedge (dx^3 + idy^3)$$

$$d(\text{vol}) = \text{Im}\Omega = -dy^1 dy^2 dy^3 + (dy^1 dx^2 dx^3 + \text{cyclic})$$

No explicit description like Newton polynomial!

Locally $\mathbb{R}^2 \times S^1$

Union of all 2-fans, thickened!

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| M5 | ○ | ○ | ○ | | | | ○ | ○ | ○ | | |
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degenerating circle fibers

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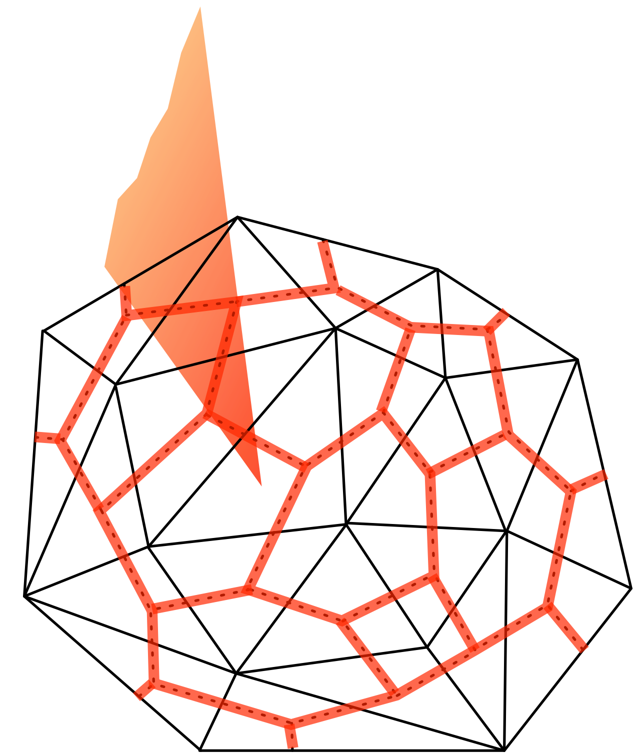

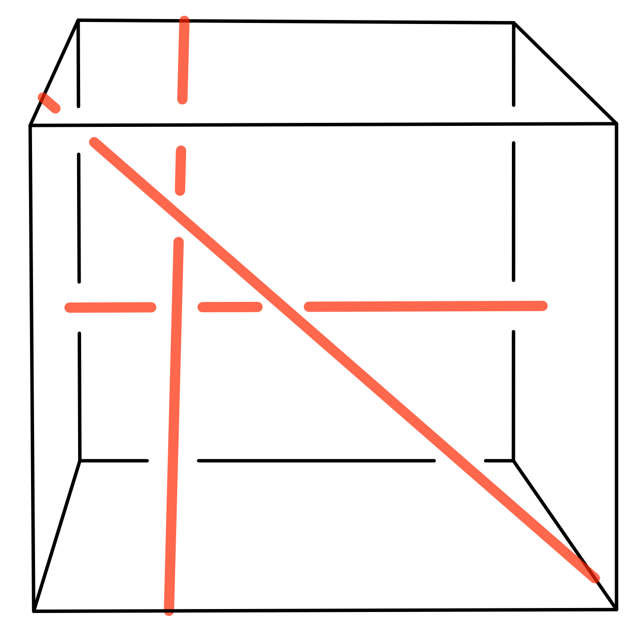

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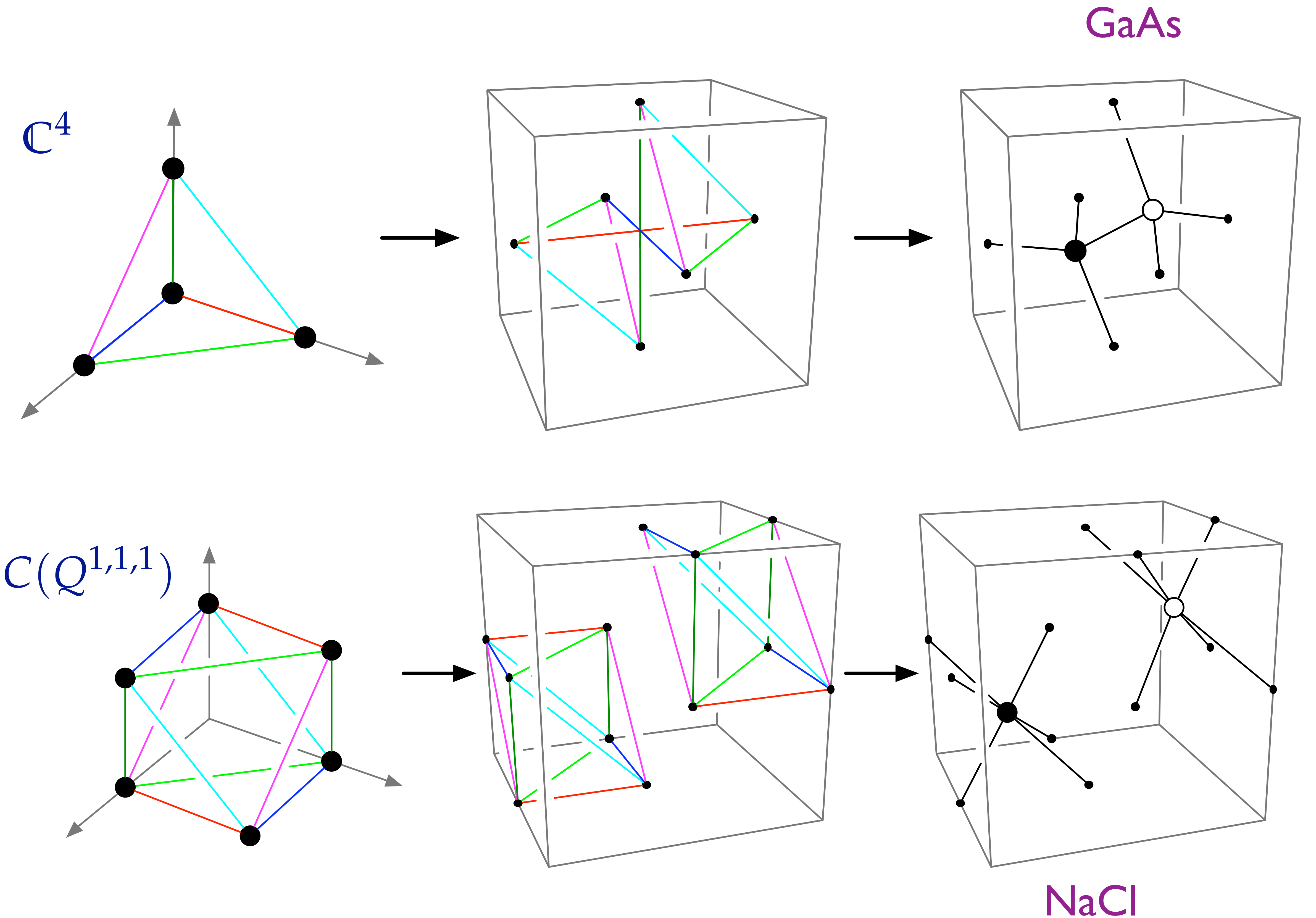
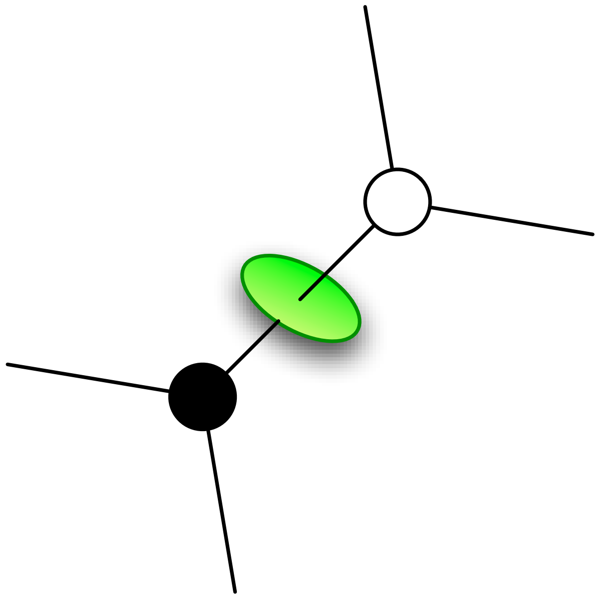
Union of all 2-fans, thickened!

