Matrix Quantum Mechanics for the Black Hole Information Paradox

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w/ J. Polchinski: ***arXiv:0801.3657***,
w/ T. Okuda and J. Polchinski: ***arXiv:0808.0530,***
(w/ D.Kabat, G. Lifschytz, D. Lowe: hep-th/0108006)

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Plan of Talk

- (Introduction: The information paradox and motivation)
- Toy Model and its SD equation
- Results at zero temperature and non-zero temperature
- Conclusion I and discussion
- Further progress: another models with $1/N^2$ corrections
- Conclusion II

(how much we can cover each depends on time we have)

The main point I want to discuss today is;

In large N theory,

'1/N expansion around N=∞'

<u>*vs*</u> *' finite N '?*

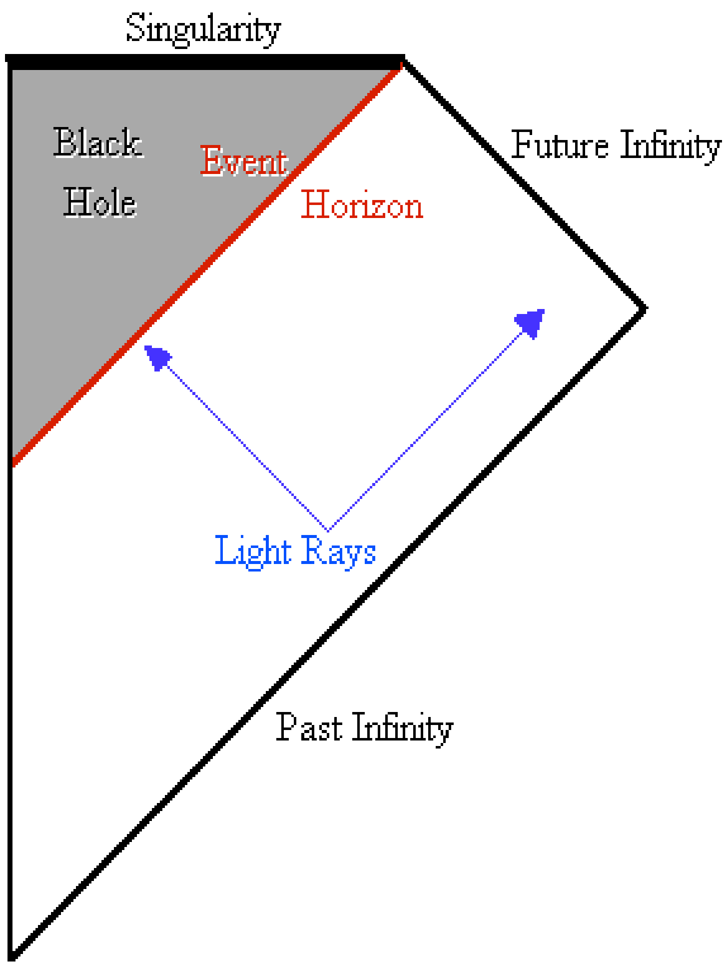
i.e.

' perturbative quantum gravity '

<u>*vs*</u> *' full quantum gravity '*

Introduction:
Black hole information paradox

Black hole evaporation & information paradox



- Before and after the formation of black hole horizon, the notion of `particles' & `vacuum' changes
- Original `vacuum' is no more vacuum after horizon formation
- It is perfectly thermal state actually black hole radiates as thermal black body radiation

Black hole evaporation & information paradox

- Hawking's argument is quite robust; it works as far as black hole horizon is formed by gravitational collapse
- Since after horizon is formed, the vacuum looks completely thermal, so black hole radiates thermally
- This thermal radiation is parameterized only by the temperature of black hole (surface gravity)
- Therefore, *all the information*, how black hole is formed etc, *are totally lost*, it seems that *pure states evolves into thermal or mixed states!?*
- But this contradicts with principles of quantum mechanics (QM): `unitarity'
- *"Do we have to give up some principle of QM once we include gravity?"*

<u>And three decades passed</u>

<u>without `*sharp*' answer</u>

<u>*Hawking*</u>: Quantum Gravity requires modification of quantum mechanics, time evolution is described by evolution of density matrix which allows pure into mixed.

$$\rho_{out} = S \rho_{in} S^\dagger$$

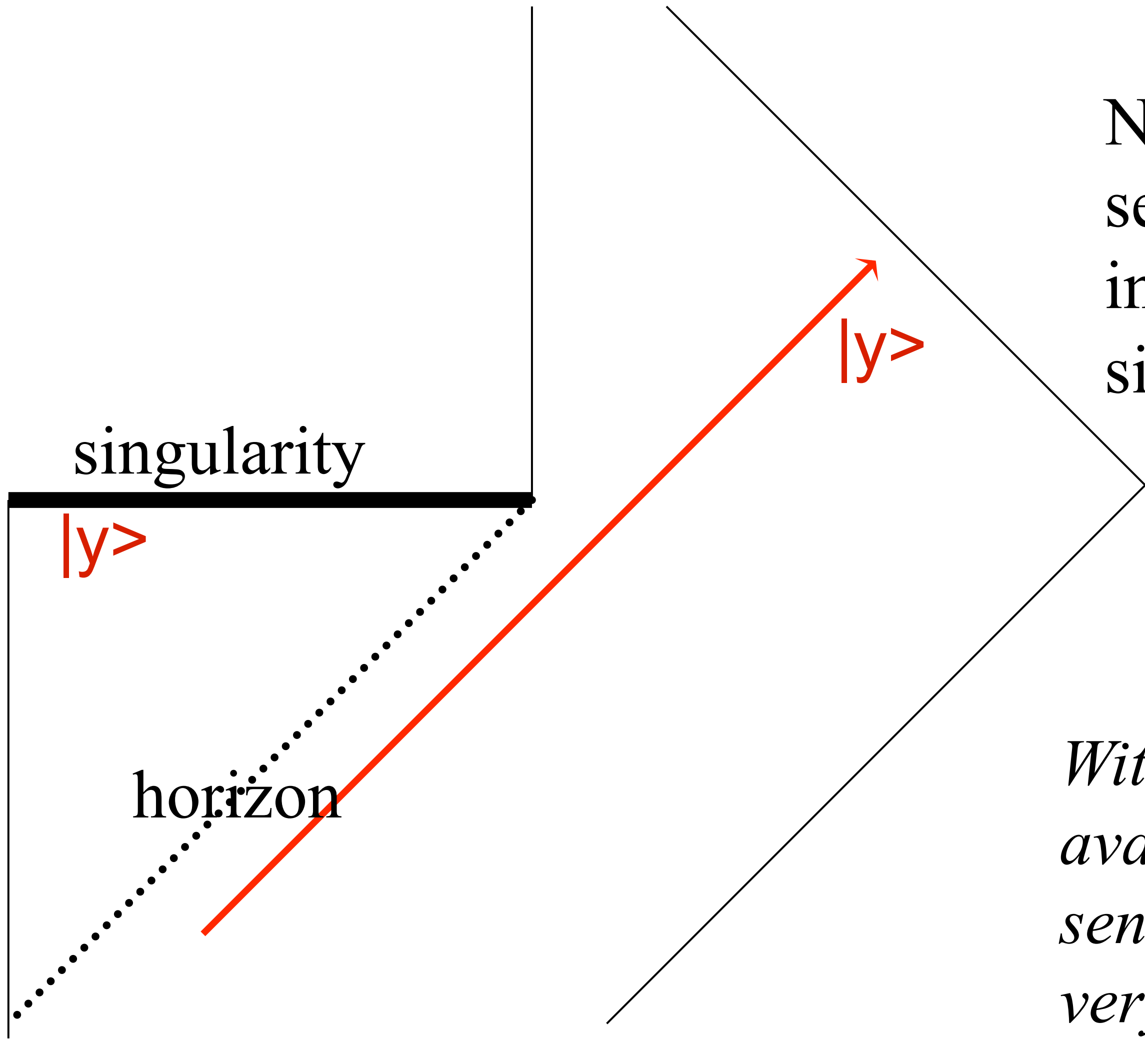
<u>*Difficulty of modifying QM*</u>

Hawking's theory can be shown to be equivalent to the QM system with random source term

$$\begin{aligned} H &= H_0 + J(t) Q \\ \text{with } J(t) J(t') &\sim \delta(t - t') \,. \end{aligned}$$

This randomness breaks time-translation invariance: energy-momentum is not conserved. (Banks, Peskin, Susskind)

And three decades passed without `*sharp*' answer



|y>->|y>|y>?
Quantum Xerox?

No observer can see physics of both inside and outside simultaneously

(Susskind)

$$\delta t \delta E \sim \hbar$$

With only low energy available, we cannot send the information in very short timescale.

<u>New development of non-pert. quantum gravity</u>

- Discovery of gauge/gravity dual (AdS/CFT)
- AdS/CFT => string theory (as quantum gravity) in <u>*asymptotic*</u> anti de Sitter (AdS) space = gauge theory without gravity
- There are huge number of evidences showing that this conjecture is correct, and this number keeps increasing!
- This duality says that quantum gravity physics in asymptotic AdS space is equivalent to the physics of some gauge theory with less dimension
- For example, the S-matrix of black hole formation/ evaporation must be unitary, since we can map this process by putting it in asymptotic AdS and consider it from dual gauge theory viewpoint, which is always <u>***unitary***</u>

Now does AdS/CFT solved the problem?

Now does AdS/CFT solved the problem?

=> In principle yes, but no (yet) in practice.

AdS/CFT should be better understood, since we don't understand

- What is wrong with the original Hawking's argument?
- If information is back, how will it be back?
- How do we see the quasi-local gravity from gauge theory?
- How do we see the non-local effects for black holes from gauge theory?
- How do we see the black hole complementarity?
- etc…

- In this talk, we will concentrate on unitary issues & information problem
- Hawking’s original argument is based on semi-classical approximation. And he showed that black hole radiate thermally, so information is lost.
- This contradicts with gauge/gravity duality, since black hole radiation is dual to unitary gauge theory evolution.
- So how things can be consistent?

