

Fluid Dynamics from Gravity II

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References : arXiv [hep-th] : 0801.3701, 0809.2596, 0809.4272

Collaborators : Nabamita Banerjee, Jyotirmoy Bhattacharya,
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Four Points to take home

- Conformal hydrodynamics admits a Weyl-covariant formalism. (Will explain soon what that means.)
- Blackholes (Large, AdS) dual to Hydrodynamic states in the boundary.
- Metric duals to general hydrodynamic states can be constructed in a well-controlled expansion.
- Novel phenomena/ Universal relations in the hydrodynamics with gravitational duals.

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Hydrodynamics - A tractable description at finite T

- **Basic Variables** : Velocity u^μ (Energy frame/Charge frame), Temperature T and Charge density n or Chemical potential μ .
- Energy/Momentum transport : $T^{\mu\nu}$ with $\nabla_\mu T^{\mu\nu} = 0$.
- Charge transport described by J^μ with $\nabla_\mu J^\mu = 0$.
- Inputs : Eqn. of State, Constitutive relations ...
- Hydrodynamics : the first step from equilibrium towards near equilibrium physics.

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Scaling in Hydrodynamics

- Scaling is a powerful concept in hydrodynamics - especially if there is an underlying CFT_d .
- Weyl transformation in hydrodynamics : $g_{\mu\nu} = e^{2\phi} \tilde{g}_{\mu\nu}$ and $u^\mu = e^{-\phi} \tilde{u}^\mu$.
- Energy scales scale as $T = e^{-\phi} \tilde{T}$ and $\mu = e^{-\phi} \tilde{\mu}$.
- Densities transform as $n = e^{-(d-1)\phi} \tilde{n}$, $J^\mu = e^{-d\phi} \tilde{J}^\mu$ and

$$T^\mu{}_\nu = e^{-d\phi} \tilde{T}^\mu{}_\nu + \text{Anomalous corrections}$$

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Weyl Covariance is useful

- Introduce in CFT_d hydrodynamics

$$\mathcal{A}_\nu \equiv u^\lambda \nabla_\lambda u_\nu - \frac{\nabla_\lambda u^\lambda}{d-1} u_\nu = \tilde{\mathcal{A}}_\nu + \partial_\nu \phi.$$

R. Loganayagam . arXiv:0801.3701 [hep-th]

- Helps in ‘Weyl-covariantly’ differentiating tensors

$$\begin{aligned} \text{If } Q_{\nu\dots}^{\mu\dots} = e^{-w\phi} \tilde{Q}_{\nu\dots}^{\mu\dots} \text{ then } \mathcal{D}_\lambda Q_{\nu\dots}^{\mu\dots} &= e^{-w\phi} \tilde{\mathcal{D}}_\lambda \tilde{Q}_{\nu\dots}^{\mu\dots} \\ \text{with } \mathcal{D}_\lambda Q_{\nu\dots}^{\mu\dots} &\equiv \nabla_\lambda Q_{\nu\dots}^{\mu\dots} + w \mathcal{A}_\lambda Q_{\nu\dots}^{\mu\dots} \\ &\quad + [g_{\lambda\alpha} \mathcal{A}^\mu - \delta_\lambda^\mu \mathcal{A}_\alpha - \delta_\alpha^\mu \mathcal{A}_\lambda] Q_{\nu\dots}^{\alpha\dots} + \dots \\ &\quad - [g_{\lambda\nu} \mathcal{A}^\alpha - \delta_\lambda^\alpha \mathcal{A}_\nu - \delta_\nu^\alpha \mathcal{A}_\lambda] Q_{\alpha\dots}^{\mu\dots} - \dots \end{aligned}$$

- $\mathcal{A}_\mu$ uniquely determined by $\mathcal{D}_\lambda g_{\mu\nu} = 0$, $u^\lambda \mathcal{D}_\lambda u_\mu = 0$ and $\mathcal{D}_\mu u^\mu = 0$.