# Brane Crsytals : examples



# Edges & Vertices

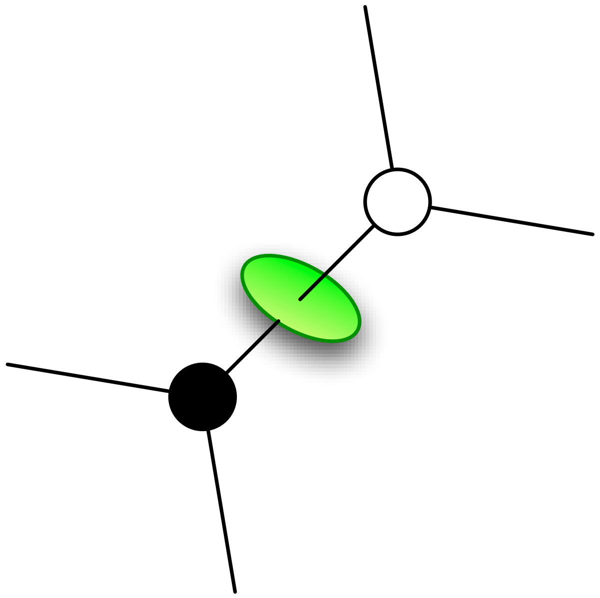


Edge : matter fields

"M2-disc" localized around M5 intersections

bipartite (two-colored) : due orientations

# Edges & Vertices



Edge : matter fields

"M2-disc" localized around M5 intersections

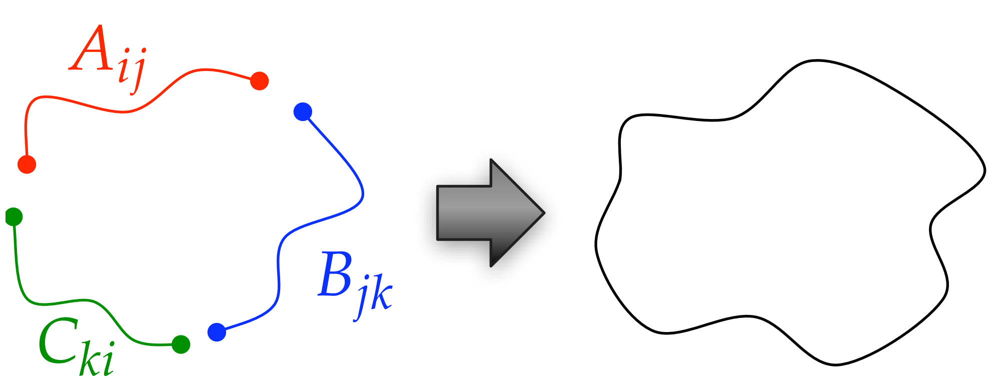
bipartite (two-colored) : due orientations

Vertex : super-potential

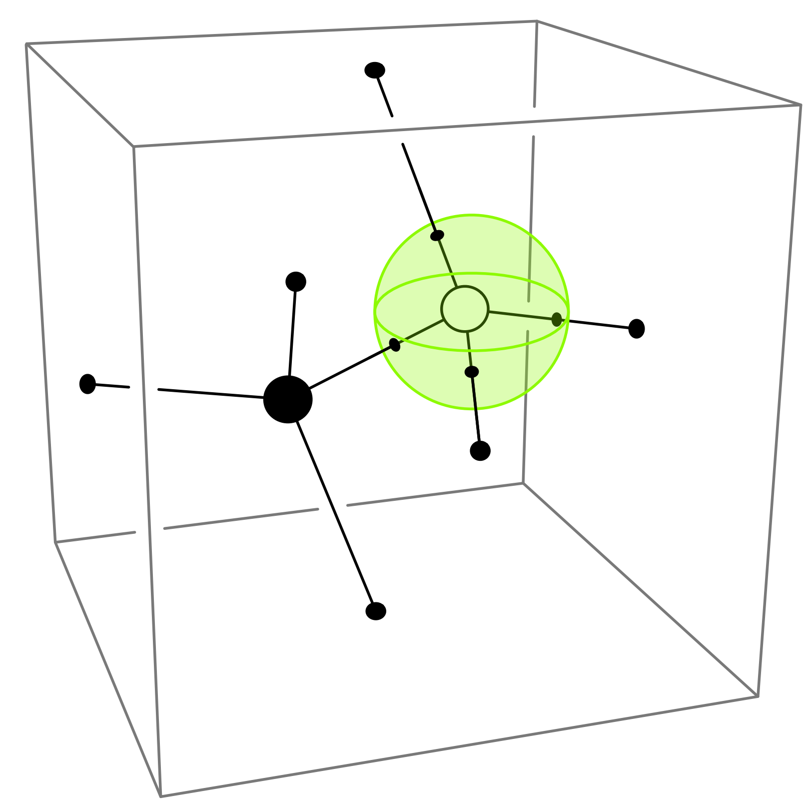
patching M2-disc to form a closed sphere

new gauge symmetry algebra?

(cf)




$= \mathrm{tr}(ABC)$



# Faces (abelian)

Face : gauge group (abelian, at least)

Open M2-brane is charged under
self-dual tensor field B on M5 world-volume

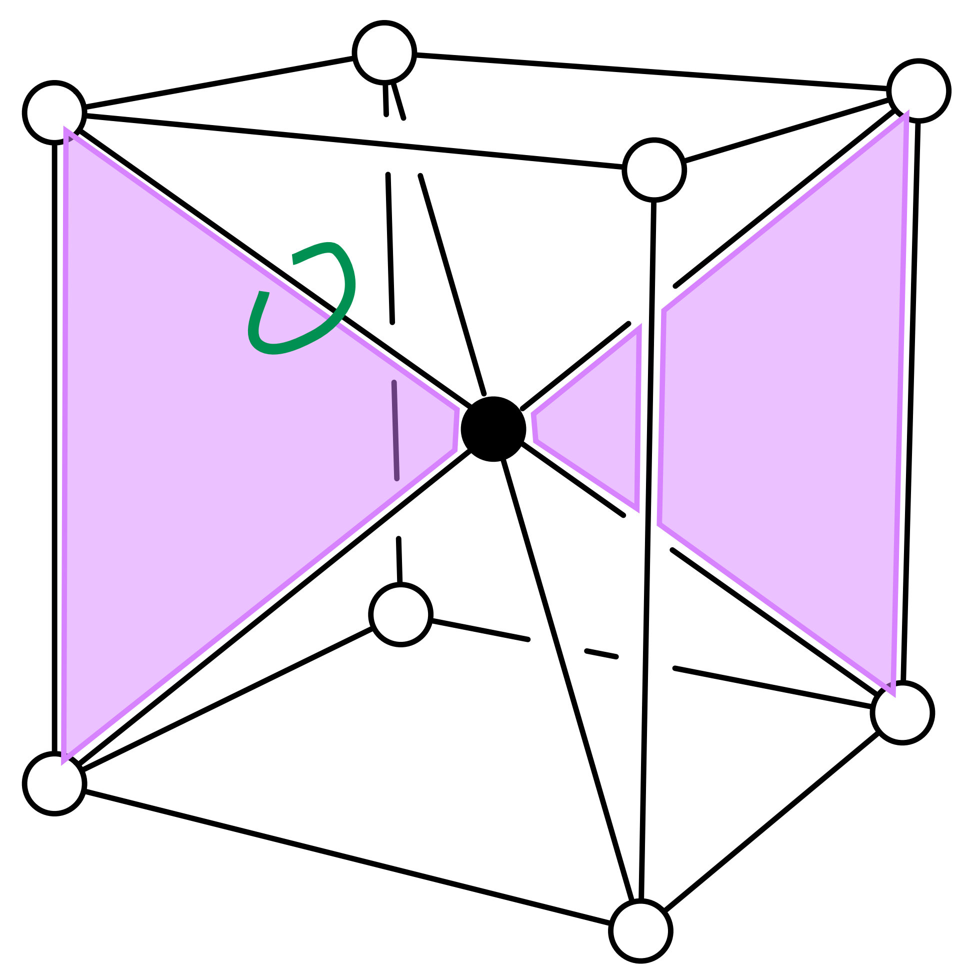
$$S_{M2} = T_{M2} \int_D d^3x \sqrt{-g} + \int_{\partial D} B$$

M5 compactified on $\mathbb{R}^{1,2} \times \mathcal{M}_3$ :

$$B = A^a(x) \wedge \omega_a(y) + \cdots$$

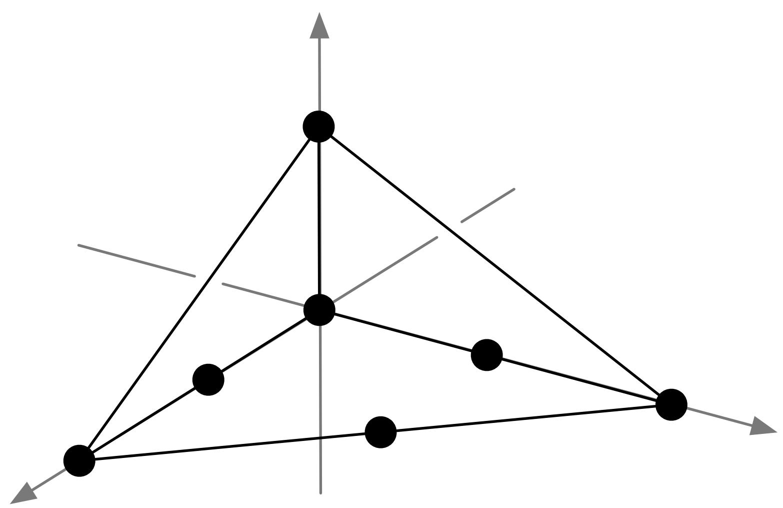
Charge matrix of M2-discs

$$Q_{ia} = \int_{C_i} \omega_a = \sharp(C_i, S_a)$$



# Moduli Space of Vacua

$\mathbb{C}/(\mathbb{Z}_2 \times \mathbb{Z}_2) \times \mathbb{C}$



| | $\phi_1$ | $\phi_2$ | $A_1$ | $A_2$ | $B_1$ | $B_2$ | $C_1$ | $C_2$ |
|---|---|---|---|---|---|---|---|---|
| $Q_1$ | 0 | 0 | + | − | − | + | 0 | 0 |
| $Q_2$ | 0 | 0 | − | + | 0 | 0 | + | − |

$$W = (\phi_1 - \phi_2)(A_1 B_1 C_1 - A_2 B_2 C_2)$$

Gauge-invariant coordinates:

$$\phi_1 = \phi_2 \ (\equiv \phi) \qquad A_1 B_1 C_1 = A_2 B_2 C_2 \ (\equiv w)$$

$$z_1 = A_1 A_2, z_2 = B_1 B_2, z_3 = C_1 C_2 \qquad z_1 z_2 z_3 = w^2$$

# Summary : Brane Crystal

- Brane Crystal model via T-duality

- Edge = matter field (M2-disc)
  Vertex = super-potential (M2-sphere)
  Face = gauge symmetry (at least in the abelian case)

- Checks & Predictions
  - moduli space of vacua
  - mesonic / baryonic spectrum
  - partial resolution / Seiberg-like duality / RG flow

- Difficulities
  - Special Lagrangian sub-manifold of $(C^*)^3$ (cf Newton polynomial)
  - Non-abelian gauge symmetry : multiple-M2 Lagrangian ??