Gauge/gravity correspondence

- AdS/CFT correspondence; $G_N \sim 1/N^2 \,, l_s^2 R_{AdS} \sim \lambda^{-1/2}$
- Semiclassical approx. is $G_N \to 0 \,, l_s^2 R_{AdS} \to 0$ with leading G_N correction *only for matter, but not for geometry*
- This means, Hawking's argument is at $N \to \infty$ theory in the dual gauge theory (with infinite 't Hooft)
- But note that in $N = \infty$, *information* "can" be lost
- This is because in this limit, we have infinite number of states for the system. System can absorb arbitrary amount of information as heat bath
- Also note that the number of states are infinity, Poincare recurrence time also becomes infinity as

  (recurrence time scale) ~ exp(S) ~ exp(N^2)
- On the contrary, if N is finite, then the field theory spectrum is discrete (on finite volume), and it evolves as QM system, so information is never lost

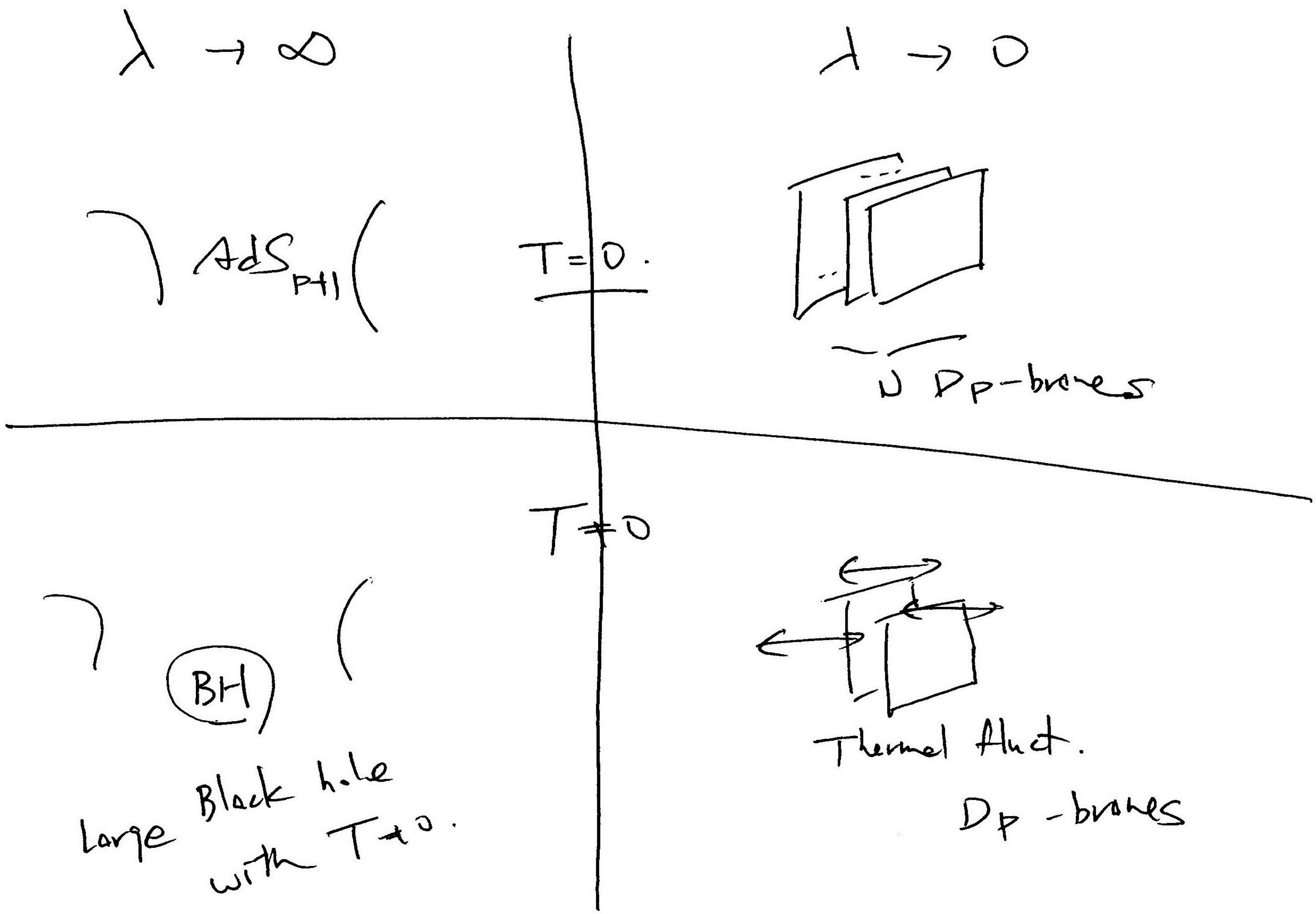
- So the question we would like to understand;
  Can we see the non-unitary black hole physics from unitary (at finite N) gauge theory, by taking $N=\infty$?
- Black hole is characterized by its horizon, where classically all information is incoming, and lost
- BH horizon makes all information (ie, correlation functions) decay exponentially at later time since information is absorbed inside the horizon
- Can we see this exponential damping/decay of correlation functions from unitary gauge theory at $N=\infty$?

- Our goal is to show this property; the exponential damping/decay of correlation function in $N = \infty$ limit, which never occurs at finite N *(Maldacena '01)*
- Note that exponential decay is not guaranteed, since power law decay is also consistent with information loss.
- The late time decay implies that system is thermalized.
- This late time decay can never been seen by perturbation theory (it is the properties of quantum chaos)

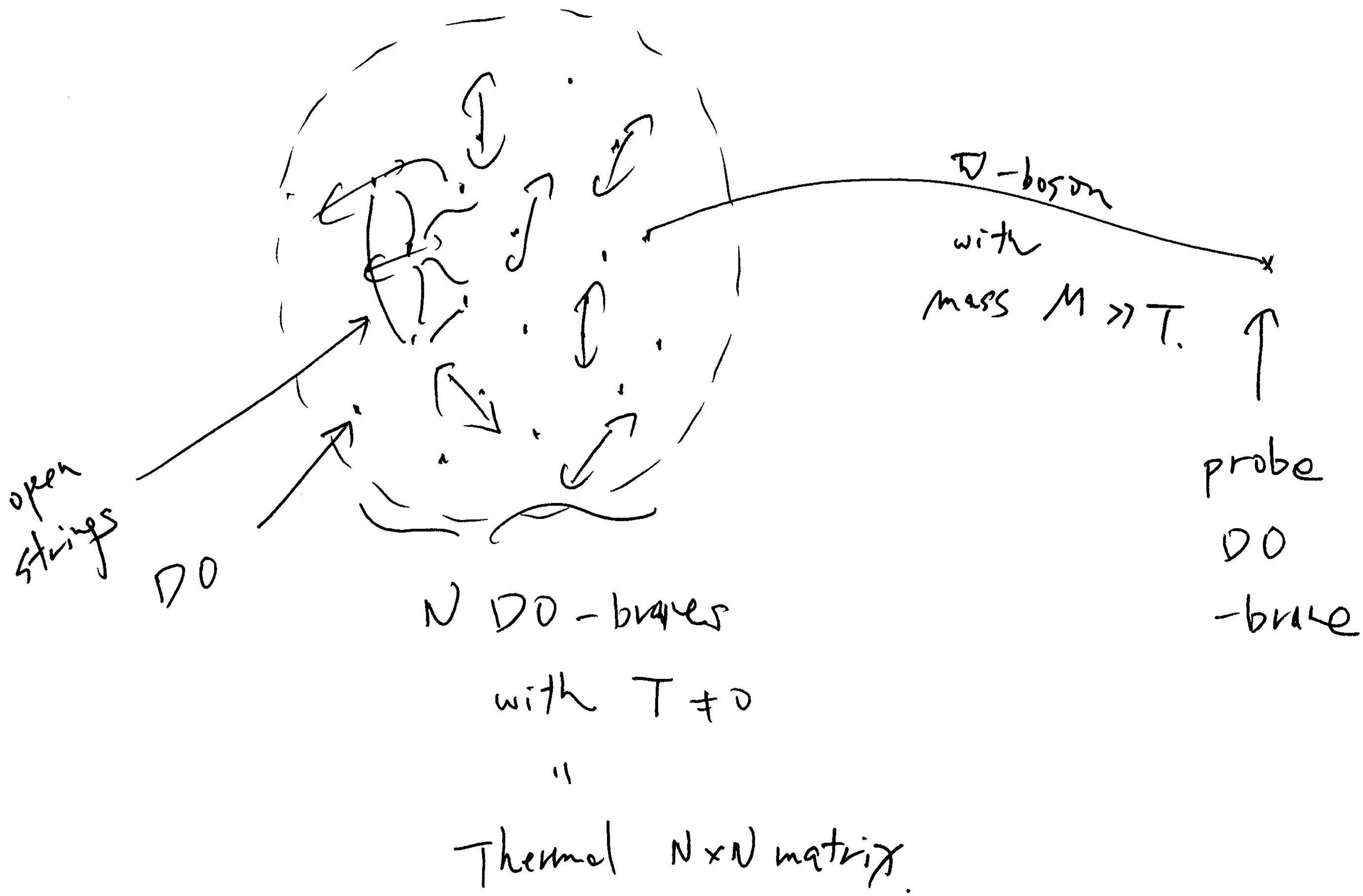
  (Liu-Festuccia '06)
- We simplify the gauge theory system as much as possible, so that we can capture *non-pertubative for* ∞

$N = \infty$ $\lambda \neq 0$

- We would like to find simple enough toy model where resumming Feynman diagrams is systematic enough so that we can see the full planner physics non-perturbatively
- If we can resum all diagrams, unitarity is guaranteed at finite N
- Our toy model is kind of reduction of D0-brane black hole with a probe D0-brane. We have one U(N) adjoint and one U(N) fundamental representation
- Here, [adjoint field] = black hole degrees of freedom and [fundamental field] = open strings or W-bosons between the black hole and a probe
- Adjoint plays the role of thermal heat bath, whose correlator are thermal one with some mass m, and since probe is away from black hole, W-bosons masses M are heavy enough
- They couple by Yukawa interaction so that U(N) indices are contracted



(Itzhaki-Maldacena-Sonnenschein-Yankielowicz)



$$U(N+1) \to U(N) \times U(1)$$

with $0 << T \leq g^{2/3} N^{1/3}$

- Fundamental mass M are heavy

  => they evolves Quantum Mech way,
- Adjoint mass m are light

  => they have thermal correlation function
- Assign the free thermal correlation function for adjoint *by hand*

  (later we discuss more on this issues)
- We would like to see how the fundamental fields evolve, through the coupling to adjoint field

  and how it can decay exponentially (quasi-normal mode) in planer limit which never happen in finite N

A Toy Matrix Quantum Mech. Model

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a + \bar{a}^\dagger \bar{a}) + g(a^\dagger X a + \bar{a}^\dagger X^T \bar{a}) \ .$$

- We focus on the following obsearvable

$\omega - M \to \omega$

$$e^{iM(t-t')} \left\langle \mathrm{T}\, a_i(t) a_j^\dagger(t') \right\rangle_T \equiv \delta_{ij} G(T, t-t') \ .$$

- Note that due to time ordering, if t < t' above quantities are zero, so this is retarded Green fn.

