Multiple M2-Brane Dynamics

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Introduction

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Its been a good year for multiple M2-branes.

Common things said in the past:

- No known suitable Lagrangian (no longer true)
- No weak coupling parameter (in fact one can arise)

We now have a complete and convincing proposal for the effective Lagrangian of n M2-branes in $\mathbb{R}^8/\mathbb{Z}_k$ for arbitrary k and n

- Lagrangians are new types of highly supersymmetric Chern-Simons matter theories in $D = 3$.
  - Constructed from a triple product rather than a Lie-bracket
- Hopefully this will lead to a big increase in our understanding of M-theory beyond supergravity
  - M2-brane CFT's 'define' M-theory in asymptotically AdS_4 backgrounds

PLAN:

Here I will aim to review the construction of the various Lagrangians and discuss some of their properties.

- Proceed in a roughly chronological and pedagogical manner.
- Many interesting and important papers and topics will not be discussed in great detail
  - *e.g.* Details of Lorentzian $\mathcal{N} = 8$ models
  - *e.g.* Detailed aspects of $AdS_4 \times \mathbb{CP}^3$ CFT duals
  - and several crazy and/or cool ideas.

Contents

Motivation

Some thought-provoking papers:

[Basu and Harvey] considered the BPS state of n coincident M2-branes ending on an M5-brane,

- They propose the BPS equation

$$\frac{dX^I}{d(x^2)} = \frac{M^3}{64\pi}\epsilon^{IJKL}[G, X^J, X^K, X^L]$$

  where $[A, B, C, D] = \frac{1}{4!}(ABCD \pm \text{cyclic combinations})$
  - Analogous to Nahm's equation
  - But did not arise as a $1/2$ susy solution of any Lagrangian

[Schwarz] looked for 3D superconformal Chern-Simons gauge theories in with no dynamical vectors

- only found $\mathcal{N} = 2$, *i.e.* 4 susys

$\mathcal{N} = 8$

What is our wish list for a theory of multiple M2-branes?

- 3D field theory with 16 susys ($\mathcal{N} = 8$)
- 8 dynamical scalars with an $SO(8)$ R-symmetry
- no dynamical gauge field
- Parity invariant
- Conformal invariance

$\mathcal{N} = 8$

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So just start from scratch:

A stack of M2-branes has 8 scalars X^I and their fermionic superpartners Ψ, $\Gamma_{012}\Psi = -\Psi$.

- We assume that these take values in some vector space

$\mathcal{N} = 8$

A natural guess for the susy algebra is, ignoring gauge symmetries, [Bagger, NL]

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \partial_\mu X^I \Gamma^\mu \Gamma^I \epsilon + [X^I, X^J, X^K]\Gamma^{IJK}\epsilon, \end{aligned}$$

where $[A, B, C]$ is totally anti-symmetric triple product.

- So our vector space needs a triple product: 3-algebra

This immediately gives a BPS Basu-Harvey equation for an M2 ending on an M5-brane:

$$\frac{dX^I}{d(x^2)} = \epsilon^{IJKL}[X^J, X^K, X^L]$$

$\mathcal{N} = 8$

Closure of the algebra implies a gauge symmetry:

$$[\delta_1, \delta_2]X^I = 2i\bar{\epsilon}_1\Gamma^\mu\epsilon_2\partial_\mu X^I + 2i\bar{\epsilon}_1\Gamma^{JK}\epsilon_2[X^I, X^J, X^K]$$

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This must be dealt with to realize the full superalgebra
We will proceed by introducing a basis T^a for $\mathcal{A}$.