# $\mathcal{N} \geq 4$ Chern-Simons and M2-branes

# Brane Crystal vs. Bagger-Lambert - 1st encounter (winter '06~'07)

- BL 3-algebra as a candidate for the "M2-disc" algebra?

# Brane Crystal vs. Bagger-Lambert - 1st encounter (winter '06~'07)

- BL 3-algebra as a candidate for the "M2-disc" algebra?
- Difficulties with $f^{abcd} A_a B_b C_c D_d$

| Brane Crystal | | Bagger-Lambert |
|---|---|---|
| polyhedra with 4, 5, 6,... faces | $\longleftrightarrow$ | products of 4, 6, 8, ... fields |
| two orientations for 4-hedron | $\longleftrightarrow$ | unique product of 4 fields |

cf) $\mathrm{tr}(ABC - CBA)$

# Brane Crystal vs. Bagger-Lambert - 2nd encounter (spring '08)

- BLG theory ...
  - can be recast as ordinary gauge theory [Raamsdonk]
  - is the first to escape the $\mathcal{N} = 3$ bound [Lee-Kao '92][Schwarz '04]

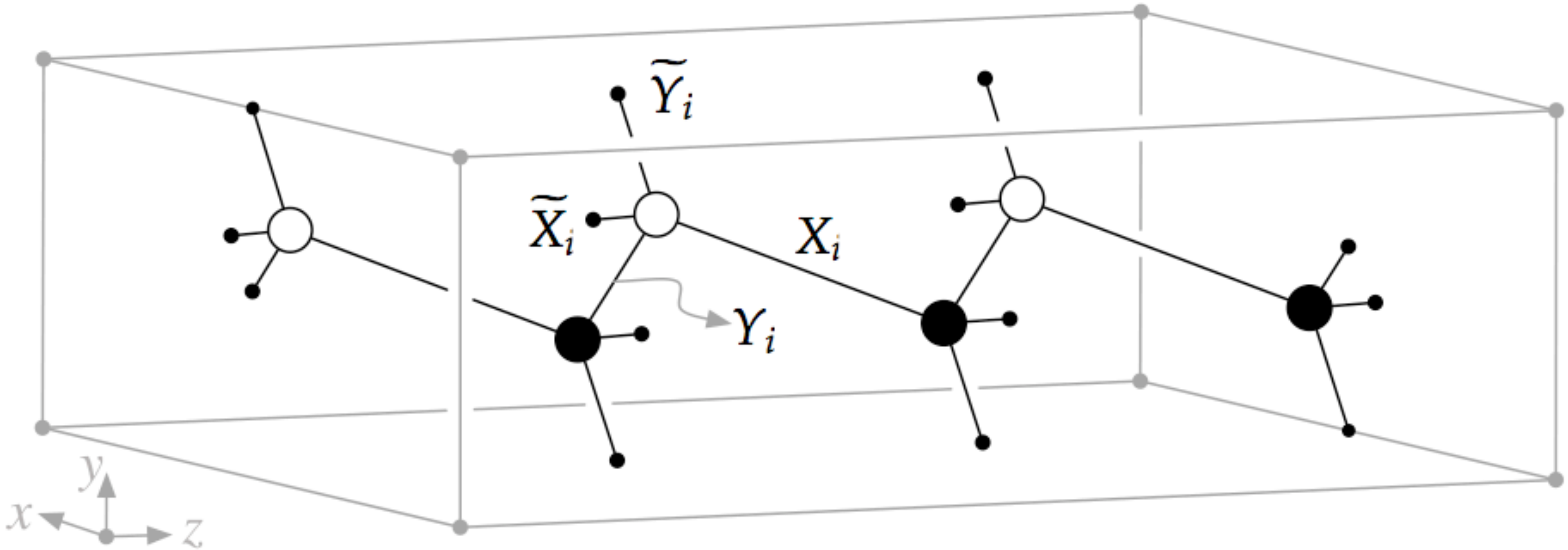
- More theories with $4 \leq \mathcal{N} \leq 8$ must exist! (orbifolds)

- Interpretation of BLG unclear
  [Lambert-Tong][Distler-Mukhi-Papageorgakis-Raamsdonk], etc.
  - String/M theory derivation of CS theories desirable.

- Any clues on CS theories from the Brane Crystal? *YES!*

# Brane Crystals for orbifolds

$(\mathbb{C}^2/\mathbb{Z}_n)^2$




$$W = \sum_{i=1}^{n} \left( X_i \widetilde{X}_i Y_i \widetilde{Y}_i - X_i \widetilde{X}_i Y_{i+1} \widetilde{Y}_{i+1} \right)$$

$$X_i \widetilde{X}_i - X_{i-1} \widetilde{X}_{i-1} = 0, \qquad Y_i \widetilde{Y}_i - Y_{i+1} \widetilde{Y}_{i+1} = 0.$$

| | $X_i$ | $\widetilde{X}_i$ |
|---|---|---|
| $Q_i$ | $+$ | $-$ |
| $Q_{i+1}$ | $-$ | $+$ |

Symmetric under exchange of $X$ and $Y$ .

Both $X$ and $Y$ should be hyper-multiplets ... of opposite type.

# $\mathcal{N} = 4$ multiplets

$\mathcal{N} = 4$ super-algebra includes $SO(4) = SU(2)_L$ x $SU(2)_R$ R-symmetry

| | (ordinary) | $SU(2)_L$ | $SU(2)_R$ | $SU(2)_L$ | $SU(2)_R$ | (twisted) |
|---|---|---|---|---|---|---|
| (hyper) | $q_\alpha$ | 2 | 1 | 1 | 2 | $\tilde{q}_{\dot\alpha}$ |
| | $\psi_{\dot\alpha}$ | 1 | 2 | 2 | 1 | $\tilde{\psi}_\alpha$ |
| (vector) | $A_\mu$ | 1 | 1 | 1 | 1 | $\tilde{A}_\mu$ |
| | $\chi_{\alpha\dot\alpha}$ | 2 | 2 | 2 | 2 | $\tilde{\chi}_{\alpha\dot\alpha}$ |
| | $s_{\dot\alpha\dot\beta}$ | 1 | 3 | 3 | 1 | $\tilde{s}_{\alpha\beta}$ |

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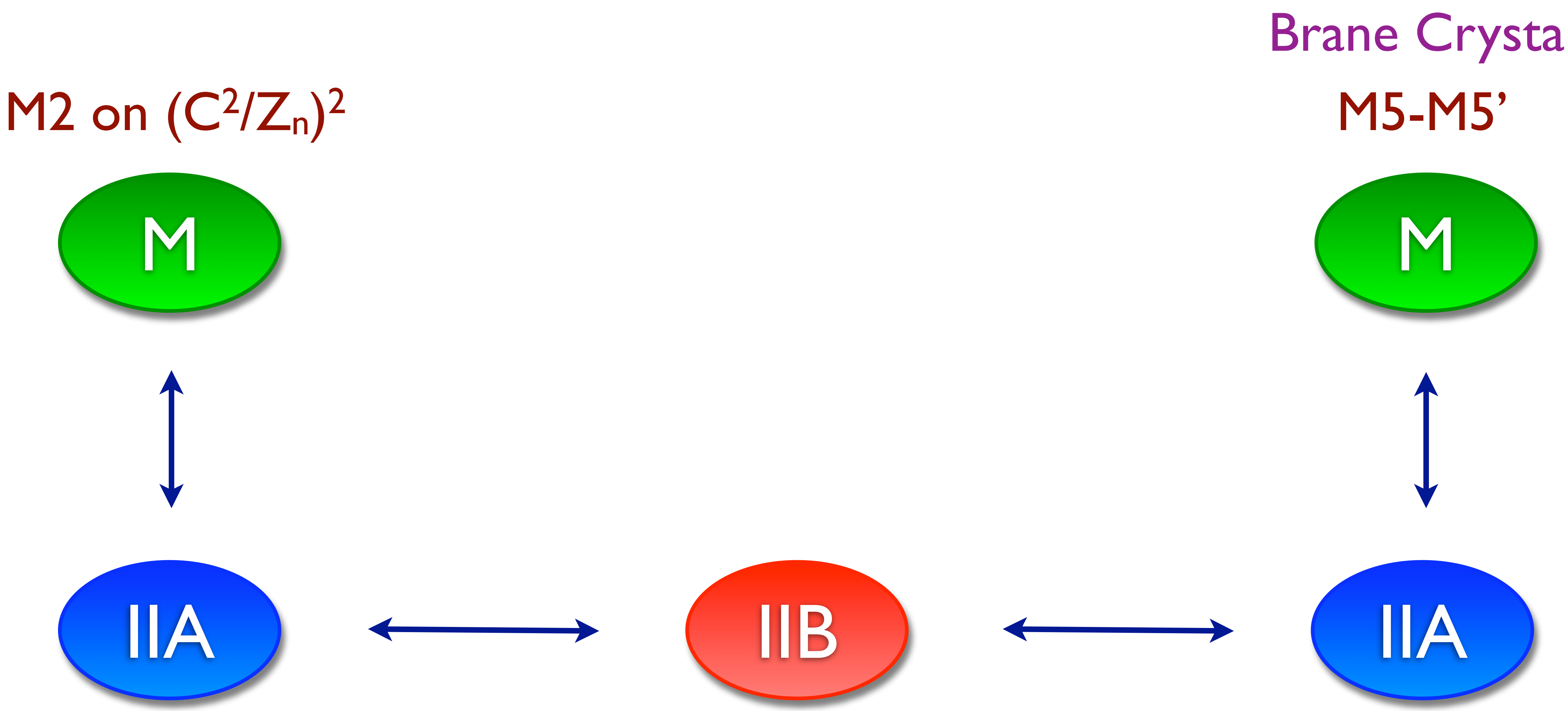
★ CS theories for M2-branes require both types of hypers !