$$G(t) = \int d\omega \tilde{G}(\omega) e^{-i\omega t}$$

- Therefore
  $\tilde{G}(\omega)$ has no pole in upper half plane

- In perturbation expansion,

$$\tilde{G}(w) = < \frac{i}{\omega + i\epsilon + gX} >_T$$

$$= \frac{i}{\omega} + \frac{i}{\omega}\sum_{n=1}^{\infty}\left(\frac{i}{\omega}\right)^n < (-igX)^n >_T$$

interaction

$$X_{ij} = \text{matrix}$$

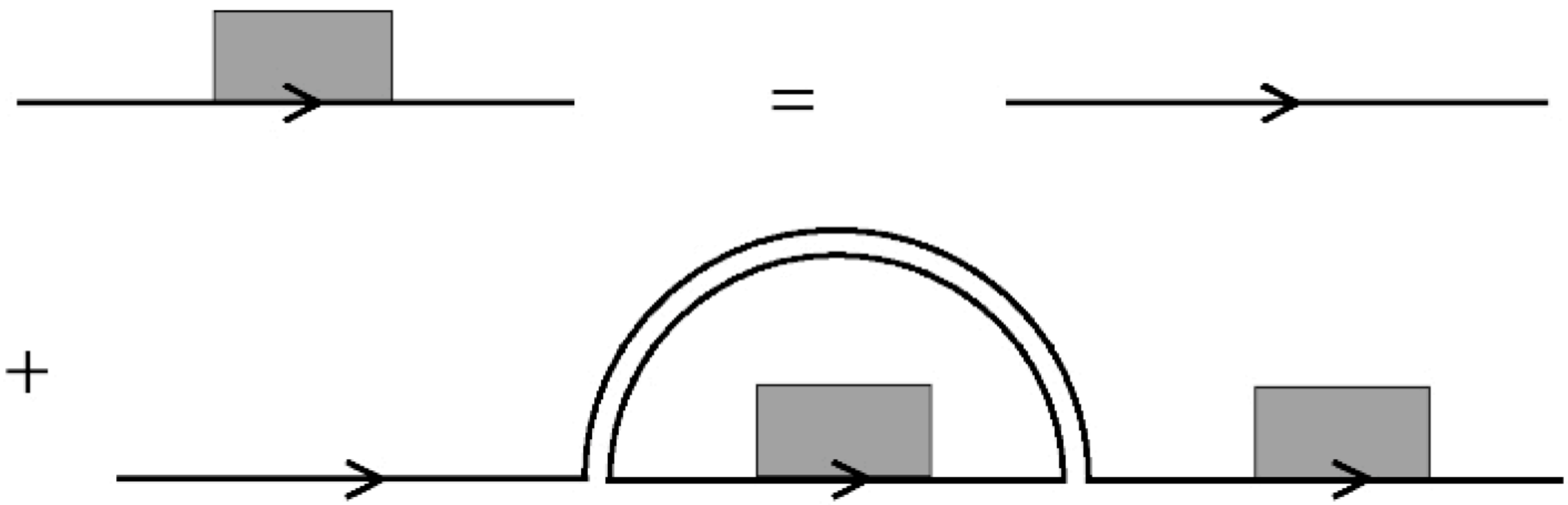
free (bare) propagator

- Where thermal sum is defined as;

op. for matrix X

$$< X >_T \equiv \mathcal{N}\, Tr < e^{-\beta m Tr(A^\dagger A)} X >

- Schwinger-Dyson (SD) equation for the fundamental field;



(We have only planar graphs)

- Mathematically

$$\tilde{G}(\omega) = \tilde{G}_0(\omega) - \lambda \tilde{G}_0(\omega)\left(\int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \tilde{G}(\omega') \tilde{K}_0(T, \omega - \omega')\right)\tilde{G}(\omega)$$

- With $\lambda = g^2 N$ and $\tilde{G}_0(w) = \dfrac{i}{\omega + i\epsilon}$

- $\tilde{K}_0$ is adjoint correlation function which we chose

$$\tilde{K}_0(T,\omega) = \frac{i}{1-e^{-m/T}}\left(\frac{1}{\omega^2-m^2+i\epsilon} - \frac{e^{-m/T}}{\omega^2-m^2-i\epsilon}\right)$$

- In zero temperature case, this reduces to free scalar propagator
- Bellow we consider zero temperature/finite temperature case of above SD eq (you will see that structure of the SD eq are totally different between zero and finite temperature)

- Since $\tilde{G}(\omega')$ has no pole in upper half plane, we can close the contour for ω' by going to the upper half plane
- As a result, we pick up only the pole from $K_0(\omega - \omega')$

$$\tilde{K}_0(T, \omega - \omega') = \frac{i}{1 - e^{-m/T}} \left(\frac{1}{(\omega - \omega')^2 - m^2 + i\epsilon} - \frac{e^{-m/T}}{(\omega - \omega')^2 - m^2 - i\epsilon} \right)$$

pole at $\omega - \omega' = m - i\epsilon$

pole at $\omega - \omega' = -m - i\epsilon$

<u>for SD equation</u>

$$\tilde{G}(\omega) = \tilde{G}_0(\omega) - \lambda \tilde{G}_0(\omega) \tilde{G}(\omega) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \tilde{G}(\omega') \tilde{K}_0(\omega - \omega')$$

- So the SD equation reduces to following recurrence eqs
- At zero temperature

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m}\tilde{G}(\omega)\tilde{G}(\omega - m)\right)$$

- At nonzero temperature

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m(1 - e^{-m/T})}\tilde{G}(\omega)\{\tilde{G}(\omega - m) + e^{-m/T}\tilde{G}(\omega + m)\}\right)$$

- Even though these two equations are similar, the structure of solutions are totally different as we will see

Zero temperature case

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m}\tilde{G}(\omega)\tilde{G}(\omega - m)\right)$$

At $m = 0$ case, SD be algebraic equation and solvable as

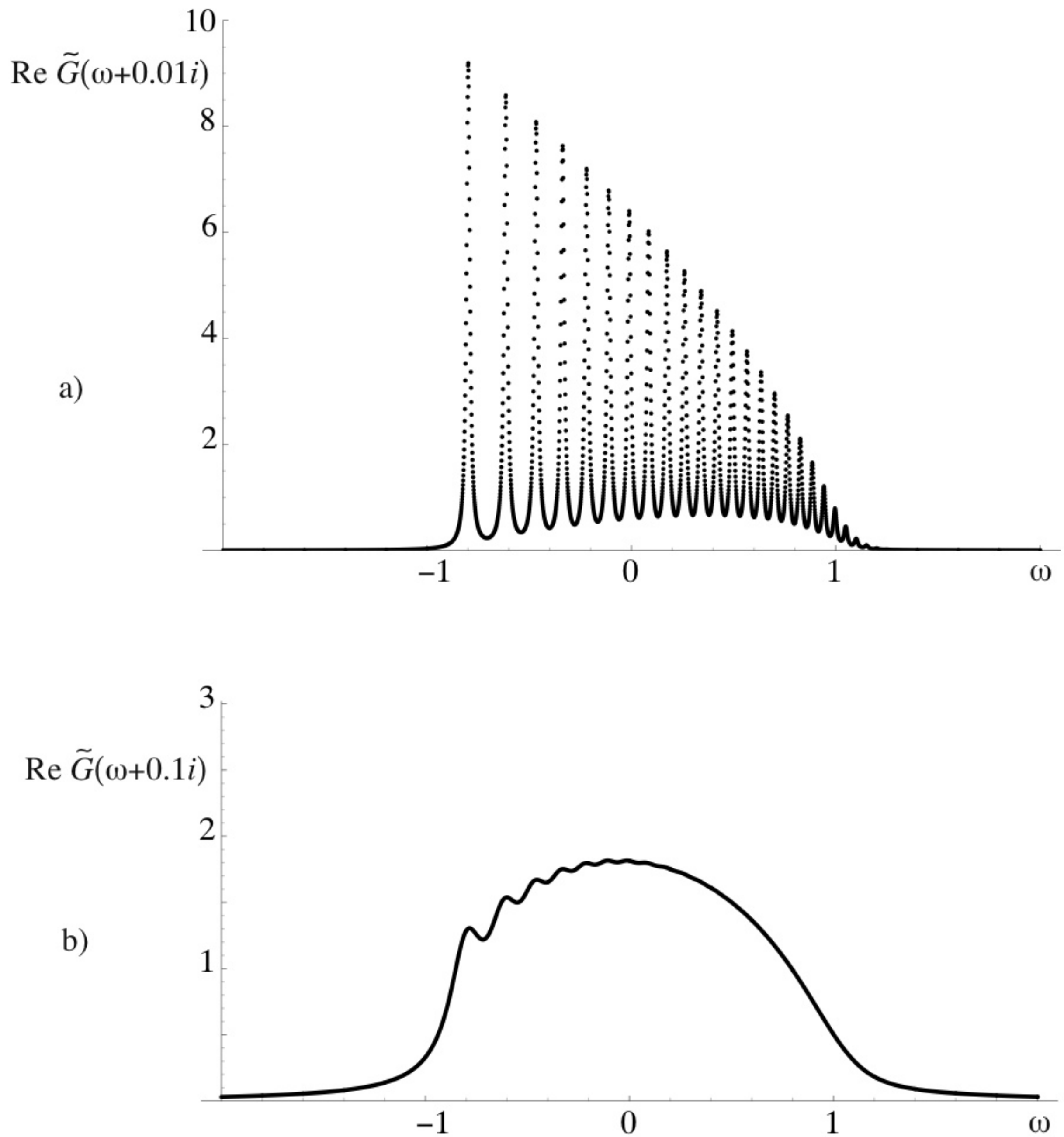
$$\tilde{G} = \frac{2i}{\omega + \sqrt{\omega^2 - \nu^2}} \quad \text{, with} \quad \nu^2 \equiv 2\frac{\lambda}{m}$$

The pole at $\omega = 0$ has been broaden into a branch cut.
This is because the mass for a is given by $g\,X$ and the distribution of X is given by Wigner semi-circle with width $\sqrt{\frac{2N}{m}}$

Zero temperature case

- The Wigner semi-circle for $m=0$ case splits up into poles at nonzero m.
- To see this, note that if there is branch cut at some $\omega = \omega_0$, then the recurrence eq. forces another branch cut at $\omega_0 + m$, $\omega_0 - m$, so we have series of branch cut by step of m.
- But this contradicts with the fact that at zero T in $\omega = \infty$, where theory reduces to free, so should approach i/ω. and no branch cut there, unless its amplitudes approaches zero
- We conclude that at zero T, spectrum is bunches of poles, no branch cut.

Real part of G(ω) for $v = 1$, $m = 0.05$ at
a) 0.01 unit
b) 0.1 unit above the real ω axis.
The poles merge into an approximate semicircle distribution



Non-zero temperature case

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m(1-e^{-m/T})}\tilde{G}(\omega)\{\tilde{G}(\omega-m)+e^{-m/T}\tilde{G}(\omega+m)\}\right)$$

Again at $m = 0$ case, with $v_T{}^2 = 2\lambda/m\ (1 - e^{-m/T})$ fixed, it gives

$$\tilde{G} = \frac{2i}{\omega + \sqrt{\omega^2 - (1+e^{-m/T})\nu_T^2}}$$

Physically eigenvalue distribution is thermally broaden, but still it is power law decay, not exponential.

Non-zero temperature case

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m(1-e^{-m/T})}\tilde{G}(\omega)\{\tilde{G}(\omega - m) + e^{-m/T}\tilde{G}(\omega + m)\}\right)$$

Spectrum representation shows negative residues are not allowed along real omega axis

$$\begin{aligned}\tilde{G}(\omega) &= \mathcal{N}\sum_{A,B}\frac{i}{\omega - E_A + E_B + i\epsilon}e^{-E_A/T}| < A|a|B > |^2 \\ &\equiv \int_{-\infty}^{\infty} d\mu F(\mu)\frac{i}{\omega - \mu + i\epsilon}\end{aligned}$$

If there is pole, pole must be sandwiched by zero both on the left and right, but this gives contradiction. Therefore the poles which we see at zero temperature are not allowed!

Non-zero temperature case

This immediately implies that the spectrum is continuous, rather than discrete poles, so there is a chance this shows quasi-normal modes

There are two possibilities

- Infinite arrays of branch cut or
- Spectrum continuously spreads all the way from - ∞ to + ∞; ie, branch cut spreads over all the real ω, and pole we found at zero temperature goes into the second Riemann sheet, complex omega Im $\omega < 0$.