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Weyl-Covariantised Curvature Tensors

Weyl covariantised Riemann tensor can be obtained from

$$\begin{aligned}
 [\mathcal{D}_\mu, \mathcal{D}_\nu] V_\lambda &= w \mathcal{F}_{\mu\nu} V_\lambda + \mathcal{R}_{\mu\nu\lambda}{}^\alpha V_\alpha \quad \text{with} \\
 \mathcal{F}_{\mu\nu} &\equiv \nabla_\mu \mathcal{A}_\nu - \nabla_\nu \mathcal{A}_\mu \\
 \mathcal{R}_{\mu\nu\lambda\sigma} &\equiv R_{\mu\nu\lambda\sigma} + \mathcal{F}_{\mu\nu} g_{\lambda\sigma} \\
 &\quad - \delta_{[\mu}^\alpha g_{\nu]} [\lambda \delta_{\sigma]}^\beta \left(\nabla_\alpha \mathcal{A}_\beta + \mathcal{A}_\alpha \mathcal{A}_\beta - \frac{\mathcal{A}^2}{2} g_{\alpha\beta} \right)
 \end{aligned}$$

where $B_{[\mu\nu]} \equiv B_{\mu\nu} - B_{\nu\mu}$ indicates antisymmetrisation. Other related tensors defined similarly - will later need

$$\mathcal{S}_{\mu\nu} \equiv \frac{1}{d-2} \left(\mathcal{R}_{\mu\nu} - \frac{\mathcal{R} g_{\mu\nu}}{2(d-1)} \right)$$

Weyl-covariant Hydrodynamics

- By construction, $\mathcal{D}_\mu u_\nu$ is traceless and transverse to the velocity.
- Split $\mathcal{D}_\mu u_\nu = \sigma_{\mu\nu} + \omega_{\mu\nu}$ where
- $\sigma_{\mu\nu}$ is the shear strain rate
 (symmetric, traceless, transverse) tensor which in viscous fluids leads to dissipation.
- $\omega_{\mu\nu}$ is the vorticity (antisymmetric, transverse) tensor which measures local rotation of the fluid element.
- The hydrodynamic equations can be written in a manifestly Weyl-covariant form ($\mathcal{W}$ is the Weyl anomaly)

$$\begin{aligned}\mathcal{D}_\mu T^{\mu\nu} &\equiv \nabla_\mu T^{\mu\nu} + \mathcal{A}^\nu (T^\mu{}_\mu - \mathcal{W}) = 0 \\ \mathcal{D}_\mu J^\mu &\equiv \nabla_\mu J^\mu + (w - d)\mathcal{A}_\mu J^\mu = 0\end{aligned}$$

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Cute, But why should you be interested ?

- AdS/CFT correspondence : Many CFT_d s have dual AdS_{d+1} gravity models.
- CFT_d Hydrodynamic states dual to Large AdS_{d+1} Blackholes.
- Coming Next : What is this duality and how does it work ?

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AdS Kerr Blackholes I

- Take the AdS_{d+1} Kerr blackholes constructed by

G.W. Gibbons, H. Lu, D.N. Page, C.N. Pope [hep-th/0404008]

- Goto Eddington Finkelstein like co-ordinates and recast the metric in the form

$$\begin{aligned}
 ds^2 = & -2u_\mu dx^\mu (dr + r \mathcal{A}_\nu dx^\nu) \\
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where b is related to the mass of the blackhole and u^μ looks like a velocity field of a fluid rotating in S^{d-1} !

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- Calculate the dual stress tensor following AdS/CFT.
- We get

$$T_{\mu\nu} = p(g_{\mu\nu} + du_\mu u_\nu) + \text{Anomalous terms}$$

with $p \equiv \frac{1}{16\pi G_{\text{AdS}} b^d}$

- We identify the dual state as a hydrodynamic state with pressure p .

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Encoding hydrodynamics around Ingoing geodesics

- How is the boundary hydrodynamic data encoded in the bulk ?
- To the zeroth approximation, hydrodynamic data on a patch in the boundary determines the metric around an ingoing null-geodesic.
- This picture can be corrected successively in a boundary derivative expansion (See Prof. Shiraz Minwalla's talk before).