- Brane Crystal on orbifolds
- BLG $\mathcal{N} = 8$ written in $\mathcal{N} = 4$ notation

# T-dual, T-dual, T-dual !

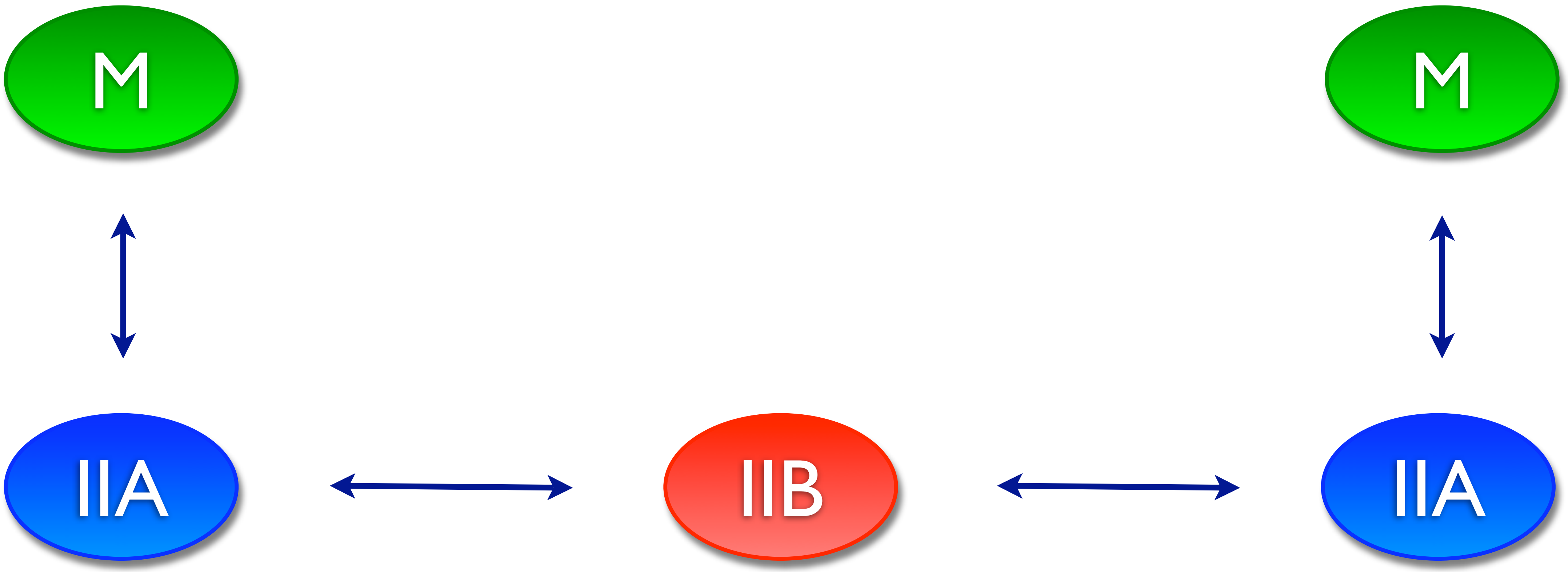


# T-dual, T-dual, T-dual !

Brane Crystal

M5-M5'

M2 on $(C^2/Z_n)^2$

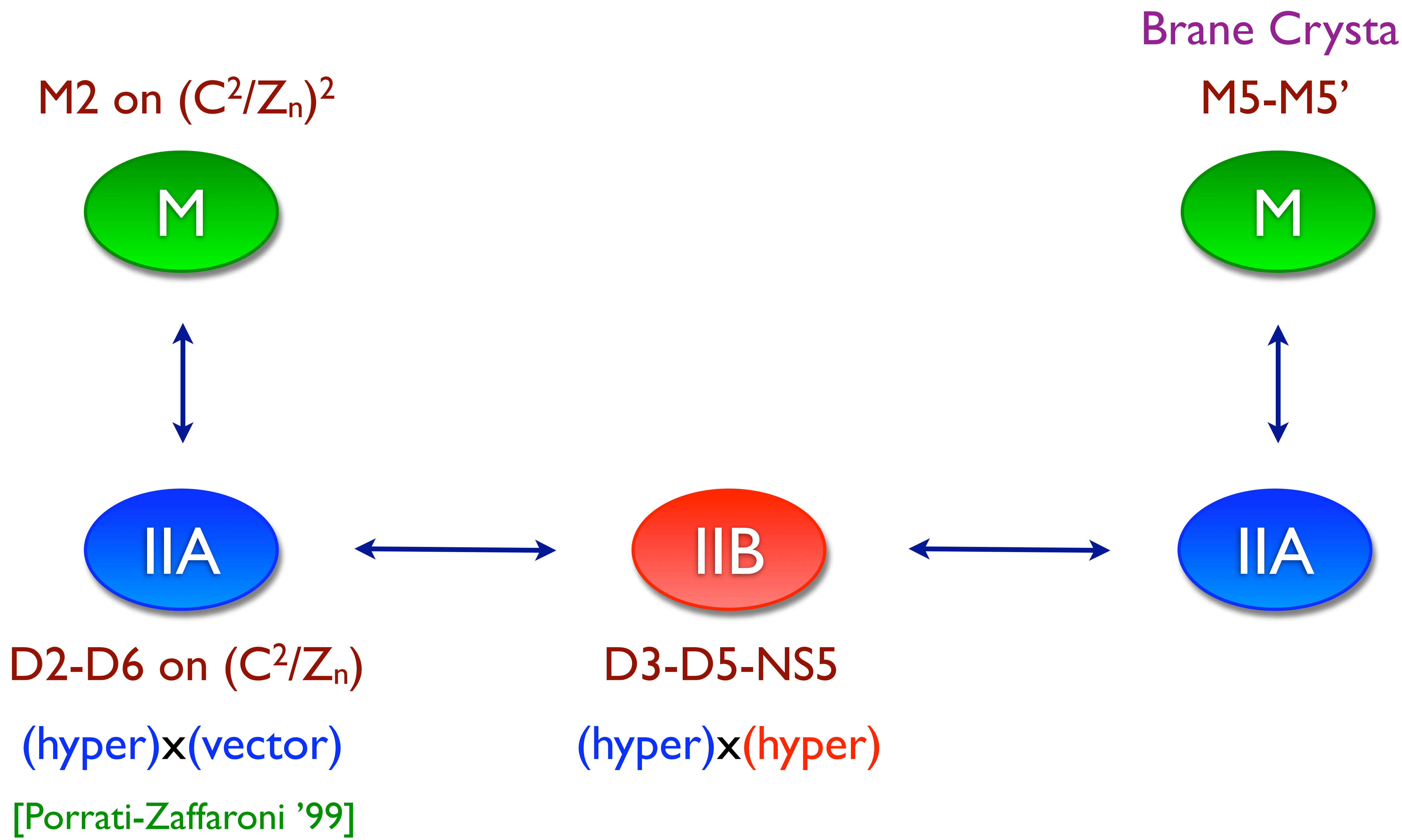



D2-D6 on $(C^2/Z_n)$

(hyper)x(vector)

[Porrati-Zaffaroni '99]