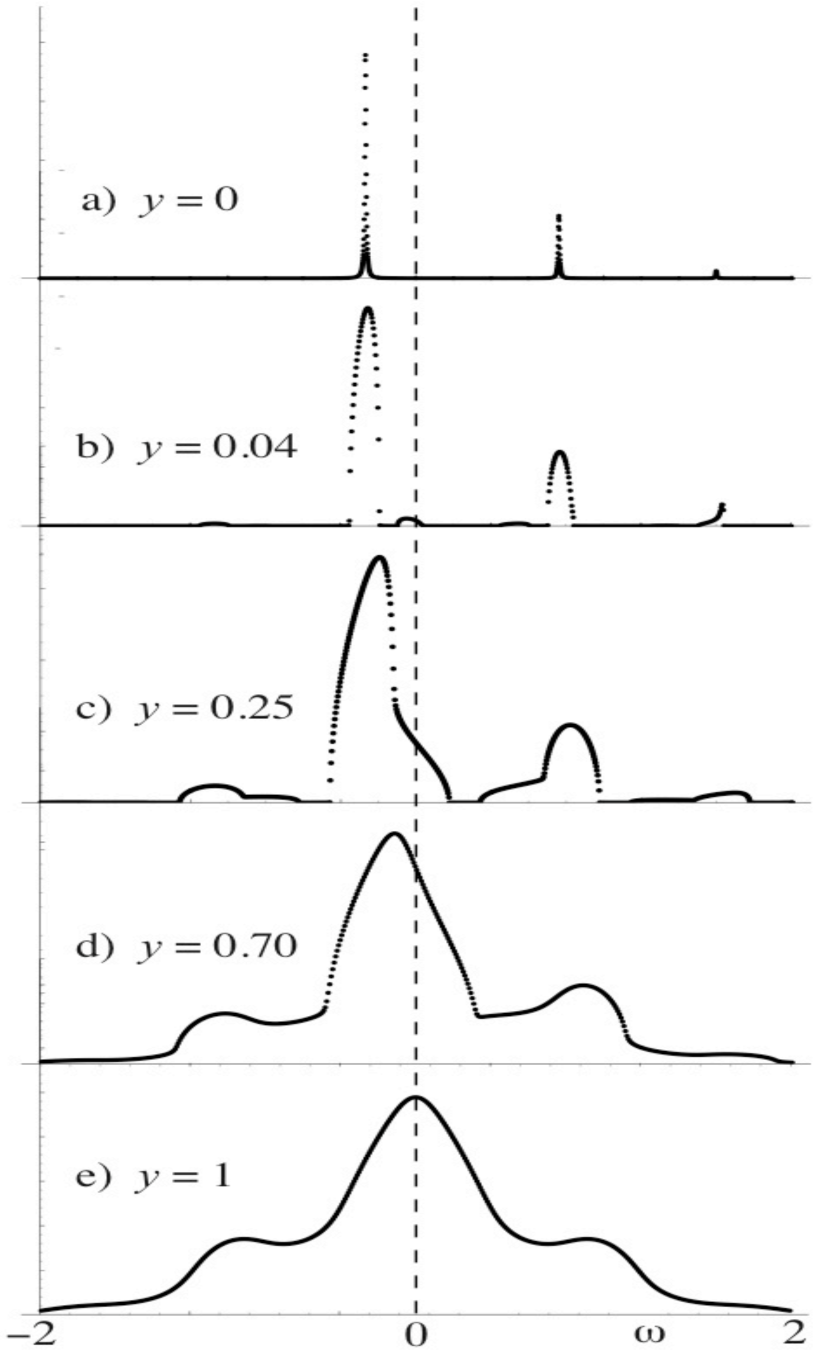
Non-zero temperature case

Although this is the model we want, it is still challenging problem to solve this eq. Because *this eq. is unstable both along increasing omega and decreasing omega!*

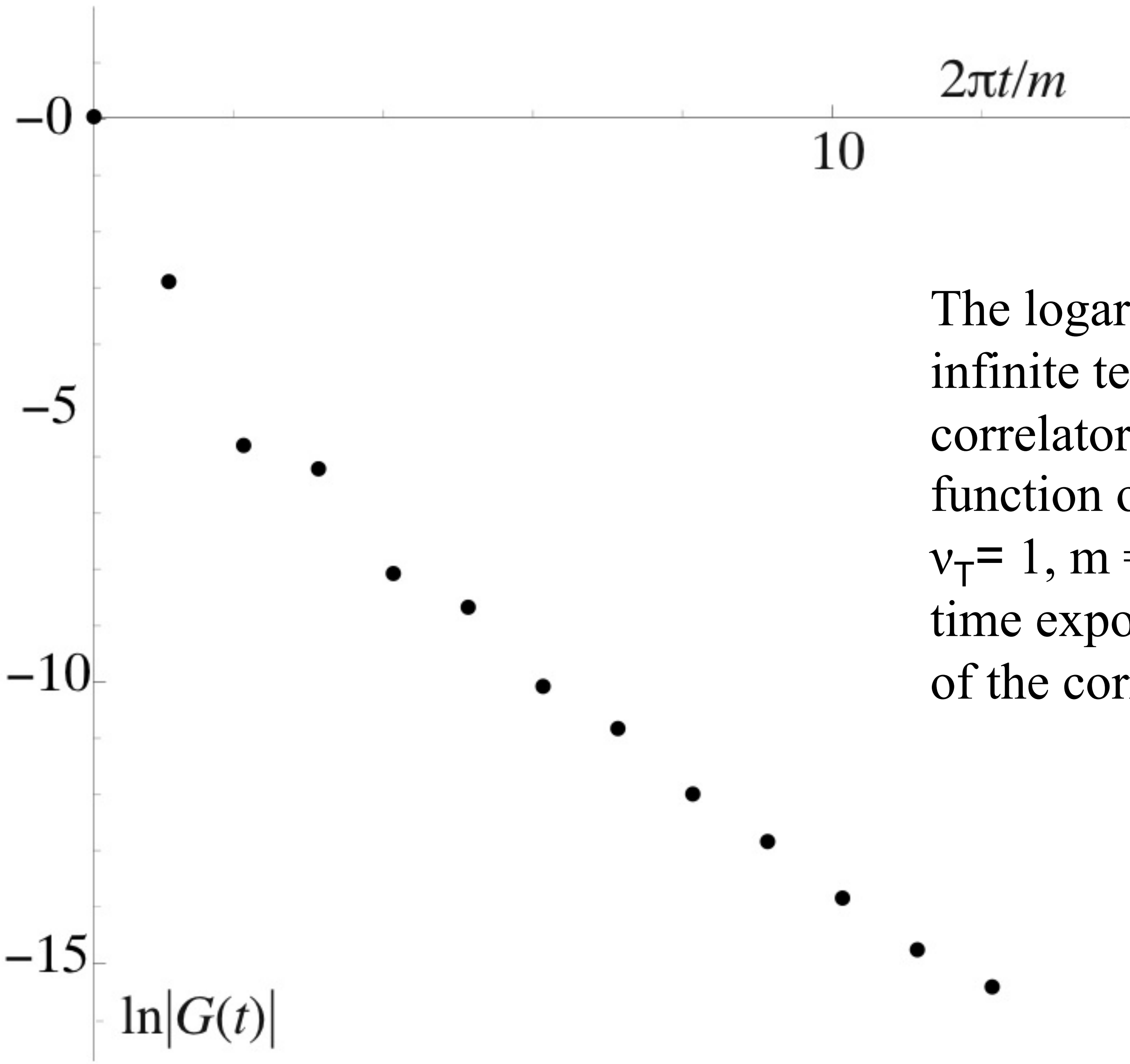
$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 - \frac{\lambda}{2m(1-e^{-m/T})}\tilde{G}(\omega)\{\tilde{G}(\omega - m) + e^{-m/T}\tilde{G}(\omega + m)\}\right)$$

Even numerically this eq. are very hard to solve!

We solved this by fixing λ/m, and taking the derivative w.r.t. T, we allow system evolves from zero temperature into finite temperature by solving differential eq w.r.t. *T*



The real part of G(ω) as spectrum density with various temperature $y = exp(-\beta m)$. The vertical axis is rescaled. The plot is ω axis for slightly above the real ω



The logarithm of infinite temperature correlator log|G(ω)| as a function of *t*, with fixed ν_T= 1, m =0.8. Late time exponential decay of the correlator is clear.

Non-zero temperature case

Asymptotic behavior of solution

In the large omega, the coupling be weaker, the propagator approaches more to free one, this means magnitude of $\pi F(\omega) = Re[G(\omega)]$ approachs 0 at large ω.

$$F(\omega - m) - \frac{4}{\mu_T^2 |\tilde{G}(\omega)|^2} F(\omega) + e^{-\beta m} F(\omega + m) = 0$$

Consistent solution with above boundary condition is;

$$\begin{aligned} F(\omega)/F(\omega - m) &\sim \mu_T^2/4\omega^2 \,, (\omega \to \infty) \\ F(\omega)/F(\omega + m) &\sim e^{-\beta m} \mu_T^2/4\omega^2 \,, (\omega \to -\infty) \end{aligned}$$

Spectral density behaves asymptotically as $|\omega|^{O(-|\omega|)}$

Conditions for quasinormal mode

- Finite mass of adjoint field; $m \neq 0$
-
- ‘t Hooft coupling is nonzero; $\lambda \neq 0$
-
- Finite temperature correlator for adjoint field; $T \neq 0$
-

Conditions for quasinormal mode

- Finite mass of adjoint field; $m \neq 0$
- Black hole microscopic degrees of freedom have finite mass, their wave function is localized at finite scale in space.
- ‘t Hooft coupling is nonzero; $\lambda \neq 0$
- To escape information into infinite phase space, mixing of states by interaction is crucial.
- Finite temperature correlator for adjoint field; $T \neq 0$
- Black hole should be at deconfinement phase.