S. Bhattacharyya, V. E. Hubeny, S. Minwalla and M. Rangamani. [arXiv:0712.2456 \[hep-th\]](https://arxiv.org/abs/0712.2456)

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Dual metric in d dimensions I

S.Bhattacharyya, R.Loganayagam,I.Mandal,S.Minwalla,A.Sharma. arxiv : 0809.4272[hep-th]

$$\begin{aligned} ds^2 = & -2u_\mu dx^\mu (dr + r\mathcal{A}_\nu dx^\nu) \\ & + \left[r^2 g_{\mu\nu} + u_{(\mu} \mathcal{S}_{\nu)\lambda} u^\lambda - \omega_\mu{}^\lambda \omega_{\lambda\nu} \right] dx^\mu dx^\nu \\ & + \frac{1}{(br)^d} \left(r^2 - \frac{1}{2} \omega_{\alpha\beta} \omega^{\alpha\beta} \right) u_\mu u_\nu dx^\mu dx^\nu \\ & + 2(br)^2 F(br) \left[\frac{1}{b} \sigma_{\mu\nu} + F(br) \sigma_\mu{}^\lambda \sigma_{\lambda\nu} \right] dx^\mu dx^\nu \\ & - 2(br)^2 \left[K_1(br) \frac{\sigma_{\alpha\beta} \sigma^{\alpha\beta}}{d-1} P_{\mu\nu} + K_2(br) \frac{u_\mu u_\nu}{(br)^d} \frac{\sigma_{\alpha\beta} \sigma^{\alpha\beta}}{2(d-1)} \right. \\ & \quad \left. - \frac{L(br)}{(br)^d} u_{(\mu} P_{\nu)}^\lambda \mathcal{D}_\alpha \sigma^\alpha{}_\lambda \right] dx^\mu dx^\nu \\ & - 2(br)^2 H_1(br) \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta} \sigma^{\alpha\beta}}{d-1} P_{\mu\nu} \right. \\ & \quad \left. + C_{\mu\alpha\nu\beta} u^\alpha u^\beta \right] dx^\mu dx^\nu \\ & + 2(br)^2 H_2(br) \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\mu\lambda} \right] dx^\mu dx^\nu \end{aligned}$$

Dual metric in d dimensions II

$$F(br) \equiv \int_{br}^{\infty} \frac{y^{d-1} - 1}{y(y^d - 1)} dy$$

$$H_1(br) \equiv \int_{br}^{\infty} \frac{y^{d-2} - 1}{y(y^d - 1)} dy$$

$$H_2(br) \equiv \frac{1}{2} F(br)^2 - \int_{br}^{\infty} \frac{d\xi}{\xi(\xi^d - 1)} \int_1^{\xi} \frac{y^{d-2} - 1}{y(y^d - 1)} dy$$

$$K_1(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \int_{\xi}^{\infty} dy y^2 F'(y)^2$$

$$K_2(br) \equiv \int_{br}^{\infty} \frac{d\xi}{\xi^2} \left[1 - \xi(\xi - 1)F'(\xi) - 2(d-1)\xi^{d-1} \right. \\ \left. + \left(2(d-1)\xi^d - (d-2) \right) \int_{\xi}^{\infty} dy y^2 F'(y)^2 \right]$$

$$L(br) \equiv \int_{br}^{\infty} \xi^{d-1} d\xi \int_{\xi}^{\infty} dy \frac{y-1}{y^3(y^d - 1)}$$

Event Horizon dynamics

Find the event horizon by following the null generators from future null infinity. (as described in the previous talk by Prof. Shiraz Minwalla.)