# T-dual, T-dual, T-dual !



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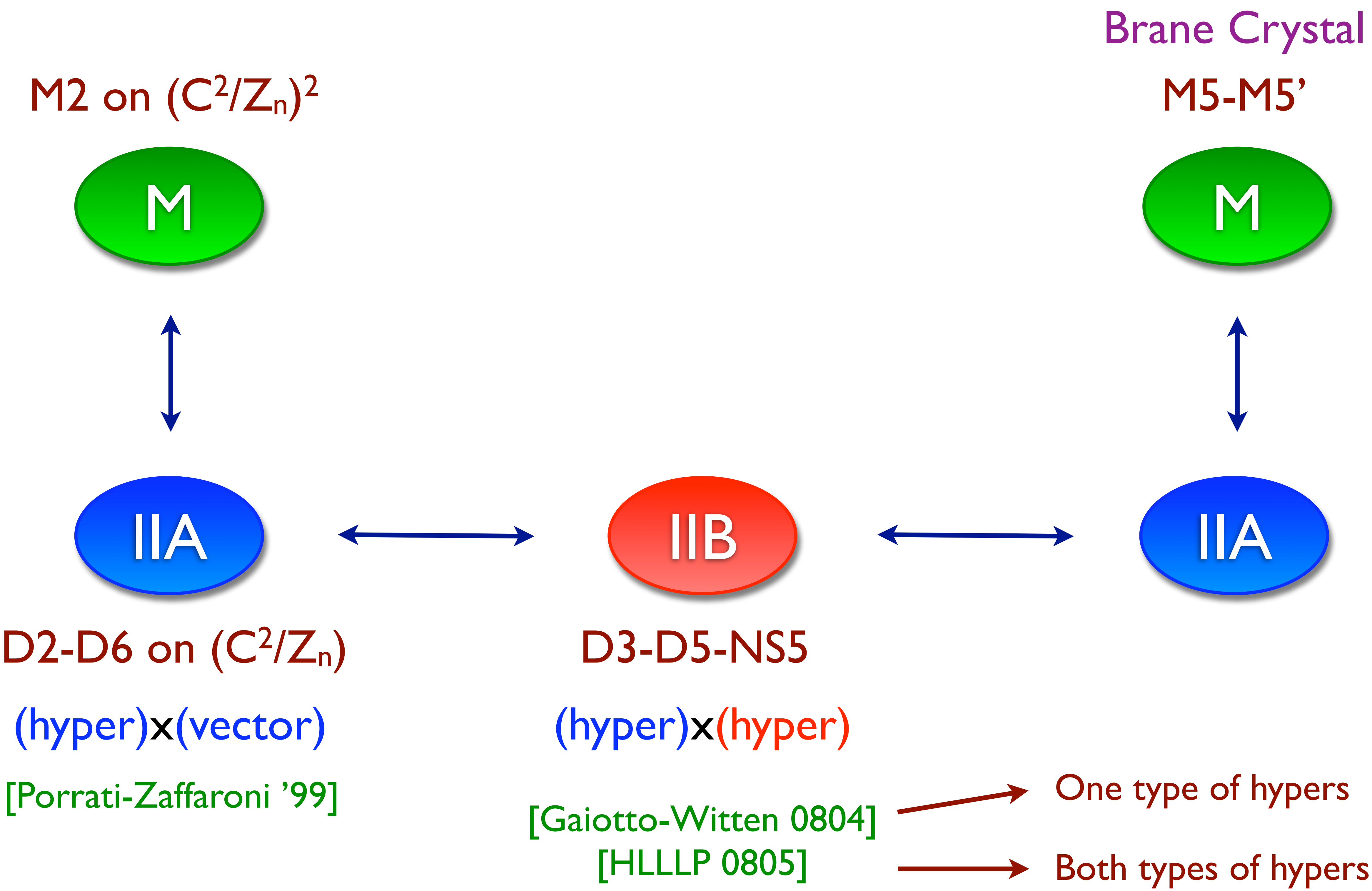

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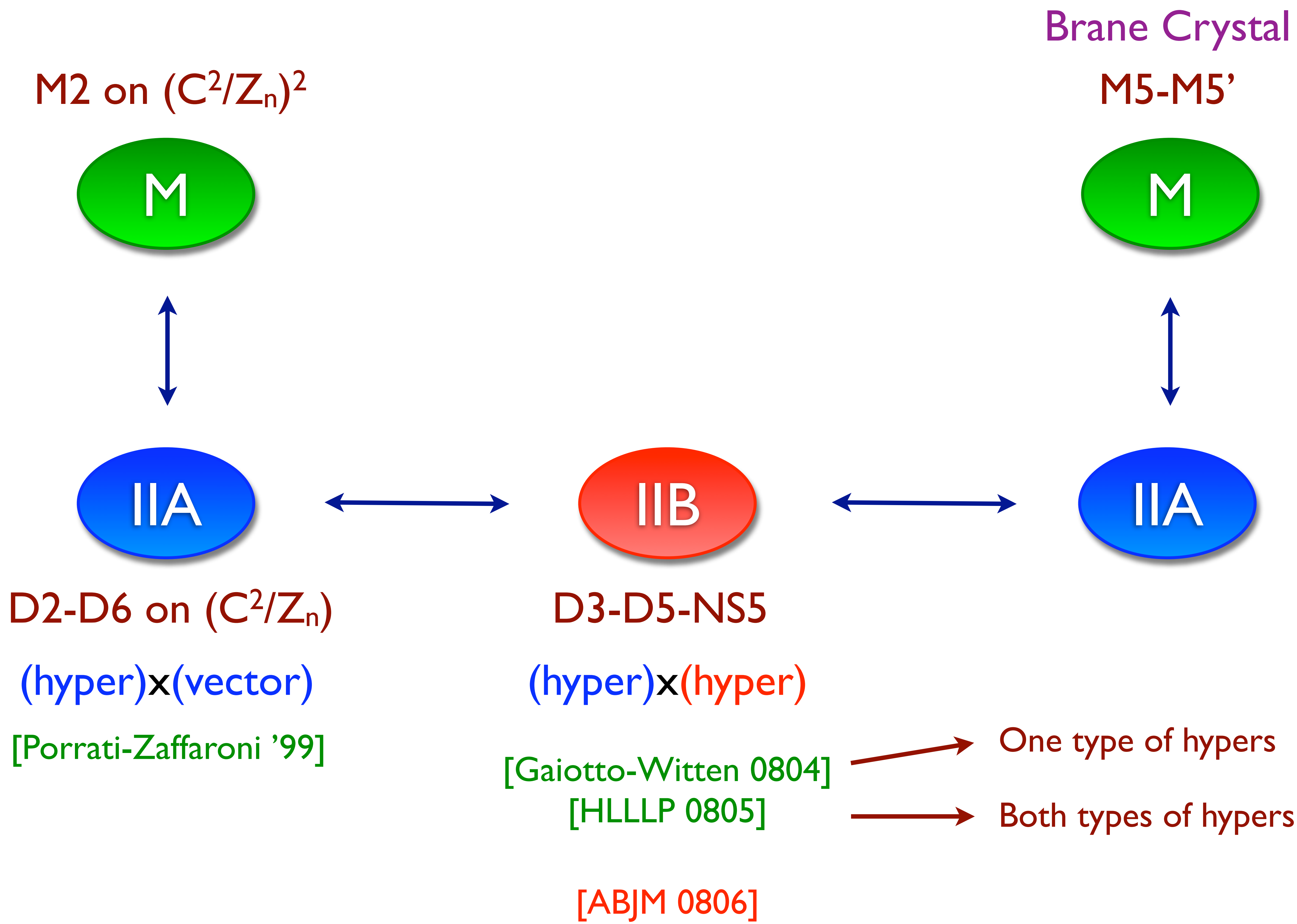


# Gaiotto-Witten - I. field content

Matter fields : $q^A_\alpha\, , \psi^A_{\dot\alpha}\, , \quad \tilde q^A_{\dot\alpha}\, , \tilde\psi^A_\alpha$

hyper-Kahler : $Sp(2n)$ with $\omega_{AB}$

reality condition : $\bar q^\alpha_A = (q^A_\alpha)^\dagger = \epsilon^{\alpha\beta}\omega_{AB} q^B_\beta$

Gauge group : $G \subset Sp(2n)$

$(t^m)^A{}_B$ satisfy $[t^m, t^n] = f^{mn}{}_p t^p , \quad \mathrm{Tr}(t^m t^n) = k^{mn}$

$$t^m_{AB} \equiv \omega_{AC}(t^m)^C{}_B = t^m_{BA}$$

Gauge field : $(A_m)_\mu$

# Gaiotto-Witten - II. condition for $\mathcal{N} = 4$ SUSY

- Begin with $\mathcal{N} = 1$ SUSY theory with $SU(2)_{\text{diag}} \subset SU(2)_L \times SU(2)_R$ global symmetry.

- Adjust the $\mathcal{N} = 1$ super-potential to restore $SU(2)_L \times SU(2)_R$.