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d \ , \qquad f^{abc}{}_d = f^{[abc]}{}_d$$

so

$$\delta X^I_d = \Lambda_{ab} f^{cab}{}_d X^I_c$$

and introduce the covariant derivative:

$$D_\mu X^I_c = \partial_\mu X^I_c - \tilde{A}_\mu{}^c{}_d X^I_c$$

$\mathcal{N} = 8$

Full superalgebra takes the form [Bagger, NL]

$$\begin{aligned} \delta X^I_d &= i\bar{\epsilon}\Gamma^I\Psi_d \\ \delta\Psi_d &= D_\mu X^I_d \Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X^I_a X^J_b X^K_c f^{abc}{}_d\Gamma^{IJK}\epsilon \\ \delta\tilde{A}_\mu{}^c{}_d &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X^I_a\Psi_b f^{abc}{}_d, \end{aligned}$$

Indeed this closes (on-shell) if f^{abcd} satisfies the fundamental identity:

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0.$$

This ensures that the gauge symmetries $\delta_\Lambda X^I_d = \Lambda_{ab} f^{cab}{}_d X^I_c$ generated by the triple product are those of a Lie-algebra with matrix representatives $\tilde{\Lambda}^c{}_d = \Lambda_{ab} f^{cab}{}_d$ acting on X^I_d.

- **N.B.** Closure was obtained first by [Gustavsson] using, but equivalent algebraic approach that gives closure

$\mathcal{N} = 8$

The invariant Lagrangian is a Chern-Simons theory [Bagger, NL]:

$$\begin{aligned}\mathcal{L} \;=\;& -\frac{1}{2}(D_\mu X^{aI})(D^\mu X^I_a) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X^I_c X^J_d \Psi_a f^{abcd} \\ & + \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right) \\ & - \frac{1}{12}\mathrm{Tr}([X^I, X^J, X^K])^2\end{aligned}$$

- Tr is an invariant trace (inner-product) on $\mathcal{A}$
- gauge invariance implies $f^{abcd} = f^{[abcd]}$
- $\tilde{A}_\mu{}^c{}_d = f^{abc}{}_d A_{\mu ab}$
- Chern-Simons term implies $f^{abc}{}_d$ is quantized

$\mathcal{N} = 8$

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Has all the expected symmetries of multiple M2-branes: 16 susys, $SO(8)$ R-symmetry, Parity (f^{abcd} is parity odd).

No continuous free parameter but weakly coupled as $f^{abc}{}_d \to 0$

$\mathcal{N} = 8$

If Tr is positive definite then there is only one finite-dimensional possibility [Nagy],[Gauntlett, Gutowski],[Papadopoulos]:

$$f^{abcd} = \frac{2\pi}{k}\varepsilon^{abcd}$$

Although examples with an infinite dimensional 3-algebra arise from the Nambu bracket.

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Although examples with an infinite dimensional 3-algebra arise from the Nambu bracket.

In this case the Lagrangian is that of an $SU(2) \times SU(2)$ Chern-Simons theory coupled to matter in the bi-fundamental.

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{4\pi}\mathrm{tr}(\tilde{A}^+ \wedge d\tilde{A}^+ + \frac{2}{3}\tilde{A}^+ \wedge \tilde{A}^+ \wedge \tilde{A}^+) \\ &\quad -\frac{k}{4\pi}\mathrm{tr}(\tilde{A}^- \wedge d\tilde{A}^- + \frac{2}{3}\tilde{A}^- \wedge \tilde{A}^- \wedge \tilde{A}^-) \end{aligned}$$

- quantization condition implies $k \in \mathbf{Z}$
- $f^{abcd} \leftrightarrow -f^{abcd}$ corresponds to switching the two $SU(2)$'s

$\mathcal{N} = 8$

What, if any, is the multiple M2-brane interpretation?

Look at the Vacuum moduli space [NL, Tong], [Distler, Mukhi, Papageorgakis, van Raamsdonk]

$$\mathcal{M}_k = \mathbb{R}^{16}/D_{2k}$$

- D_{2k} - dihedral group
- $\mathcal{M}_1 = \mathbb{R}^8/\mathbb{Z}_2 \times \mathbb{R}^8/\mathbb{Z}_2$
  - vacuum moduli space of an $SO(4)$ gauge theory
- $\mathcal{M}_2 = \left(\mathbb{R}^8/\mathbb{Z}_2 \times \mathbb{R}^8/\mathbb{Z}_2\right)/\mathbb{Z}_2$
  - vacuum moduli space of an $SO(5)$ gauge theory

Two 2-branes on $\mathbb{R}^8/\mathbb{Z}_2$

No clear picture for $k > 2$ (although for $k = 3$ one finds the vacuum moduli space of a G_2 gauge theory).