S. Bhattacharyya, V.E. Hubeny, R. Loganayagam, G. Mandal, S. Minwalla, T. Morita, M. Rangamani, H.S. Reall arxiv:

0803.2526[hep-th]

$$r_H = \frac{1}{b} + b \left(h_1 \sigma_{\alpha\beta} \sigma^{\alpha\beta} + h_2 \omega_{\alpha\beta} \omega^{\alpha\beta} + h_3 \mathcal{R} \right) + \dots$$

where
$$h_1 = \frac{2(d^2 + d - 4)}{d^2(d-1)(d-2)} - \frac{K_{2H}}{d(d-1)}$$

$$h_2 = -\frac{d+2}{2d(d-2)} \quad \text{and} \quad h_3 = -\frac{1}{d(d-1)(d-2)}$$

with
$$K_{2H} = \int_1^\infty \frac{d\xi}{\xi^2} \left[1 - \xi(\xi-1)F'(\xi) - 2(d-1)\xi^{d-1} \right. \\ \left. + 2 \left((d-1)\xi^d - (d-2) \right) \int_\xi^\infty dy y^2 F'(y)^2 \right]$$

Matches with the event horizons of the AdS_{d+1} Kerr metrics.

Entropy current from the event horizon

Pullback of the Horizon area form gives the entropy current.

$$4G_{AdS}b^{d-1}J_S^\mu = u^\mu + b^2u^\mu \left[A_1 \sigma_{\alpha\beta}\sigma^{\alpha\beta} + A_2 \omega_{\alpha\beta}\omega^{\alpha\beta} + A_3 \mathcal{R} \right] \\ + b^2 \left[B_1 \mathcal{D}_\lambda \sigma^{\mu\lambda} + B_2 \mathcal{D}_\lambda \omega^{\mu\lambda} \right] + \dots$$

with

$$A_1 = \frac{2}{d^2}(d+2) - \frac{K_{1H}d + K_{2H}}{d}, \quad A_2 = -\frac{1}{2d}, \quad B_2 = \frac{1}{d-2}$$

$$B_1 = -2A_3 = \frac{2}{d(d-2)}$$

$$K_{1H}d + K_{2H} = \int_1^\infty \frac{d\xi}{\xi^2} \left[1 - \xi(\xi-1)F'(\xi) - 2(d-1)\xi^{d-1} \right. \\ \left. + 2\left((d-1)\xi^d + 1\right) \int_\xi^\infty dy y^2 F'(y)^2 \right]$$

$$\text{and } T\mathcal{D}_\mu J_S^\mu = 2\eta \left[\sigma_{\mu\nu} + \frac{1}{2} (A_1 b d + \tau_\omega) u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} \right]^2 + \dots \geq 0.$$

Hydrodynamic energy - momentum tensor from gravity

$$T_{\mu\nu} = p(g_{\mu\nu} + du_\mu u_\nu) - 2\eta\sigma_{\mu\nu} - 2\eta\tau_\omega \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \omega_\mu{}^\lambda \sigma_{\lambda\nu} + \omega_\nu{}^\lambda \sigma_{\mu\lambda} \right] + 2\eta b \left[u^\lambda \mathcal{D}_\lambda \sigma_{\mu\nu} + \sigma_\mu{}^\lambda \sigma_{\lambda\nu} - \frac{\sigma_{\alpha\beta} \sigma^{\alpha\beta}}{d-1} P_{\mu\nu} + C_{\mu\alpha\nu\beta} u^\alpha u^\beta \right]$$

with

$$b = \frac{d}{4\pi T} \quad ; \quad p = \frac{1}{16\pi G_{\text{AdS}} b^d}$$

$$\eta = \frac{s}{4\pi} = \frac{1}{16\pi G_{\text{AdS}} b^{d-1}} \quad \text{and} \quad \tau_\omega = b \int_1^\infty \frac{y^{d-2} - 1}{y(y^d - 1)} dy$$

Universal relations between transport coefficients - how universal are they ?

Universal forms

- Some universal relations among transport coefficients hold true in any two derivative theory with scalars and abelian vectors.