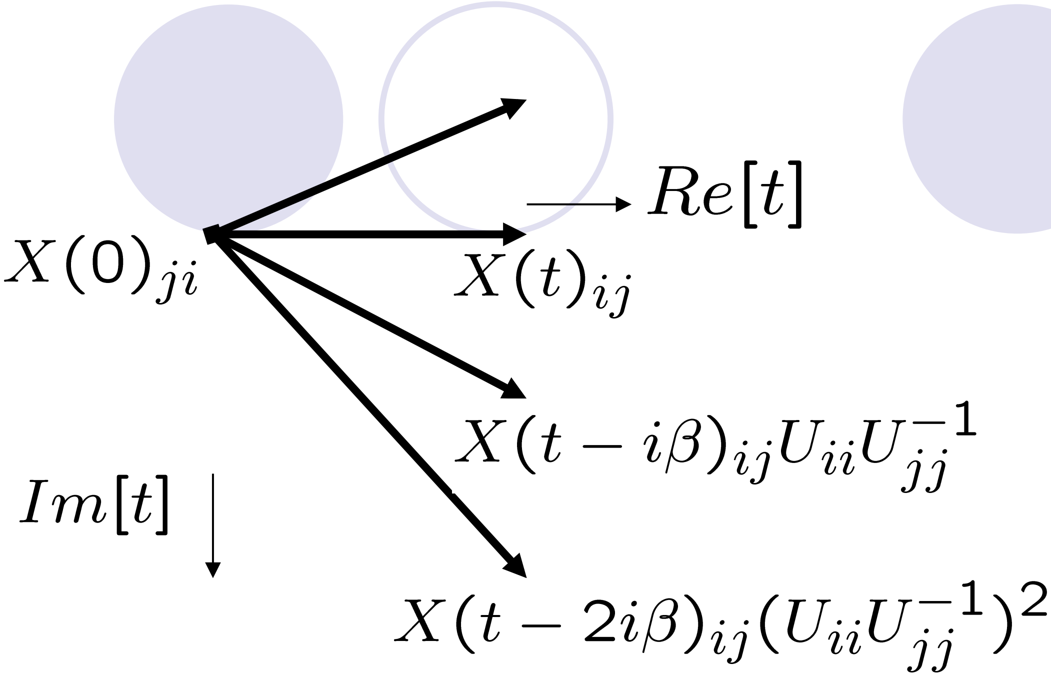
Hawking-Page transition and Confinement/Deconfinement transition

- Confinement/deconfinement transition is expected to be connected with Hawking-Page transition (thermal AdS/AdS black hole) in gauge/gravity duality
- In $N = \infty$ at confinement phase, degrees of freedom are glueball (gauge singlet, closed strings). Their propagate freely at $N = \infty$ whatever 't Hooft coupling we take. So at this case, theory is in practice free, even though 't Hooft is nonzero.
- At deconfinement phase, degrees of freedom are gluons (gauge non-singlet, open strings, or `string bits'). They still interact at $N = \infty$, if 't Hooft coupling are nonzero.

- The system which has dynamical adjoint gauge fields shows Hagedorn transition, which is confinement/deconfinement transition.
- For example, in d=4, $\mathcal{N}$ =4 SYM shows this transition (Sundborg, Aharony et al)
- This transition is characterized by how the VEV of Polyakov loop operator along time direction changes S

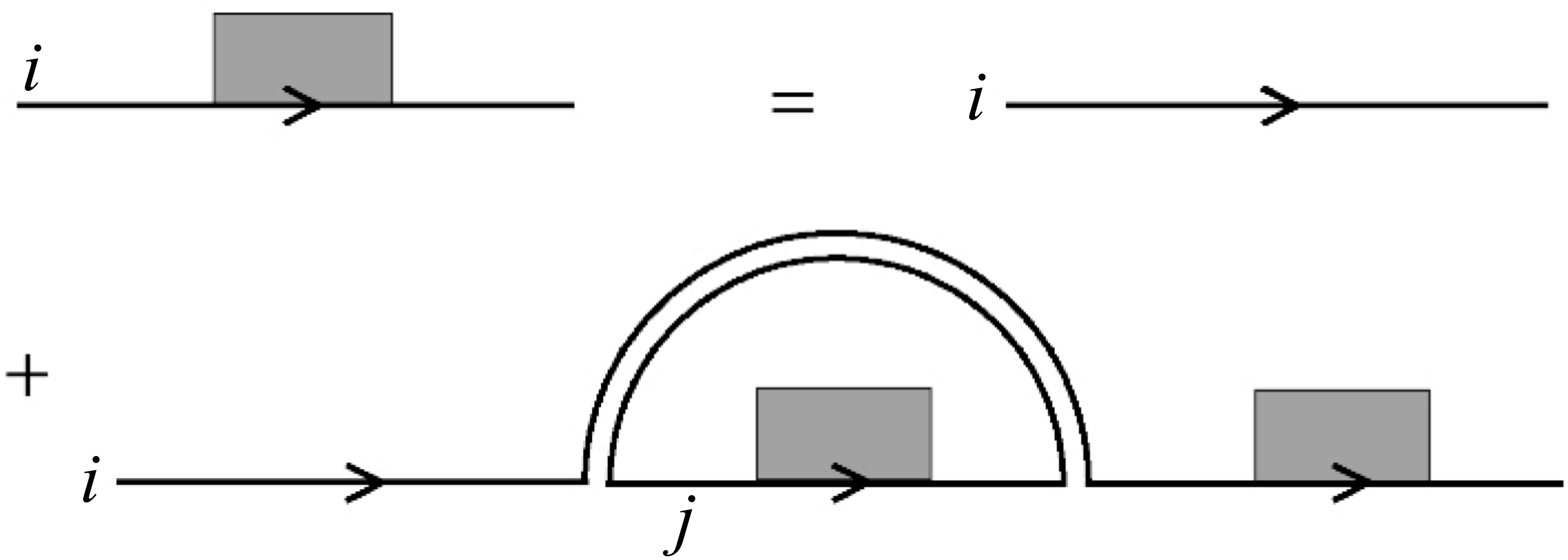
$$< U >_{ij} = e^{\int dt < A_0 >_{ij}} = e^{i\beta < A_0 >_{ii} \delta_{ij}}$$

Zero mode, can be diagonalized



- The finite temperature adjoint correlator is given by summing over infinite mirror image separated by -ib
- In SD eq for fundamental field a_i , X correlator contributes after summing over j

- At confinement phase, U is uniformly distributed, $\sum_j U_{jj}^{-1} = 0$
  So the adjoint thermal propagator reduces to the zero temperature one, thermal effect cancel out
- At deconefinement phase, especially at very high temperature, U is localized, delta-functionally peaked, then the adjoint thermal propagator reduces to the nonzero temperature propagator we used because $U_{ij} = e^{i\theta}$, $U_{ii}U_{jj}^{-1} = 1$


i
=
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- So the zero temperature case we studied in our model corresponds to the confinement phase in the real gauge/gravity duality, and the nonzero temperature case, especially very high temperature case, corresponds to precisely the very high temperature deconfinement phase in the real gauge/gravity duality.
- The fact that we don't see quasinormal mode when adjoint X has zero temperature propagator means that we cannot find quasinormal mode when adjoint X is at the confinement phase (=thermal AdS phase).

Conclusions. Part I

Conditions for quasinormal mode

- Finite mass of adjoint field; $m \neq 0$
- Black hole microscopic degrees of freedom have finite mass, their wave function is localized at finite scale in space.
- ‘t Hooft coupling is nonzero; $\lambda \neq 0$
- To escape information into infinite phase space, mixing of states by interaction is crucial.
- Finite temperature correlator for adjoint field; $T \neq 0$
- Black hole should be at deconfinement phase.

Conclusion

Infinite N, deconfinement phase, and non-pert. λ are responsible for classical black holes

- System can contain infinite information
- Discrete spectrum at pert. λ be continuous by strongly-coupled (= non-pert.) λ effects
- Poincare recurrence never occurs at finite timescale
- Quasinormal mode (exponential damping/decay) is seen due to infinite phase space in deconfinement phase
- Properties of black hole shows up only at infinite N, in finite N, spectrum is discrete, always recurrence appears

Discussion

- To restore the information, finiteness of N is crucial.
- As far as we use the semi-classical approximation, we never restore the information.
- The information loss occurs only at (semi-)classical gravity. In full quantum gravity, we expect 'horizon-like' boundary where information flows only along one side never occur.
- At infinite N, spectrum is continuous, but at finite N it is collections of delta functional peak.
- The continuous spectrum at nonzero m, T, g^2N and infinite N should split into poles at finite $\boldsymbol{N}$

  with spacing dE ~ exp(-O(N^2))

finite N vs. $N = \infty$

- If we measure the spectrum precisely (measuring each delta functional peak at fixed omega) for the black holes, then we are able to distinguish each microstate of black hole, and this effect is very large, not order $exp(-N^2)$

  (Balasubramanian, Marolf, Rozali)
- On the other hand, if we neglect the detail of spectrum, and measure the spectrum in rough way, then we see as if black hole has continuous spectrum.
- In principle, how accurate can we measure the spectrum?

finite N vs. N $= \infty$

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- On the other hand, if we neglect the detail of spectrum, and measure the spectrum in rough way, then we see as if black hole has continuous spectrum.
- In principle, how accurate can we measure the spectrum?
- Uncertainty principle $dE\ dt \geq h \sim 1/N^{\#}$ prohibits $dE \sim exp(-N^2)$ precision measurement in time scale of black hole evaporation $dt \sim M^{\#} \sim N^{2\#}$, so Hawking's argument hold
- As far as we observe 2pt fn., this seem inevitable conclusion

finite N vs. N $= \infty$

- For SU(N) field theory at $N = \infty$,
  is theory unitary?
- N plays the role of IR cut-off of the phase space volume. With finite N, phase space volume is finite
- The reason why we see exponential damping is the same as correlators decay exponentially at late time, for the system which has literary infinite volume

To understand furthermore…

- As we see, thermalization occurs due to the nonperturbative effect by interaction
- On the other hand, our Schwinger-Dyson equation is too complicated to solve analytically <u>even at planar limit</u>
- So <u>non-planar corrections</u> are even more difficult
- We also want to understand N^2 point correlation, but difficult
- To understand better, we would like to have system which shows more analytical control: but still complicated enough to show information loss physics, i.e., continuous spectrum from Schwinger-Dyson equation
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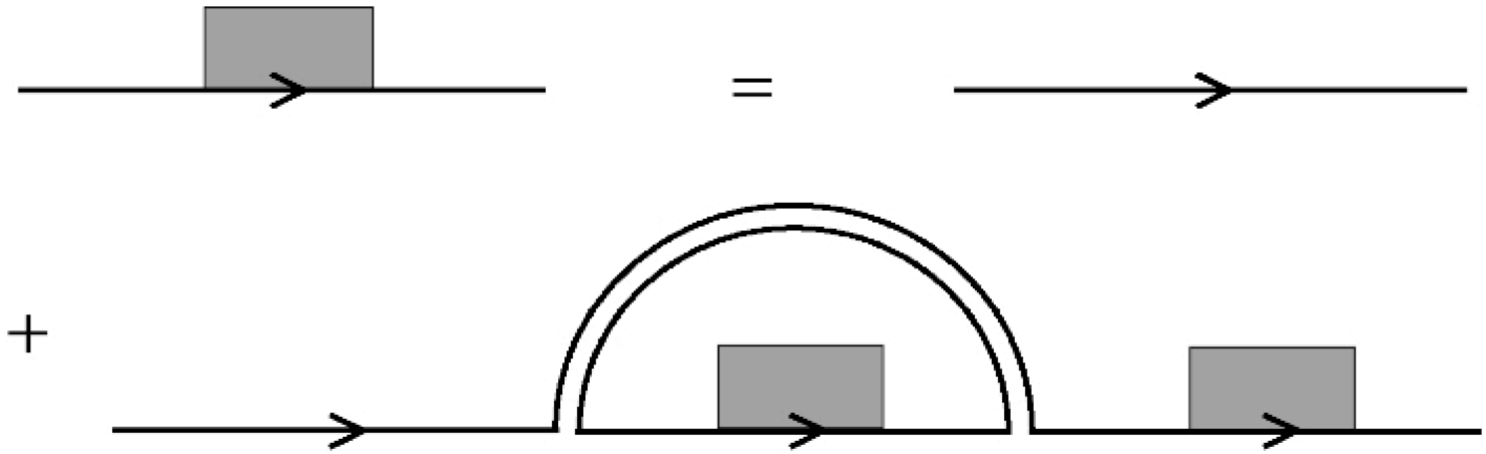
- We also want to understand N^2 point correlation, but difficult
- To understand better, we would like to have system which shows more analytical control: but still complicated enough to show information loss physics, i.e., continuous spectrum from Schwinger-Dyson equation
- So are there any way to build better model? for that, let's look at our models more closely