$\mathcal{N} = 8$

What does one expect for two M2-branes on orbifold $\mathbb{R}^8/\mathbb{Z}_2$?

- $\mathcal{N} = 8$, $SO(8)$ R-symmetry and parity
- two possible orbifolds depending on the value of discrete torsion [Sethi],[Berkooz,Kapustin]:
  - $O(4)$ gauge group
  - $SO(5)$ gauge group
- The $SO(5)$ agrees with what we find for $k = 2$
- The $k = 1$ $SO(4)$ case should be $O(4)$.

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  - $SO(5)$ gauge group
- The $SO(5)$ agrees with what we find for $k = 2$
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So the bottom line is that it's not clear what the interpretation is

- for $k = 2$ there is agreement with M-theory.

$\mathcal{N} = 8$

What is the relation to D2-branes and SYM [Mukhi,Papageorgakis]?

- Give a vev to a field, *e.g.* $v = \langle X_4^8 \rangle$ and expand

$$\mathcal{L} = k\mathcal{L}_0(X_4^I) + \frac{k}{v^2}\mathcal{L}_{SU(2)\ SYM}(X_{a\neq 4}^{I\neq 8}, A_\mu) + \mathcal{O}(kv^{-3})$$

- scalars $X_{a\neq 4}^8$ become a dynamical $SU(2)$ connection A_μ.
- identify $v^2 = kg_{YM}^2$

So far out on the Coulomb branch we recover SYM theory and a perturbative limit as $k \to \infty$, $v \to \infty$.

$\mathcal{N} = 8$

Clearly one needs to generalize these models:

There are infinitely many models with a Lorentzian signature metric [Gomis, Milanesi, Russo], [Benvenuti, Rodriguez-Gomez, Tonni, Verlinde], [Ho, Imamura, Matuso]:

- The resulting Lagrangian (same as above) has a $B \wedge F$ form and no free parameters

Can gauge away the negative mode [Bandres, Lipstein, Schwarz], [Gomis, Rodriguez-Gomez, van Raamsdonk, Verlinde]

- Classically equivalent to $\mathcal{N} = 8$ SYM but with manifest $SO(8)$ R-symmetry and conformal invariance which are spontaneously broken: $g^2_{YM} = \langle X^I_+ \rangle$ (see also [Ezhuthachan,Mukhi,Papageorgakis])

Won't discuss anymore here: See Sunil's talk.

$\mathcal{N} = 6$

The existence of an orbifold, whatever it may be, gives the weak coupling expansion that leads to a Lagrangian formulation

- Hence to proceed we need to look for a suitable orbifold:

There is an $\mathbb{R}^8/\mathbb{Z}_k$ orbifold that preserves 12 susys ($\mathcal{N} = 6$)

$$\begin{pmatrix} Z^1 \\ Z^2 \\ Z^3 \\ Z^4 \end{pmatrix} \sim \begin{pmatrix} \omega & & & \\ & \omega & & \\ & & \omega^{-1} & \\ & & & \omega^{-1} \end{pmatrix} \begin{pmatrix} Z^1 \\ Z^2 \\ Z^3 \\ Z^4 \end{pmatrix} \qquad \omega = e^{2\pi i/k}$$

- $SO(8) \rightarrow SU(4) \times U(1)$

$\mathcal{N} = 6$

The key construction is by [Aharony, Bergman, Jafferis and Maldacena]

- Only impose $\mathcal{N} = 6$ and an $SU(4) \times U(1)$ R-symmetry in the Lagrangian.
- Constructed $U(n) \times U(n)$ Chern-Simons Matter theories at level $(k, -k)$
- Vacuum moduli space is $\mathrm{Sym}^n(\mathbb{R}^8/\mathbb{Z}_k)$
- Describes n M2-branes in this $\mathbb{R}^8/\mathbb{Z}_k$ orbifold.