M.Haack and A.Yarom arXiv:0811.1794 [hep-th]

- What other universal relations lie behind gravity and what do they imply ?
- Can we construct an effective mesoscopic model with some degrees of freedom at the boundary which reproduces these forms for energy momentum transport ?
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Novelties : Anomalous transport

- Now, will turn to an interesting phenomena in hydrodynamics with a global charge.
- Focus on charged solutions with Einstein-Maxwell-Chern-Simons theory in the bulk.
- General AdS_5 dual is known : Solution too long to be presented here

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Defining the gravity theory

- Focus on two-derivative theory of gravity in five dimensions with asymptotically AdS boundary conditions
- Interested in matter content that allows consistent truncation to Einstein-Maxwell Chern-Simons system
- E.g. IIB SUGRA in $\text{AdS}_5 \times S^5$ the equal R-charge sector.
- Truncated action is

$$S = \frac{1}{16\pi G_{\text{AdS}}} \int \left[\sqrt{-g_5} (R + 12) - \frac{1}{2} \mathbf{F} \wedge *_5 \mathbf{F} + \frac{2\kappa}{3} \mathbf{A} \wedge \mathbf{F} \wedge \mathbf{F} \right]$$

with $\kappa = (2\sqrt{3})^{-1}$ for the IIB SUGRA.

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R-charged Blackholes in hydrodynamic coordinates (Charge frame velocity)

General Blackhole Solution with mass m , charge q , Angular Velocities ω_1, ω_2

Z. W. Chong, M. Cvetič, H. Lu and C. N. Pope . arXiv:hep-th/0506029

$$\begin{aligned}
 ds^2 = & -2u_\mu dx^\mu (dr + r \mathcal{A}_\nu dx^\nu) \\
 & + \left[r^2 g_{\mu\nu} + u_{(\mu} \mathcal{S}_{\nu)\lambda} u^\lambda - \omega_\mu{}^\lambda \omega_{\lambda\nu} \right] dx^\mu dx^\nu \\
 & + \left[\left(\frac{2m}{\rho^2} - \frac{q^2}{\rho^4} \right) u_\mu u_\nu + \frac{q}{2\rho^2} u_{(\mu} l_{\nu)} \right] dx^\mu dx^\nu \\
 \mathbf{A} = & \frac{\sqrt{3}q}{\rho^2} u_\mu dx^\mu \quad ; \quad \rho^2 \equiv r^2 + \frac{1}{2} \omega_{\alpha\beta} \omega^{\alpha\beta} \quad ; \quad l_\mu \equiv \epsilon_{\mu\nu\lambda\sigma} u^\nu \omega^{\lambda\sigma}
 \end{aligned}$$

Can be reproduced in order by order derivative expansion -
checked upto first order against the general solution.

The Anomalous transport

- Calculate the stress tensor

$$T_{\mu\nu} = p(g_{\mu\nu} + 4u_\mu u_\nu) + \frac{n}{\sqrt{3}} l_{(\mu} u_{\nu)} \\
+ \frac{1}{64\pi G_{\text{AdS}}} \left(R^{\alpha\beta} R_{\alpha\mu\beta\nu} - \frac{R^2}{12} g_{\mu\nu} \right) \\
J_\mu = nu_\mu \quad \text{where} \quad l_\mu \equiv \epsilon_{\mu\nu\lambda\sigma} u^\nu \omega^{\lambda\sigma} ; \\
p \equiv \frac{m}{8\pi G_{\text{AdS}}} \quad \text{and} \quad n \equiv \frac{\sqrt{3}q}{8\pi G_{\text{AdS}}}$$

- Question : What is the new term in the stress tensor ?
- Suggests an additional anomalous transport of energy in $\mathcal{N}=4$ SYM hydrodynamics.

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More about the anomalous transport

- A rotating $\mathcal{N}=4$ SYM fluid (charged equally under all three cartans of $\text{SO}(6)$) has an energy flow along the axis of rotation !
- This term can be traced back to the bulk Chern-Simons coupling .
- Is it related to the R-charge anomaly ?(Note that R-charge field strength in this case is zero- so the anomaly is turned off.)
- This term is crucial for the correct thermodynamics of the rotating blackholes
- How do we understand it directly from the boundary ?

S. Bhattacharyya, S. Lahiri, R. Loganayagam and S. Minwalla arXiv:0708.1770 [hep-th]

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 - 1 Metric dual to arbitrary fluid configurations(Uncharged, Arbitrary dim.) known. Charged duals known for some cases.
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