$\boxed{\textit{Part II talk}}$



$$
\begin{aligned}
\tilde{G}(\omega) &= \frac{i}{\omega} \\
&- \left(\frac{i}{\omega}\right)\lambda \frac{\tilde{G}(\omega - m) + e^{-m/T}\tilde{G}(\omega + m)}{2m(1 - e^{-m/T})}\tilde{G}(\omega)
\end{aligned}
$$

- We pick up only the pol$\tilde{K}_0(\omega - \omega')$

$$\tilde{K}_0(T, \omega - \omega') = \frac{i}{1 - e^{-m/T}} \left(\frac{1}{(\omega - \omega')^2 - m^2 + i\epsilon} - \frac{e^{-m/T}}{(\omega - \omega')^2 - m^2 - i\epsilon} \right)$$

pole at $\omega - \omega' = m - i\epsilon$

pole at $\omega - \omega' = -m - i\epsilon$

<u>for SD equation</u>

$$\tilde{G}(\omega) = \tilde{G}_0(\omega) - \lambda \tilde{G}_0(\omega) \tilde{G}(\omega) \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \tilde{G}(\omega') \tilde{K}_0(\omega - \omega')$$

Part II:
Another Toy Matrix Q.M. Model

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a + \bar{a}^\dagger \bar{a}) + g(a^\dagger X a + \bar{a}^\dagger X^T \bar{a}) \ .$$

- The interaction can be written as $H_{int} = \frac{g}{\sqrt{2m}} a^\dagger (A^\dagger + A) a$

  (where A and $A^\dagger$ is annihi. and creation op. for adjoint field)
- Our interaction does not preserve the # of adjoint field. As we saw in part I talk, each adjoint propagator induces pole shift for fundamental propagator as

$$\frac{1}{\omega + i\epsilon} \to \frac{1}{\omega + nm + i\epsilon} \text{, with } n = 0, \pm 1, \pm 2, \ldots$$

- Thus, each Feynman graph gives only the pole shift on real w, but if re-sum over all planar graphs, then spectrum changes, this is because pert. l expansion is singular at l -> 0.

Part II: Another Toy Matrix Q.M. Model

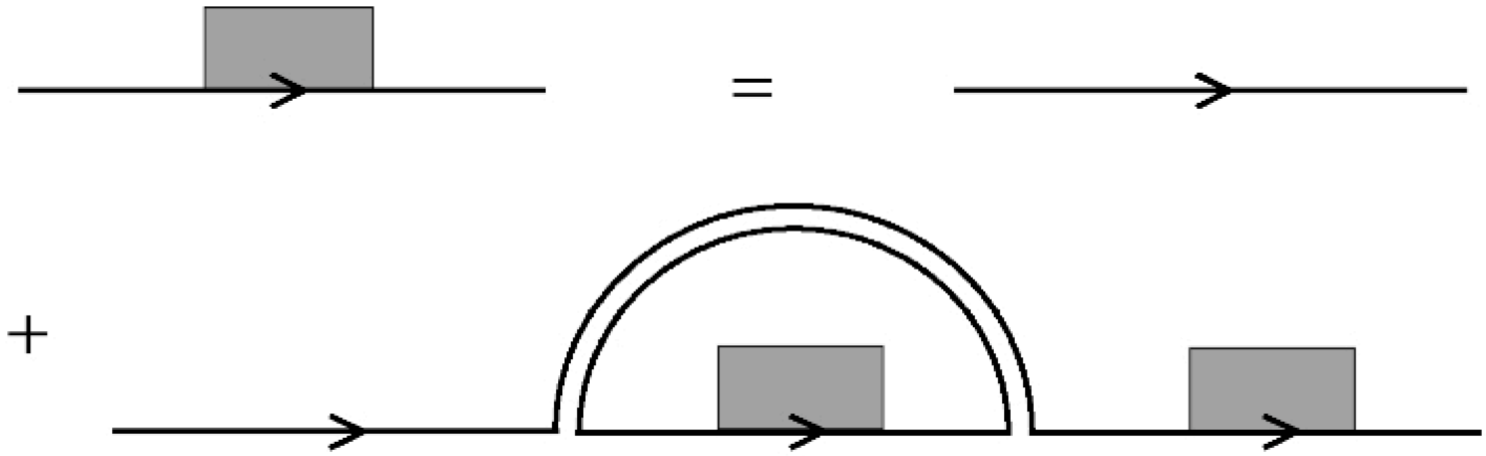
- Pert. l expansion is singular because higher order graphs gives more higher order poles
- Higher singular poles occurs when *n* alternates *n= 0, 1, -1.*

$$\frac{1}{\omega + nm + i\epsilon}\ \text{, with}\ n = 0, \pm 1$$

- This is the point where λ pert. breaks down by singular graphs

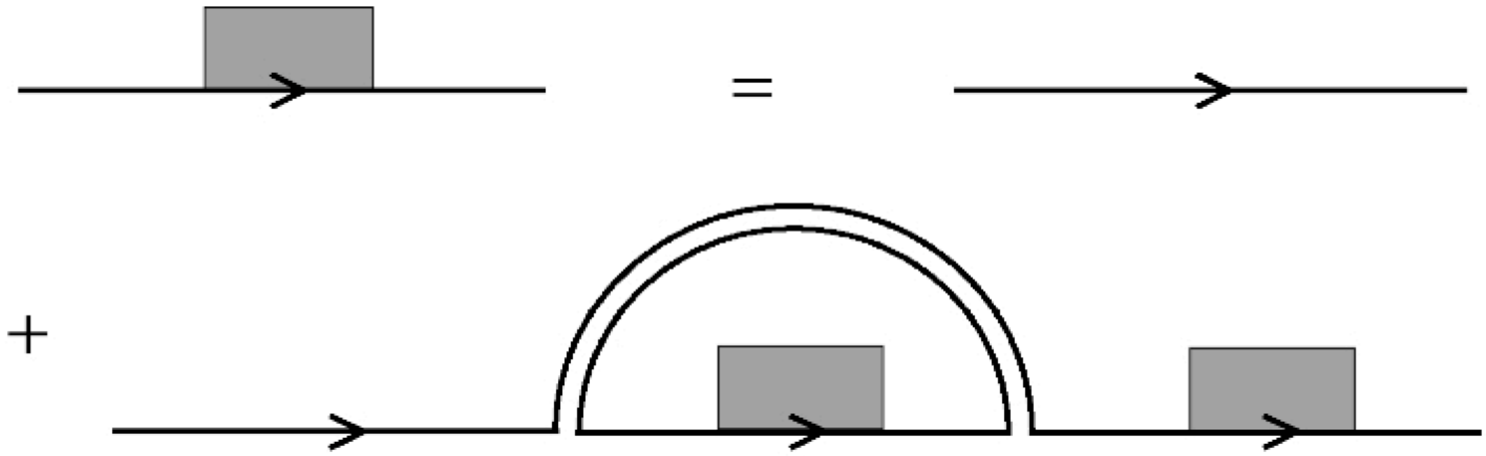
=> We would like to modify our interaction so that only singular graphs survive

- This turns out that we modify the model such that SD eq is polynomial eq. and we gain more analytical control
- For this, we path integrate out the intermediate fundamental field as follows;



$$
\begin{aligned}
\tilde{G}(\omega) &= \frac{i}{\omega} \\
&- \left(\frac{i}{\omega}\right)\lambda \frac{\tilde{G}(\omega - m) + e^{-m/T}\tilde{G}(\omega + m)}{2m(1 - e^{-m/T})}\tilde{G}(\omega)
\end{aligned}
$$

$$
\begin{aligned}
H_{eff} &\rightarrow (g/\sqrt{2m})^2 a^\dagger (A + A^\dagger)\frac{1}{H_0 - E}(A + A^\dagger)a \\
&\rightarrow h a^\dagger (A^\dagger A + A A^\dagger + A A + A^\dagger A^\dagger)a
\end{aligned}
$$



$$
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\end{aligned}
$$

$$
\begin{aligned}
H_{eff} &\rightarrow (g/\sqrt{2m})^2 a^\dagger (A+A^\dagger)\frac{1}{H_0-E}(A+A^\dagger)a \\
&\rightarrow h a^\dagger (A^\dagger A + \cancel{AA^\dagger} + \cancel{AA} + \cancel{A^\dagger A^\dagger})a
\end{aligned}
$$

Just for simplicity

Part II: Another Toy Matrix Q.M. Model

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a) + h(a^\dagger A^\dagger A a)$$

interaction is modified

$$H_{int} = h\, a^{j\dagger} A_j^{\dagger k'} A_{k'}{}^i a_i$$

with $i, j = 1, 2, ..., N$; $k' = 1, 2, ..., N'$, a_i is fundamental rep'n of $SU(N)$, and $A_{k'}{}^i$ is bi-fundamental rep'n of $SU(N')$ x $SU(N)$

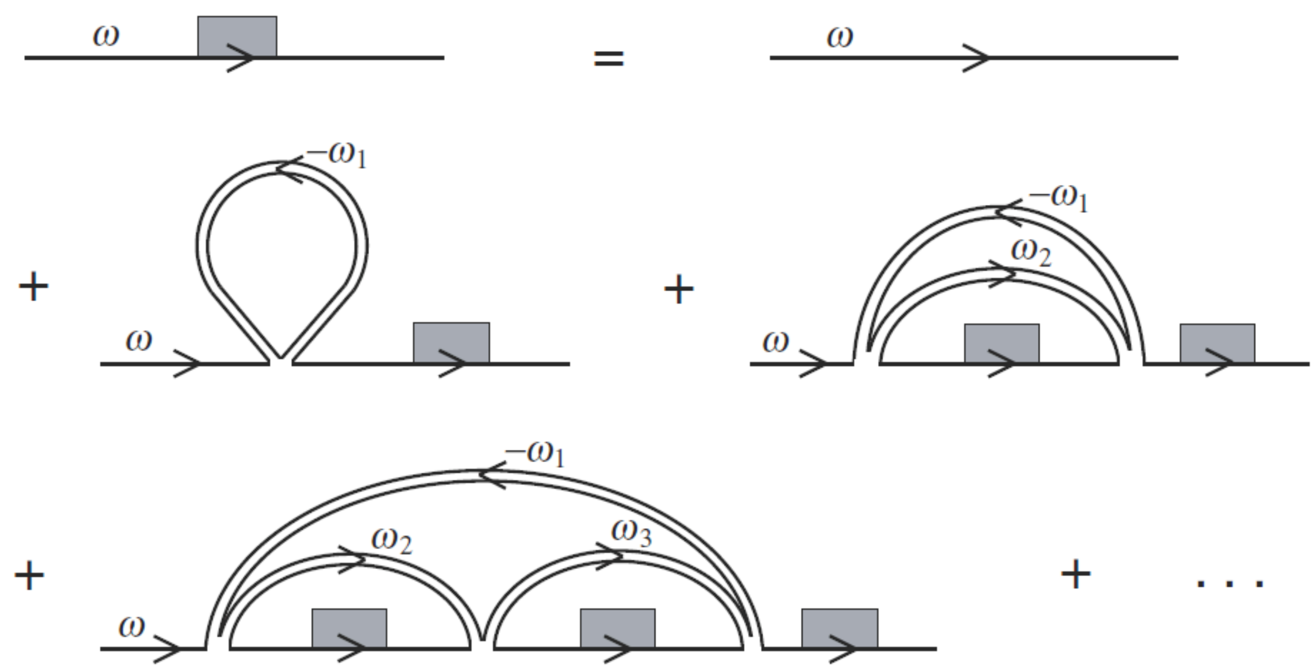
- Gain: more symmetry, more analytic control
- Price: dynamics is more constraint, system may not show enough chaotic property for thermalization

interaction preserves the adjoint field number

Again, we calculate the following observable

$$e^{iM(t-t')}\left\langle \mathrm{T}\, a_i(t)a_j^\dagger(t')\right\rangle_T \equiv \delta_{ij}G(T,t-t')\ .$$