$\mathcal{N} = 6$

$$
\begin{array}{ll|ccccccccc}
 & & & & & & & & & & \\
\mathrm{IIB} & D3: & 1 & 2 & 3 & & & & & & \\
 & NS5: & 1 & 2 & & 4 & 5 & 6 & & & \\
 & (1,k)5: & 1 & 2 & & & & & 7 & 8 & 9
\end{array}
$$

$$\Downarrow \mathrm{T-duality\ along}\ x^3$$

$$
\begin{array}{llccccccccc}
\mathrm{IIA} & D2: & 1 & 2 & & & & & & & \\
 & KK: & & & \hat{3} & & & & 7 & 8 & 9 \\
 & KK/D6: & & & \hat{3} & 4_\theta & 5_\theta & 6_\theta & 7_\theta & 8_\theta & 9_\theta
\end{array}
$$

$$\Downarrow \mathrm{lift\ to\ M-theory}$$

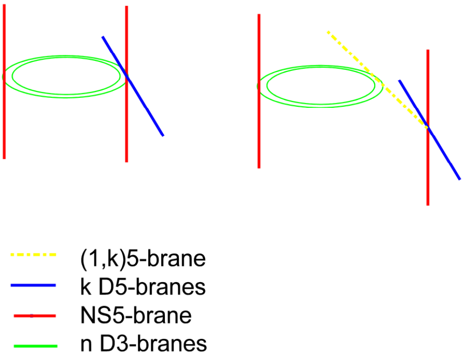
$$
\begin{array}{llcccccccccc}
\mathrm{M-theory} & M2: & 1 & 2 & & & & & & & & \\
 & KK: & & & \hat{3} & & & & 7 & 8 & 9 & \hat{10} \\
 & KK: & & & \hat{3} & 4_\theta & 5_\theta & 6_\theta & 7_\theta & 8_\theta & 9_\theta & \hat{10}
\end{array}
$$

$\mathcal{N} = 6$

The final configuration is just n M2s in a non-trivial curved background preserving 3/16 susys.

- Metric can be written explicitly [Gauntlett, Gibbons,Papadopoulos, Townsend]
- smooth except where the centre's ($U(1)$ fixed points) intersect
- Taking the near horizon limit gives n M2's in $\mathbb{R}^8/\mathbb{Z}_k$.
- Fraction of preserved susy's is enhanced to 12/16.

$\mathcal{N} = 6$



In terms of the D3-brane SYM worldvolume theory:

- Integrating out $D5 - D3$-strings and flowing to IR gives a $U(n) \times U(n)$ CS theory with level $(k, -k)$ coupled to bi-fundamental matter.
- $\mathcal{N} = 3$ is enhanced to $\mathcal{N} = 6$

$\mathcal{N} = 6$

This construction can be further generalized to include discrete torsion $H_4(\mathbb{R}^8/\mathbb{Z}_k) = \mathbb{Z}^k$ [Aharony, Bergman, Jafferis]:

- $U(m) \times U(n)$ CS theory with level $(k, -k)$ coupled to bi-fundamental matter
- conjectured that $|m - n| \leq k$
  -*e.g.* $n = m$, $n = m + 1$,...,$n = m + k - 1$
  -$n = m + k$ is equivalent to $n = m$
  -always strongly coupled

$\mathcal{N} = 6$

N.B. In these models spacetime states that carry non-trivial $U(1)$ charges are represented by non-local operators made of of Wilson lines.

$$
\begin{aligned}
e.g. \qquad \mathcal{O} &= tr(Z^A Z_B^\dagger ....) \qquad \text{has } U(1) \text{ charge } 0 \\
\mathcal{O} &= tr(Z^A Z^B ....) \qquad \text{is not gauge invariant}
\end{aligned}
$$

- For $k = 1, 2$ there should be enhanced $\mathcal{N} = 8$ supersymmetry that is not manifest
- For $k = 1$ there is a free translational mode that is not manifest.