- The SO(4) $R$-symmetry together with $\mathcal{N} = 1$ SUSY generate a full $\mathcal{N} = 4$ structure.

$$k_{mn} t^m_{(AB} t^n_{C)D} = 0 \quad \text{("fundamental identity")}$$

- Adding twisted hypers : more terms in Lagrangian, but no more constraints. [HLLLP]

# General $\mathcal{N} = 4$ Lagrangian : Summary

$$\begin{aligned}
\mathcal{L} \;=\;& \varepsilon^{\mu\nu\lambda}\left(k_{mn}A_m^m\partial_\nu A_\lambda^n + \frac{1}{3}f_{mnp}A_\mu^m A_\nu^n A_\lambda^p\right) \\
&+\omega_{AB}\left(-\epsilon^{\alpha\beta}Dq_\alpha^A Dq_\beta^B + i\epsilon^{\dot\alpha\dot\beta}\psi_{\dot\alpha}^A \not{D}\,\psi_{\dot\beta}^B\right) + \tilde\omega_{AB}\left(-\epsilon^{\dot\alpha\dot\beta}D\tilde q_{\dot\alpha}^A D\tilde q_{\dot\beta}^B + i\epsilon^{\alpha\beta}\tilde\psi_\alpha^A \not{D}\,\tilde\psi_\beta^B\right) \\
&-ik_{mn}\epsilon^{\alpha\beta}\epsilon^{\dot\gamma\dot\delta}j_{\alpha\dot\gamma}^m j_{\beta\dot\delta}^n - ik_{mn}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\gamma\delta}\tilde j_{\dot\alpha\gamma}^m \tilde j_{\dot\beta\delta}^n + 4ik_{mn}\epsilon^{\alpha\gamma}\epsilon^{\dot\beta\dot\delta}j_{\alpha\dot\beta}^m \tilde j_{\dot\delta\gamma}^n \\
&+ik_{mn}\left(\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\tilde\mu_{\dot\alpha\dot\beta}^m \psi_{\dot\gamma}^A t_{AB}^n \psi_{\dot\delta}^B + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mu_{\alpha\beta}^m \tilde\psi_\gamma^A \tilde t_{AB}^n \tilde\psi_\delta^B\right) \\
&-\frac{1}{12}f_{mnp}(\mu^m)^\alpha{}_\beta(\mu^n)^\beta{}_\gamma(\mu^p)^\gamma{}_\alpha - \frac{1}{12}f_{mnp}(\tilde\mu^m)^\alpha{}_\beta(\tilde\mu^n)^\beta{}_\gamma(\tilde\mu^p)^\gamma{}_\alpha \\
&+\frac{1}{2}(\tilde\mu^{mn})^{\dot\gamma}{}_{\dot\gamma}(\mu_m)^\alpha{}_\beta(\mu_n)^\beta{}_\alpha + \frac{1}{2}(\mu^{mn})^\gamma{}_\gamma(\tilde\mu_m)^{\dot\alpha}{}_{\dot\beta}(\tilde\mu_n)^{\dot\beta}{}_{\dot\alpha},
\end{aligned}$$

$$\mu_{\alpha\beta}^m \equiv t_{AB}^m q_\alpha^A q_\beta^B, \quad j_{\alpha\dot\gamma}^m \equiv q_\alpha^A t_{AC}^m \psi_{\dot\gamma}^C$$
$$\tilde\mu_{\dot\alpha\dot\beta}^m \equiv \tilde t_{AB}^m \tilde q_{\dot\alpha}^A \tilde q_{\dot\beta}^B, \quad \tilde j_{\dot\alpha\alpha}^m \equiv \tilde q_{\dot\alpha}^A \tilde t_{AB}^m \tilde\psi_\alpha^B$$

(moment map multiplets)

## Classification by super-algebra [Gaiotto-Witten]

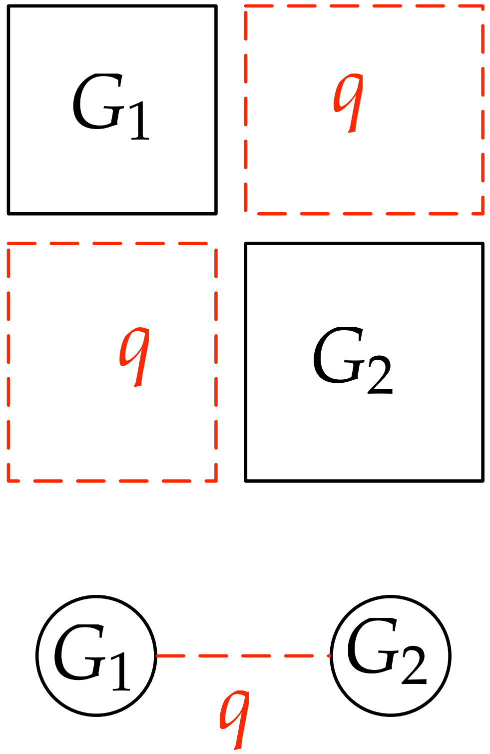
Fundamental identity is “solved” by (auxiliary) super-algebras :

$$[M^m, M^n] = f^{mn}{}_p M^p, \quad [M^m, Q_A] = Q_B (t^m)^B{}_A, \quad \{Q_A, Q_B\} = t^m_{AB} M_m.$$

$$[\{Q_A, Q_B\}, Q_C] + (\text{cyclic}) = 0 \quad \Longleftrightarrow \quad k_{mn} t^m_{(AB} t^n_{C)D} = 0$$

Super-algebras classify $\mathcal{N} \geq 4$ theories completely!

Typical examples : $U(N|M)$, $OSp(N|M)$

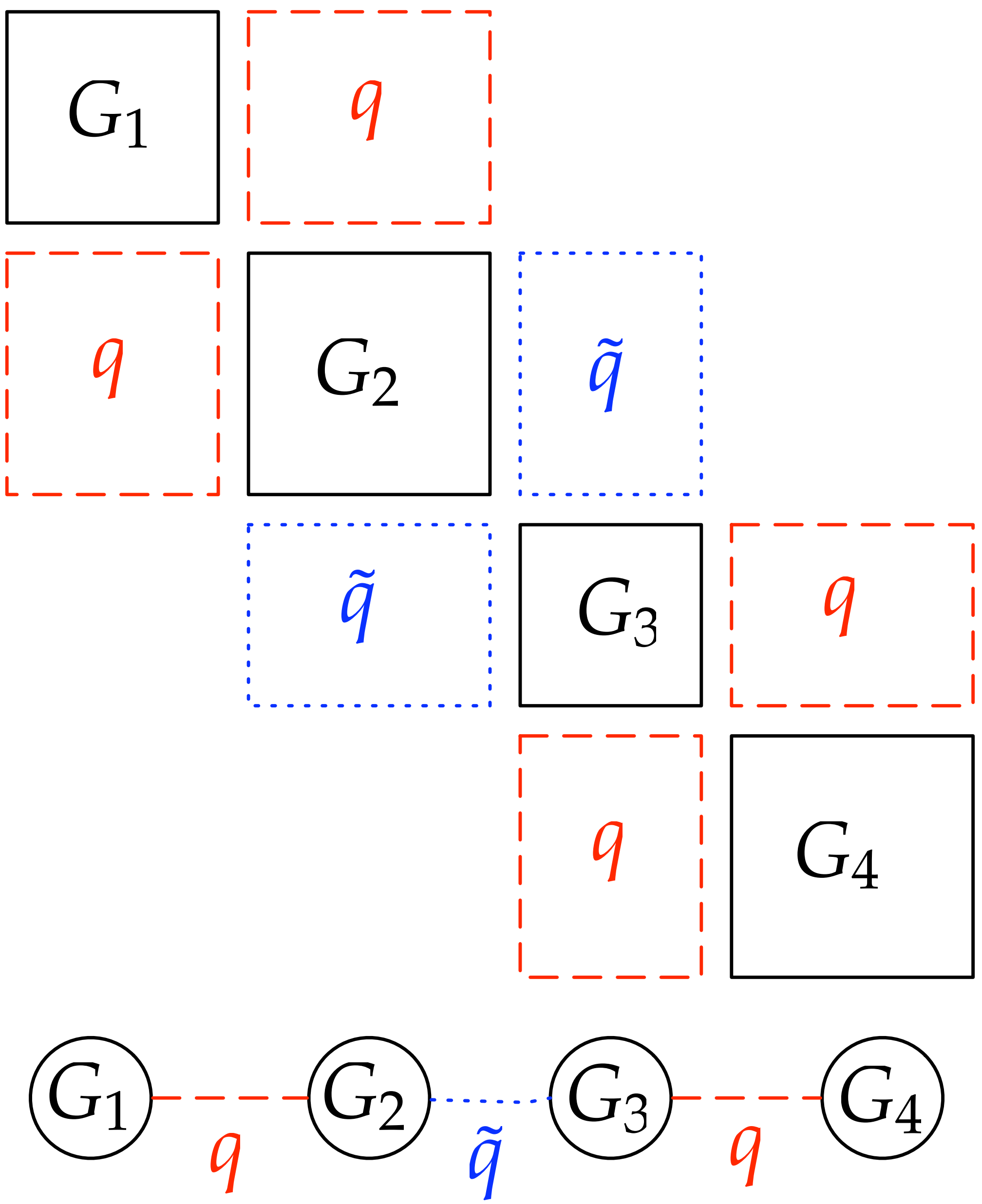


# Classification and linear quiver theories [Gaiotto-Witten][HLLLP]

Hypers and twisted hypers should independently form super-algebra structure.

Generically, they form quivers with the two types of hypers alternating between gauge groups.

The quiver can either have open ends or form a closed loop.