- In large *N* planar limit, SD equation becomes;



1. Schwinger-Dyson approach

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a) + g(a_i^\dagger A_\alpha^{\dagger i} A_j^\alpha a^j)\,;$$

SD equation is written as;

$$\tilde{G}(\omega) = \frac{i}{\omega}\Big(1 + S[\tilde{G}(\omega)]\,\tilde{G}(\omega)\Big)$$

- *In planar case, S is calculated as*

$$S[\tilde{G}] = \sum_{n=0}^{\infty} y \left(\frac{-ihN}{1-y}\right)^{n+1} \tilde{G}(\omega)^n$$
$$= -iy\lambda_y/(1+i\lambda_y\tilde{G}(\omega))\,.$$

- *Here we used,*

$$y = e^{-m\beta}\,, \lambda_y \equiv hN/1-y\,,$$

$$< TA(t)^i_{k'}A^\dagger(0)^{l'}_j >_\beta = \delta^i_j\delta^{l'}_{k'}Tr < e^{-\beta H}e^{iHt}A(0)e^{-iHt}A(0) >$$
$$= \frac{e^{-imt}}{1-y}(\theta(t)+y\theta(-t))$$
$$= \int d\omega e^{-i\omega t}L_0(\omega)$$

$$L_0(\omega) = \frac{i}{1-y}\left(\frac{1}{\omega-m+i\epsilon} - \frac{y}{\omega-m-i\epsilon}\right)$$

- The solution is λ = $h\,N$

$$\tilde{G}(\omega) = \frac{i(1-y)}{2\omega\lambda}(\lambda + \omega - \sqrt{(\omega-\omega_+)(\omega-\omega_-)})$$
$$\omega_\pm = \lambda\frac{1+y\pm 2\sqrt{y}}{1-y}$$

- With branch cut its late time behavior is power law decay as

$$G(t) \propto t^{-3/2} \text{ as } t \to \infty$$

- Due to higher symmetry, the dynamics is less chaotic, show no quasinormal mode, but at least correlator decay in late time.

because we neglect
$a^\dagger(A^\dagger A^\dagger + AA)a$

- Our goal in this II talk is to understand $1/N^2$ expansion, and how bulk pic appears in this more solvable model

We solved this model in 3 different ways;

- 1. Schwinger-Dyson approach
- 2. Loop variable approach
- 3. Summing over Young Tableaux approach

1. Schwinger-Dyson approach

$$H = \frac{1}{2}\mathrm{Tr}(\Pi^2) + \frac{m^2}{2}\mathrm{Tr}(X^2) + M(a^\dagger a) + g(a_i^\dagger A_\alpha^{\dagger i} A_j^\alpha a^j)\,;$$

SD equation is written as;

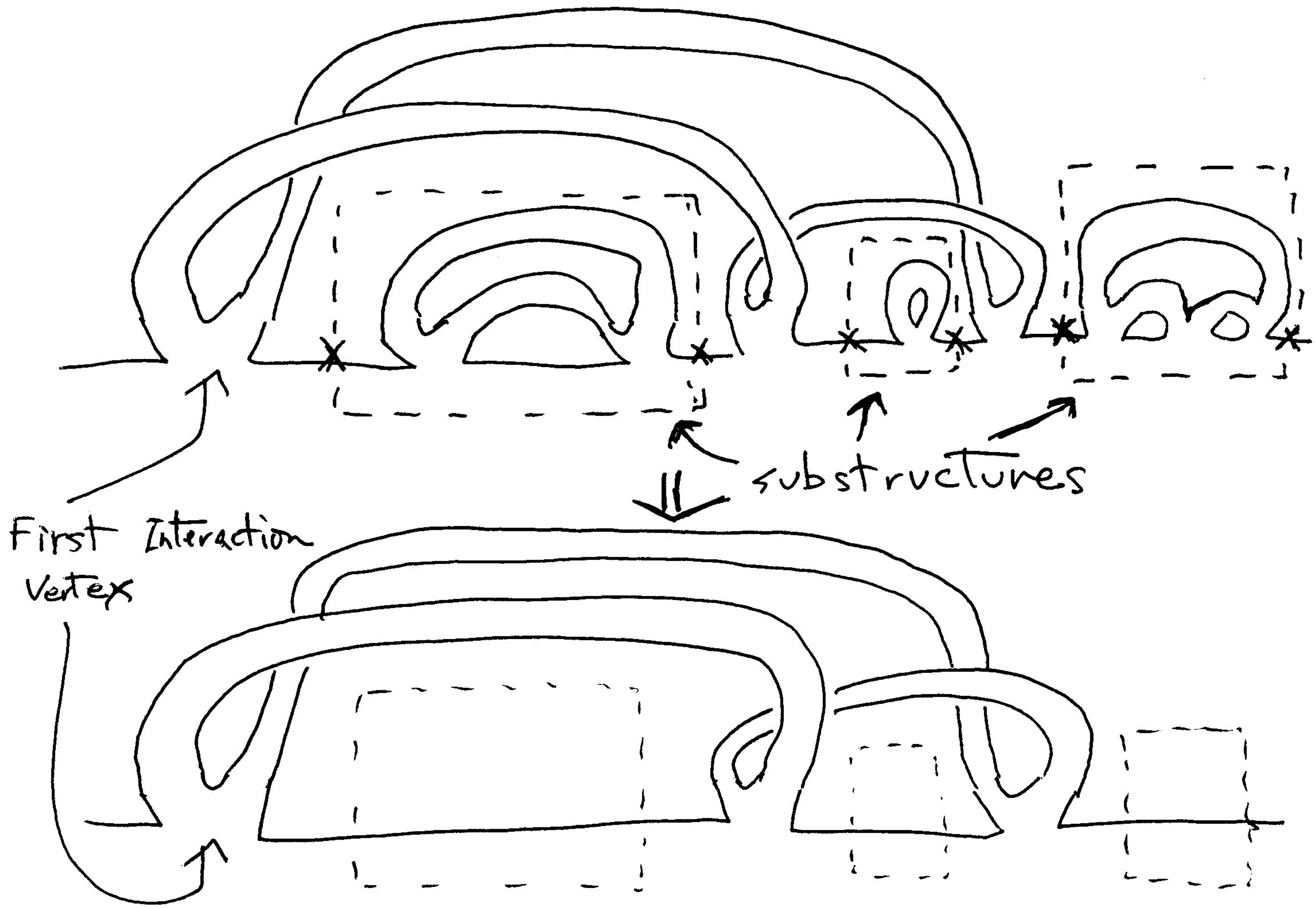
$$\tilde{G}(\omega) = \frac{i}{\omega}\Big(1 + S[\tilde{G}(\omega)]\,\tilde{G}(\omega)\Big)$$

- *S is sum over self energy graphs*
- *S is sum over 1PI*

=> If not, double-counting due to manifest G(w) on r.h.s.

- *S is sum over 2PI (=no substructure for fundamental propagator)*

=> fundamental propagators which contain substructures are already taken into account by dressed propagator)

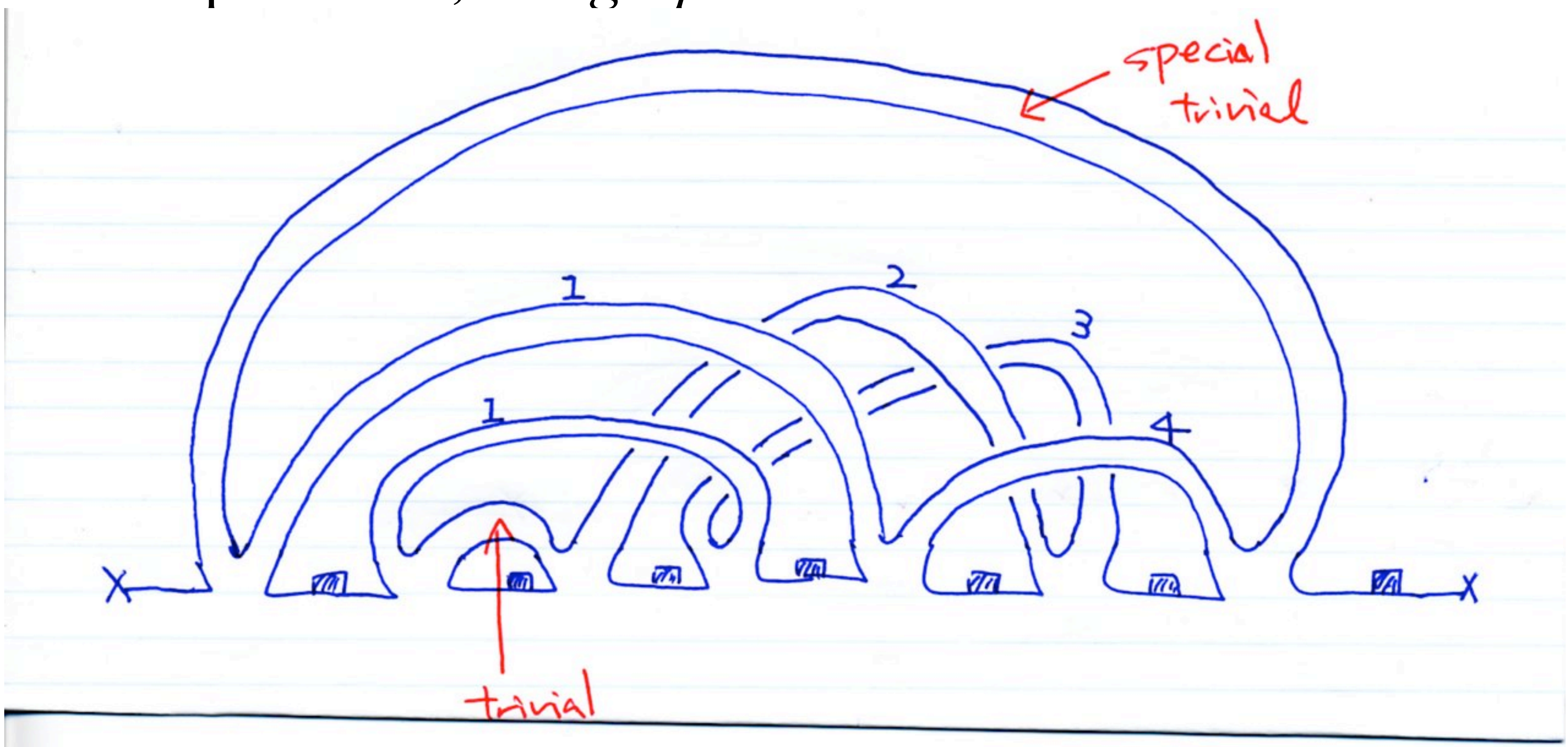

substructures
First Interaction Vertex

It can be shown that generic self-energy graph takes the following form:

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 + S[\tilde{G}(\omega)]\,\tilde{G}(\omega)\right)$$

$$\frac{i}{\omega}S[\tilde{G}(\omega)]\tilde{G}(\omega) \;\; = \sum_{1PI,2PI graphs} \frac{iy^b}{\omega}\left(\frac{-ihN\tilde{G}(\omega)}{1-y}\right)^v \left(\frac{1}{N^2}\right)^g$$

- An example of *1PI, 2PI graph*



It can be shown that generic self-energy graph takes the following form:

$$\tilde{G}(\omega) = \frac{i}{\omega}\left(1 + S[\tilde{G}(\omega)]\,\tilde{G}(\omega)\right)$$

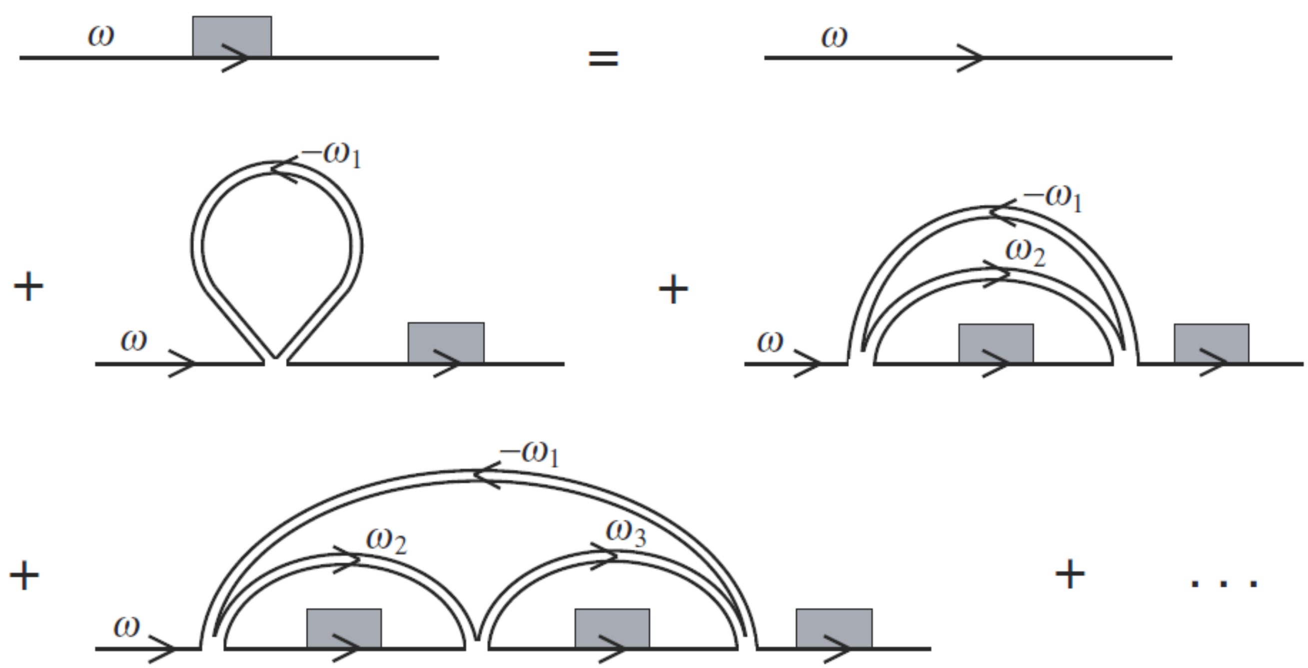
$$S[\tilde{G}]\tilde{G} \;=\; \sum_{1PI,2PIgraphs} y^b \left(\frac{-ihN\tilde{G}(\omega)}{1-y}\right)^v \left(\frac{1}{N^2}\right)^g$$

- *We now need to sum over genus 1 graph S[G(w)]*
- *Using the property that we can always embed these on disc with one handle, we classify graphs*
- *<u>The contributions of 'trivial propagators' are geometric sum,</u> as seen in planar case, its effects are taken into account by*

$$h \to h_{eff} = h/(1 + i\lambda_y \tilde{G}(\omega))$$

In planar case we obtained;

$$
\begin{aligned}
S[\tilde{G}(\omega)]\tilde{G}(\omega) &= \sum_{n=0}^{\infty} y \left(\frac{-ihN}{1-y}\right)^{n+1} \tilde{G}(\omega)^n \\
&= -iy\lambda_y/(1+i\lambda_y\tilde{G}(\omega))\,.
\end{aligned}
$$



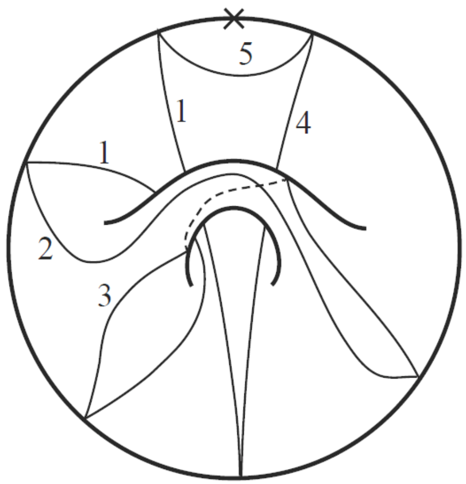
Counting genus 1 graphs:

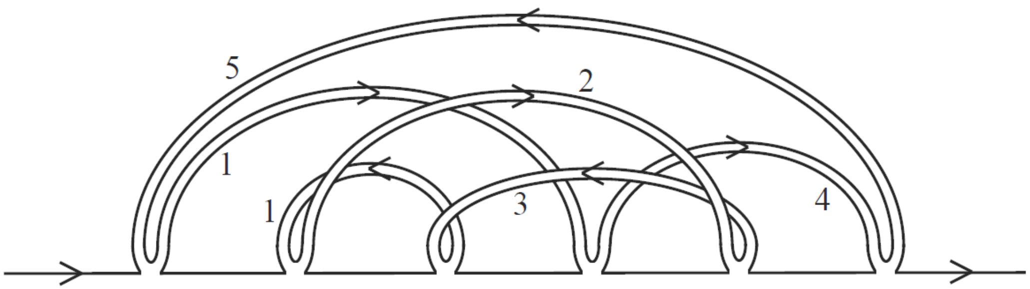
- 1PI, 2PI, and NO trivial propagator graphs are classified by how many non-trivial adjoint propagators graph has;

$$n_1 \geq 1, n_2 \geq 1,$$
$$n_3 \geq 0, n_4 \geq 0,$$
$$n_5 = \{0, 1\}$$

- Ordering gives;

$$(n_1, n_2, n_3, n_4)$$
$$= (OOOE)$$
$$= (EOOO)$$
$$= (OEEO)$$
$$= (EEEE)$$





It can be shown that generic self-energy graph takes the following form:

$$\tilde{G}(\omega) = \frac{i}{\omega}\Big(1 + S[\tilde{G}(\omega)]\,\tilde{G}(\omega)\Big)$$

$$\begin{aligned} S[\tilde{G}(\omega)]\tilde{G}(\omega) &= \sum_{1PI,2PIgraphs} y^b\left(-i\lambda_y\tilde{G}(\omega)\right)^v\left(\frac{1}{N^2}\right)^g \\ &= \sum_{1PI,2PI,NTPgraphs} y^b\left(\frac{-i\lambda_y\tilde{G}(\omega)}{1+i\lambda_y\tilde{G}(\omega)}\right)^v\left(\frac{1}{N^2}\right)^g . \end{aligned}$$

- *Given each set of integers (n1,n2,n3,n4,n5), it is straightforward to evaluate the values b and v as a function of these n's*
- *summing over graphs are equivalent to summing over these integers*

$$
\begin{aligned}
& S[\tilde{G}(\omega)]\tilde{G}(\omega)|_{g=1} \\
= & \sum_{1PI,2PI,NTPgraphs} y^b \left(\frac{-i\lambda_y \tilde{G}(\omega)}{1+i\lambda_y \tilde{G}(\omega)}\right)^v (\frac{1}{N^2})^1 . \\
= & \frac{1}{N^2} \sum_{1PI,2PI,NTPgraphs} y^b W[\tilde{G}]^v \\
= & \frac{1}{N^2} \sum_{OOOE} (W^n + W^{n+1}) y^{(n+1)/2} \\
+ & \frac{1}{N^2} \sum_{EOOO} (W^n + W^{n+1}) y^{(n+1)/2} \\
+ & \frac{1}{N^2} \sum_{OEEO} (W^n y^{(n+2)/2} + W^{n+1} y^{n/2}) \\
+ & \frac{1}{N^2} \sum_{EEEE} (W^n y^n/2 + W^{n+1} y^{(n+2)/2}) \\
= & \frac{1}{N^2} \frac{W[\tilde{G}]^3 y^2 (1+W[\tilde{G}])^2 (1+W[\tilde{G}]y)}{(1-W[\tilde{G}]^2 y)^4}
\end{aligned}
$$

$$\tilde{G}(T,\omega) = \tilde{G}^{(0)}(T,\omega) + \frac{1}{N^2}\tilde{G}^{(1)}(T,\omega) + \mathcal{O}\left(\frac{1}{N^4}\right)$$

$$\tilde{G}(T,\omega) = \frac{i}{\omega} + \frac{i}{\omega} S_{\mathrm{I}^*}(T,\omega,h',G)\,\tilde{G}(T,\omega)$$

$$x_0 = -i\lambda_y \tilde{G}^{(0)}(\omega)$$

$$\tilde{G}^{(1)}(T,\omega) = \frac{iy^2 x_0^3 (1-x_0)^4 (1-x_0[1-y])}{(1-2x_0+x_0^2[1-y])^4(\omega[1-x_0]^2-\lambda_y y)}\,.$$

- Leading $1/N^2$ correction for the spectrum $r = ImG(w + ie)$ from $g=1$ graphs does not change the branch point

Conclusion: Part II

- In *our* toy model, $1/N^2$ corrections does not help to restore the information come back
- This is expected since non-pert. width of $e^{(-N^2)} << 1/N^2$ precision spectrum is crucial
- One goal is to obtain some solvable model where we can obtain *full $1/N^2$ expansion* and re-sum that in systematic way: so that we can obtain the finite N effect
- Systematic understanding of $1/N^2$ expansion is necessary
- More analysis should be done to understand better

More comments…

- In our model, we gave temperature T for the adjoint (black hole), therefore, black hole is in thermal state (mixed states)
- In the large N limit, there is no difference between canonical ensemble and microcanonical, but in finite N, they are different
- In order to understand black hole physics in canonical ensemble, we should not give temperature to adjoint, but rather just give energy to form black hole
- From $(N+2)$ D0-quantum theory, we can describe two D0-prove head-on collision to form black hole in AdS (or M-theory) background, if their kinetic energy is large enough as $O(N^2)$. It will be interesting to understand this better, for example, how fast can matrix be thermalized?

Thank You