$\mathcal{N} = 6$

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For the AdS_4 dual one can write S^7 as a Hopf fibration over $\mathbb{CP}^3$

- $\mathbb{Z}_k$ acts only on the S^1 fibres
- Can reduce to $AdS_4 \times \mathbb{CP}^3$ vacua of type IIA
- $\lambda_{'t\ Hooft} = n/k$

$\mathcal{N} = 6$

The [MP] relation to SYM can be viewed as moving all M2's far away from the fixed point so that

$$\mathbb{R}^8/\mathbb{Z}_k \underbrace{\longrightarrow}_{v \to \infty} \mathbb{R}^7 \times S^1/\mathbb{Z}_k \underbrace{\longrightarrow}_{k \to \infty} \mathbb{R}^7$$

i.e. compactification to type IIA in the large v, k limit

$\mathcal{N} = 6$

Following the logic of the $\mathcal{N} = 8$ construction let us obtain the most general Lagrangian with $\mathcal{N} = 6$ susy, conformal invariance and an $SU(4) \times U(1)$ R-symmetry.

- scalars $Z^A_a \in \mathbf{4}_1$ of $SU(4) \times U(1)$
  -complex conjugates $\bar{Z}_{A\bar{a}} \in \bar{\mathbf{4}}_{-1}$
- fermions $\psi_{Aa} \in \bar{\mathbf{4}}_1$ of $SU(4) \times U(1)$
- susys $\epsilon_{AB} \in \mathbf{6}_0$ of $SU(4) \times U(1)$
  - $(\epsilon_{AB})^* = \epsilon^{AB} = \frac{1}{2}\varepsilon^{ABCD}\epsilon_{CD}$
- complex conjugation raises/lowers and A-index and flips the $U(1)$ charge
- Trace form $h^{\bar{a}b} = \mathrm{Tr}(T^{\bar{a}}, T^b)$

$\mathcal{N} = 6$

Starting from the most general form for the susy's one finds [Bagger, NL]

$$
\begin{aligned}
\delta Z^A_d &= i\bar{\epsilon}^{AB}\psi_{Bd} \\
\delta\psi_{Bd} &= \gamma^\mu D_\mu Z^A_d \epsilon_{AB} + f^{ab\bar{c}}{}_d Z^C_a Z^A_b \bar{Z}_{C\bar{c}}\epsilon_{AB} + f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}}\epsilon_{CD} \\
\delta\tilde{A}_\mu{}^c{}_d &= -i\bar{\epsilon}_{AB}\gamma_\mu Z^A_a \psi^B_{\bar{b}} f^{ca\bar{b}}{}_d + i\bar{\epsilon}^{AB}\gamma_\mu \bar{Z}_{A\bar{b}}\psi_{Ba} f^{ca\bar{b}}{}_d
\end{aligned}
$$

Provided that

$$
f^{ab\bar{c}}{}_e f^{ef\bar{g}}{}_d = f^{af\bar{g}}{}_e f^{eb\bar{c}}{}_d + f^{bf\bar{g}}{}_e f^{ae\bar{c}}{}_d - f_{\bar{e}}{}^{f\bar{g}\bar{c}} f^{ab\bar{e}}{}_d
$$

and

$$
f^{ab\bar{c}\bar{d}} = -f^{ba\bar{c}\bar{d}}, \qquad f^{*\bar{c}\bar{d}ab} = f^{ab\bar{c}\bar{d}}.
$$

N.B Recover the $\mathcal{N} = 8$ theory when f^{abcd} is real and totally anti-symmetric