# Enhancement to $\mathcal{N}=5$

- A necessary condition
  two types of hypers form a single $\mathcal{N}>4$ multiplet :
  same representation under the gauge group

- For $\mathcal{N}=5$, this is sufficient, too! $SU(2)\times SU(2)\subset USp(4)\approx SO(5)$

$$\Phi^A_\alpha=\begin{pmatrix} q^A_\alpha \\ \tilde{q}^A_{\dot\alpha}\end{pmatrix}\,,\quad \Psi^A_\alpha=\begin{pmatrix} \tilde{\psi}^A_\alpha \\ \psi^A_{\dot\alpha}\end{pmatrix}\;;\quad C^{\alpha\beta}=\begin{pmatrix}\epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot\alpha\dot\beta}\end{pmatrix}\,.$$

$$\mathcal{M}^m_{\alpha\beta}\equiv t^m_{AB}\Phi^A_\alpha\Phi^B_\beta\,,\qquad \mathcal{M}^{mn}_{\alpha\beta}\equiv(t^m t^n)_{AB}\Phi^A_\alpha\Phi^B_\beta\,,\qquad \mathcal{J}^m_{\alpha\beta}\equiv t^m_{AB}\Phi^A_\alpha\Psi^B_\beta\,.$$

- General $\mathcal{N}=5$ Lagrangian :

$$\begin{aligned}\mathcal{L} \;=\;& \omega_{AB}C^{\alpha\beta}\left(-D\Phi^A_\alpha D\Phi^B_\beta+i\Psi^A_\alpha D\Psi^B_\beta\right)-ik_{mn}C^{\alpha\beta}C^{\gamma\delta}\left(\mathcal{J}^m_{\alpha\gamma}\mathcal{J}^n_{\beta\delta}-2\mathcal{J}^m_{\alpha\gamma}\mathcal{J}^n_{\delta\beta}\right)\\ &+\frac{1}{15}f_{mnp}(\mathcal{M}^m)^\alpha{}_\beta(\mathcal{M}^n)^\beta{}_\gamma(\mathcal{M}^p)^\gamma{}_\alpha+\frac{3}{10}(\mathcal{M}^{mn})^\gamma{}_\gamma(\mathcal{M}_m)^\alpha{}_\beta(\mathcal{M}_n)^\beta{}_\alpha\,,\end{aligned}$$

# Enhancement to $\mathcal{N} = 6$

- The matter fields are in a pseudo-real representation

  $\left(\text{Recall}: \bar{q}_A^\alpha = (q_\alpha^A)^\dagger = \epsilon^{\alpha\beta}\omega_{AB}q_\beta^B .\right)$

- Further enhancement to $\mathcal{N} = 6$ occurs if and only if $\mathcal{R} = R + \overline{R}$.

$$(\Phi_\alpha^A)_{\mathcal{N}=5} = \begin{pmatrix} \Phi_\alpha^A \\ C_{\alpha\beta}\bar{\Phi}_A^\beta \end{pmatrix}, \quad (\Psi_\alpha^A)_{\mathcal{N}=5} = \begin{pmatrix} C_{\alpha\beta}\Psi^{\beta A} \\ -\bar{\Psi}_{\alpha A} \end{pmatrix}; \quad (\omega_{AB})_{\mathcal{N}=5} = \begin{pmatrix} 0 & \delta_A{}^B \\ -\delta^A{}_B & 0 \end{pmatrix}.$$

| | $\Phi_\alpha^A$ | $\bar{\Phi}_A^\alpha$ | $\Psi^{\alpha A}$ | $\bar{\Psi}_{\alpha A}$ |
|---|---|---|---|---|
| Gauge | $R$ | $\bar{R}$ | $R$ | $\bar{R}$ |
| $SO(6)_R$ | $\mathbf{4}$ | $\bar{\mathbf{4}}$ | $\bar{\mathbf{4}}$ | $\mathbf{4}$ |

$$(t^A{}_B)_{\mathcal{N}=5} = \begin{pmatrix} t^A{}_B & 0 \\ 0 & -t^B{}_A \end{pmatrix}, \quad (t^m)^A_B (t_m)^C_D + (t^m)^A_D (t_m)^C_B = 0 .$$

# Enhancement to $\mathcal{N} = 6$

- $USp(4) \subset SU(4) \approx SO(6)$

$$(\mathcal{M}^m)^\alpha{}_\beta = -(M^m)^\alpha{}_\beta - C^{\alpha\delta} C_{\beta\gamma} (M^m)^\gamma{}_\delta ,$$

$$(\mathcal{M}^{mn})^\alpha{}_\beta = -(M^{mn})^\alpha{}_\beta + C^{\alpha\delta} C_{\beta\gamma} (M^{nm})^\gamma{}_\delta , \qquad C^{\alpha\beta} C^{\gamma\delta} + C^{\alpha\gamma} C^{\delta\beta} + C^{\alpha\delta} C^{\beta\gamma} = \epsilon^{\alpha\beta\gamma\delta} .$$

$$(\mathcal{J}^m)_{\alpha\beta} = (J^m)_{\alpha\beta} - C_{\alpha\gamma} C_{\beta\delta} (\bar{J}^m)^{\gamma\delta} ,$$

- General $\mathcal{N} = 6$ Lagrangian :

$$\begin{aligned}\mathcal{L} &= -D\bar{\Phi}^\alpha_A D\Phi^A_\alpha + i\bar{\Psi}_{\alpha A} D\Psi^{\alpha A} \\ &\quad + \frac{i}{4} \left[ 2(\bar{J}_m)^{\alpha\beta} (J^m)_{\alpha\beta} - 4(\bar{J}_m)^{\alpha\beta} (J^m)_{\beta\alpha} + \epsilon^{\alpha\beta\gamma\delta} (J_m)_{\alpha\beta} (J^m)_{\gamma\delta} + \epsilon_{\alpha\beta\gamma\delta} (\bar{J}_m)^{\alpha\beta} (\bar{J}^m)^{\gamma\delta} \right] \\ &\quad - \frac{1}{12} f_{mnp} (M^m)^\alpha{}_\beta (M^n)^\beta{}_\gamma (M^p)^\gamma{}_\alpha + \frac{1}{4} (M^{mn})^\alpha{}_\beta (M_m)^\beta{}_\gamma (M_n)^\gamma{}_\alpha .\end{aligned}$$

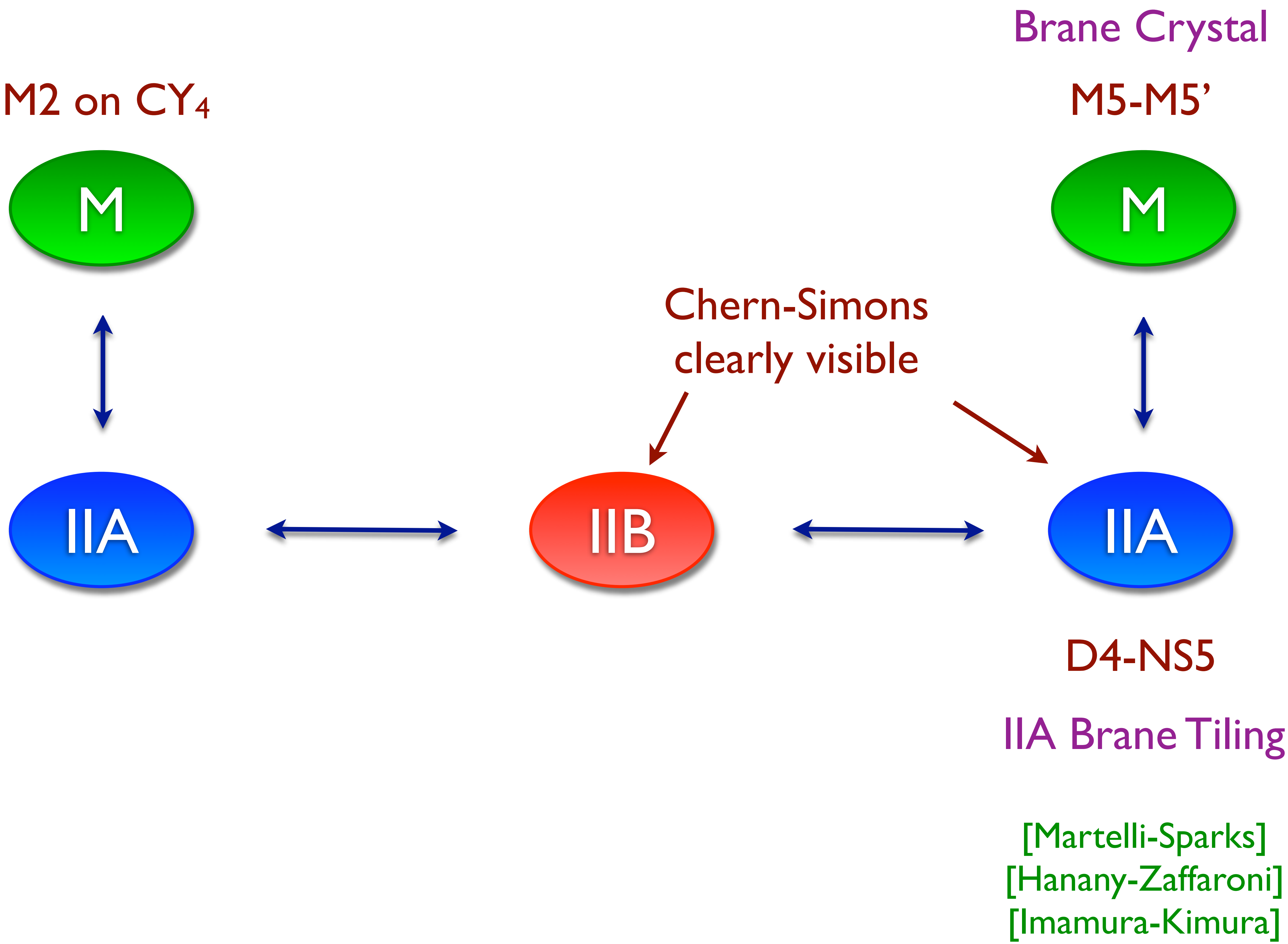
- Super-algebras give classification of all $\mathcal{N} = 5, 6$ theories.