$\mathcal{N} = 6$

The Lagrangian has a similar form to the $\mathcal{N} = 8$ case [Bagger, NL]:

$$\begin{aligned}\mathcal{L} &= -D^\mu \bar{Z}^a_A D_\mu Z^A_a - i\bar{\psi}^{Aa}\gamma^\mu D_\mu \psi_{Aa} - \Upsilon^{CD}_{Bd}\bar{\Upsilon}^{Bd}_{CD} + \mathcal{L}_{CS} \\ &\quad -if^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi_{Aa}Z^B_b\bar{Z}_{B\bar{c}} + 2if^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi_{Ba}Z^B_b\bar{Z}_{A\bar{c}} \\ &\quad +\frac{i}{2}\varepsilon_{ABCD}f^{ab\bar{c}\bar{d}}\bar{\psi}^A_{\bar{d}}\psi^B_{\bar{c}}Z^C_a Z^D_b - \frac{i}{2}\varepsilon^{ABCD}f^{cd\bar{a}\bar{b}}\bar{\psi}_{Ac}\psi_{Bd}\bar{Z}_{C\bar{a}}\bar{Z}_{D\bar{b}}\end{aligned}$$

where

$$\Upsilon^{CD}_{Bd} = f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}} - \frac{1}{2}\delta^C_B f^{ab\bar{c}}{}_d Z^E_a Z^D_b \bar{Z}_{E\bar{c}} + \frac{1}{2}\delta^D_B f^{ab\bar{c}}{}_d Z^E_a Z^C_b \bar{Z}_{E\bar{c}}.$$

and

$$\mathcal{L}_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{ab\bar{c}\bar{d}}A_{\mu\bar{c}b}\partial_\nu A_{\lambda\bar{d}a} + \frac{2}{3}f^{ac\bar{d}}{}_g f^{ge\bar{f}\bar{b}}A_{\mu\bar{b}a}A_{\nu\bar{d}c}A_{\lambda\bar{f}e}\right).$$

$\mathcal{N} = 6$

As before $f^{ab\bar{c}}{}_d$ also defines a triple product:

$$[X, Y; \bar{Z}]_d = f^{ab\bar{c}}{}_d X_a Y_b \bar{Z}_{\bar{c}}$$

- gauge symmetry $\delta_\Lambda Z_d^A = \Lambda_{\bar{c}b} f^{ab\bar{c}}{}_d Z_a^A$ acts as a derivation

One natural class of solutions: Let X, Y, Z be complex $n \times m$ matrices

$$[X, Y; \bar{Z}] = \frac{2\pi}{k}(XZ^\dagger Y - YZ^\dagger X)$$

- Gauge symmetry is $\delta X = MX - XN$ with $M \in u(m)$ and $N \in u(n)$
  - $SU(m) \times SU(n)$ Chern-Simons with matter in bi-fundamental
- gives the [ABJM] and [ABJ] models by gauging the $U(1)$ global symmetry

$\mathcal{N} = 6$

$$
\begin{aligned}
\mathcal{L} \;=\; & -\mathrm{tr}(D^\mu Z_A^\dagger D_\mu Z^A) - i\,\mathrm{tr}(\bar\psi^{A\dagger}\gamma^\mu D_\mu \psi_A) - V + \mathcal{L}_{CS} \\
& - i\lambda\,\mathrm{tr}(\bar\psi^{A\dagger}\psi_A Z_B^\dagger Z^B - \bar\psi^{A\dagger} Z^B Z_B^\dagger \psi_A) \\
& + 2i\lambda\,\mathrm{tr}(\bar\psi^{A\dagger}\psi_B Z_A^\dagger Z^B - \bar\psi^{A\dagger} Z^B Z_A^\dagger \psi_B) \\
& + i\lambda\varepsilon_{ABCD}\,\mathrm{tr}(\bar\psi^{A\dagger} Z^C \psi^{B\dagger} Z^D) - i\lambda\varepsilon^{ABCD}\,\mathrm{tr}(Z_D^\dagger \bar\psi_A Z_C^\dagger \psi_B)\ .
\end{aligned}
$$

with $\mathcal{L}_{CS} = k\mathcal{L}_{CS}(u(n)) - k\mathcal{L}_{CS}(u(m))$ and

$$
V = \mathrm{tr}\left|[Z^A, Z^B; \bar Z_C] - \frac{1}{2}\delta_C^A [Z^B, Z^E, \bar Z_E] + \frac{1}{2}\delta_C^B [Z^A, Z^E, \bar Z_E]\right|^2
$$

see also [Benna,Klebanov,Klose,Smedback], [Hosomichi,Lee,Lee,Lee,Park], [Bandres,Lipstein,Schwarz]

N.B. there are other possibilities: $U(1) \times Sp(2n)$ [HLLLP]
- classified by [Schnabl, Tachikawa].

Mass Deformations

Turning on a background 4-form flux should lead to a 'Myers' effect and induce mass terms preserving all supersymmetry.

- Vacua correspond to M2's 'blown-up' into M5's on fuzzy S^3 [Benna],[Bagger,NL]

There is a deformation of the $\mathcal{N} = 8$ theory that breaks $SO(8) \rightarrow SO(4) \times SO(4)$ [Gomis, Salim, Passerini], [Hosomichi,Lee and Lee]

Let us look for mass deformations of $\mathcal{N} = 6$

- one mass breaks $SU(4) \times U(1) \rightarrow SU(2) \times SU(2) \times U(1)$ [HLLLP], [Gomis,Rodriguez-Gomez,van Raamsdonk, Verlinde]

Mass Deformations

Start by considering the most general $U(1)$-preserving deformation of the fermion variation:

$$\delta_m \psi_{Aa} = \delta \psi_{Aa} + m_A{}^B \epsilon_{BD} Z_a^D + \epsilon_{AB} m'^B{}_C Z_a^C$$

δZ_a^A and $\delta \tilde{A}_\mu{}^c{}_d$ unchanged.

- Closure on Z_a^A must generate an $SU(4) \times U(1)$ rotation: $\Rightarrow m'^B{}_C = 0$
- Closure on $\tilde{A}_\mu{}^c{}_d$ must vanish $\Rightarrow (m^*)_A{}^B = m^B{}_A$
- Closure on ψ_{Ad} must generate an $SU(4) \times U(1)$ rotation on-shell: $\Rightarrow m^B{}_B = 0$ and

$$\begin{aligned} 0 = \gamma^\mu D_\mu \psi_{Cd} \quad &+ \quad f^{ab\bar{c}}{}_d \psi_{Ca} Z_b^D \bar{Z}_{D\bar{c}} - 2 f^{ab\bar{c}}{}_d \psi_{Da} Z_b^D \bar{Z}_{C\bar{c}} \\ &- \quad \varepsilon_{CDEF} f^{ab\bar{c}}{}_d \psi_c^D Z_a^E Z_b^F + m_C{}^D \psi_{Dd} \end{aligned}$$

Mass Deformations

So it would seem that we require m to be a traceless Hermitian matrix.

- However the variation of the Fermion equation of motion only gives a consistent Bosonic equations of motion if ${m_A}^B {m_B}^C = \mu^2 \delta_A^C$
- Up to an $SU(4)$ rotation we can take

$$ {m_A}^B = \mu \begin{pmatrix} 1_{2\times 2} & 0 \\ 0 & -1_{2\times 2} \end{pmatrix} $$

- Thus the $SU(2) \times SU(2)$ mass is the unique deformation
- This implies that the $SO(4) \times SO(4)$ deformation of the $\mathcal{N} = 8$ theory is also unique

Mass Deformations

The Lagrangian is the same as before with the fermionic mass $\mathcal{L}_m = i m_A{}^B \bar{\psi}^A_a \psi_B{}^a$ and

$$\begin{aligned} \Upsilon^{CD}_{Bd} &= f^{ab\bar{c}}{}_d Z^C_a Z^D_b \bar{Z}_{B\bar{c}} - \frac{1}{2}\delta^C_B f^{ab\bar{c}}{}_d Z^E_a Z^D_b \bar{Z}_{E\bar{c}} + \frac{1}{2}\delta^D_B f^{ab\bar{c}}{}_d Z^E_a Z^C_b \bar{Z}_{E\bar{c}} \\ &\quad + \frac{1}{2} m_B{}^D Z^C_d - \frac{1}{2} m_A{}^C Z^D_d \end{aligned}$$