Alternative approaches : [Bagger-Lambert][Schnabl-Tachikawa][Bergshoeff et al]

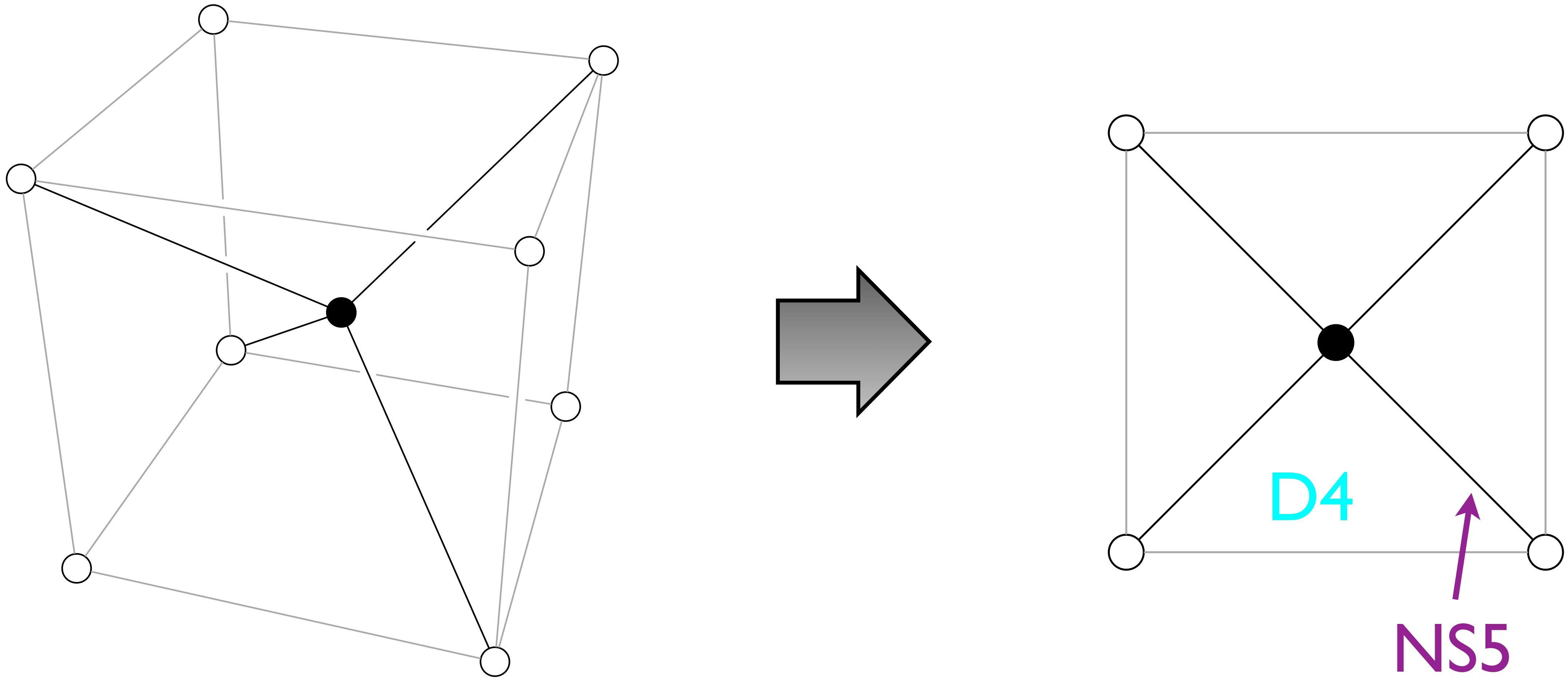
# Explicit examples

- $(\mathcal{N}=8)$ : BLG
- $(\mathcal{N}=6)$ : ABJM $U(N)\times U(N);\ O(2)\times Sp(2N)$, etc.
- $(\mathcal{N}=5)$ : $O(2N)\times Sp(2N)$, etc. [HLLLP]
- $(\mathcal{N}=4)$ : linear $U$ or $OSp$ quivers, etc.
  [HLLLP][Benna-Klebanov-Klose-Smedback][Imamura-Kimura][Terashima-Yagi]

# Back to Brane Tiling



# IIA Brane Tiling [Martelli-Sparks][Hanany-Zaffaroni][Imamura-Kimura]



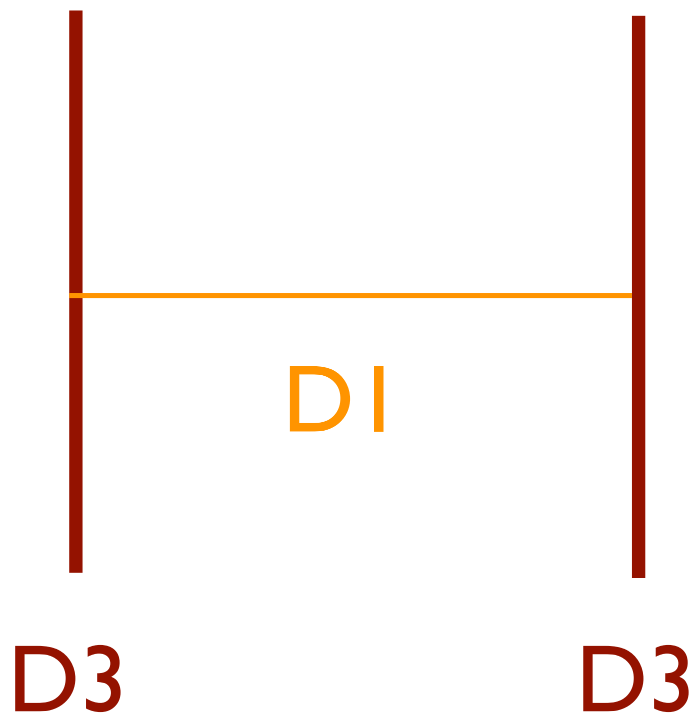

$$S = \frac{1}{2\pi} \int_{\partial \mathrm{D4}} A \wedge dA \wedge d\phi$$

scalar on NS5 ~ $X^{11}$ coordinate

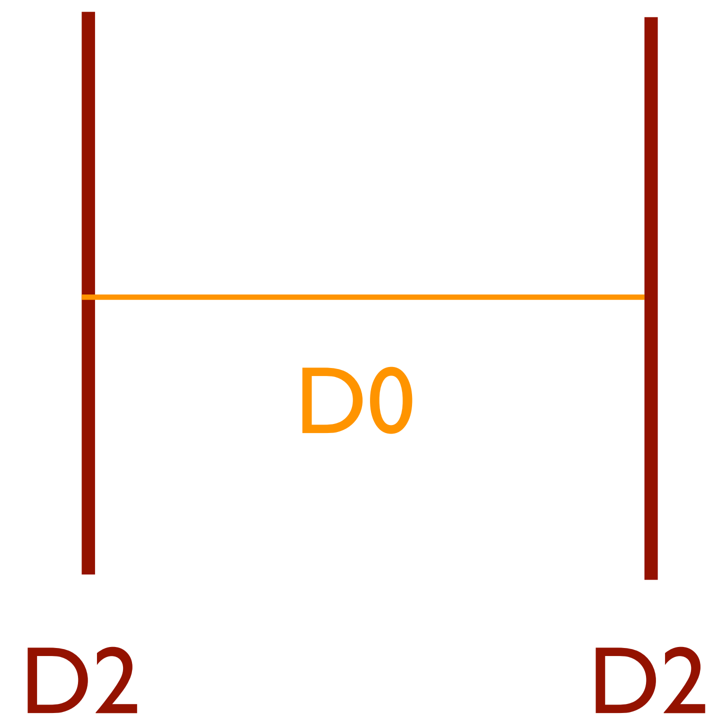
# Instanton effects in ABJM

# Instantons in 3-dim Yang-Mills

Monopole in 4-dim




Instanton in 3-dim




Idea : compare world-volume theory and bulk theory

- A non-perturbative test of the ABJM proposal!

# Instantons in ABJM

| ABJM model | 11 / 10-dim bulk |
|---|---|
| Classical instanton action | D0 : DBI + RR |
| Higgs mechanism | Strings stretched between D2's |
| Zero-instanton one-loop | Super-graviton exchange |
| One-instanton one-loop | D0 exchange |

subtleties due to imaginary value of Chern-Simons action, etc.

# Instantons in ABJM

| ABJM model | | 11 / 10-dim bulk |
|---|---|---|
| Classical instanton action | [HLLLPY] | D0 : DBI + RR |
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# Instantons in ABJM

| ABJM model | 11/10-dim bulk |
|---|---|
| Classical instanton action | D0 : DBI + RR |
| Higgs mechanism | Strings stretched between D2's |
| Zero-instanton one-loop [Baek-Hyun-Jang-Yi] | Super-graviton exchange |
| One-instanton one-loop | D0 exchange |

[HLLLPY]

subtleties due to imaginary value of
Chern-Simons action, etc.

# Discussions

# Summary

- Brane Crystals
- Superconformal Chern-Simons theories
- Instanton effects in ABJM theory

# Discussions

- Beyond ordinary gauge symmetry? (connection to M5)
- More on instantons.
- Vortex solitons and relation to NR AdS/CFT?
- Beta deformation and fuzzy three-torus
- Topological twisting of CS-matter theories :
  relation to pure-CS and Rozansky-Witten theories?

Thank you