- Vacuum solutions:

$$[Z^A, Z^B; \bar{Z}_C] = m_C{}^A Z^B - m_C{}^B Z^A$$

  constructed by [G,R-G,vR,V]
- Agrees with expectations at large k but more numerous than expected at finite k

Conclusions, Comments and Problems

Conclusions:

- There are Lagrangians to describe n M2's in $\mathbb{R}^8/\mathbb{Z}_k$ for any n, k.
  - including Lagrangians with $\mathcal{N} = 8$ and $SO(8)$ R-symmetry.
- Weak coupling arises by an orbifold
- Note that the Lagrangian for M2's in flat $\mathbb{R}^8$ is strongly coupled
  - Not all symmetries are manifest when $k = 1, 2$:
    - $\mathcal{N} = 6$ becomes $\mathcal{N} = 8$
    - For $k = 1$ there should be a free centre of mass mode
  - Quantum aspects can be very different from Classical predictions
    - *e.g.* degeneracy of massive vacua are not in agreement [G,R-G,vR,V]
    - Discrete quantities can change as a function of k

Conclusions, Comments and Problems

Comment: Is the 3-algebra structure important?

On the one hand:

- The Lagrangian can be expressed as a Chern-Simons gauge theory with matter fields whose interactions are made from matrix products.

On the other hand:

- Supersymmetry says that the entire Lagrangian is fixed by giving a 3-algebra
- Dynamical fields are controlled by a triple-product and not the Lie-bracket
- Higher derivative corrections of D2-branes are also determined by a 3-algebra when lifted to Lorentzian M2-brane Lagrangians [Alishahiha,Mukhi]

Problems:

- Understand the enhancement to $\mathcal{N}=8$ in [ABJM],[ABJ] and translational symmetry when $k=1$
- Understand the $d.o.f \sim f(\lambda)n^2$
  - at weak coupling $f \sim 1$ so $d.o.f \sim n^2$.
  - at strong coupling $f \sim 1/\sqrt{\lambda} = \sqrt{k/n}$ so $d.o.f \sim n^{\frac{3}{2}}$.
- What is the role of the $SU(2) \times SU(2)$, $\mathcal{N}=8$ Theory?
  - note that there are now two proposals for the $\mathbb{R}^8/\mathbb{Z}_2$ orbifold with discrete torsion
- What is the role of the Lorentzian $\mathcal{N}=8$ Theories

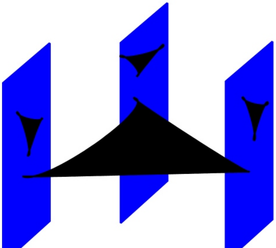
Conclusions, Comments and Problems

More generally: what can this tell us about M-theory? M5-branes? *e.g.* Consider the $SU(2) \times SU(2)$, $\mathcal{N} = 8$ model:

One finds that the mass formula for states in these vacua is [NL, Tong]

$$M = \frac{4\pi}{k} A \qquad A = \frac{1}{2} |z_1^7 \bar{z}_2^8 - z_1^8 \bar{z}_2^7|$$

- A is the area of the triangle with vertices on the M2's and the fixed point
- higher k orbifolds do preserve this area formula
  - Novel M-theory orbifold?



Conclusions, Comments and Problems

Note that we get an enhanced gauge symmetry whenever this triangle degenerates: the M2-branes become collinear.

- A similar effect happens for D2-branes in $D = 10$:
  - at the origin of the scalar moduli space there is an unbroken gauge symmetry and the branes are strongly coupled.
  - however they can be separated in the eleventh dimension (but are collinear)