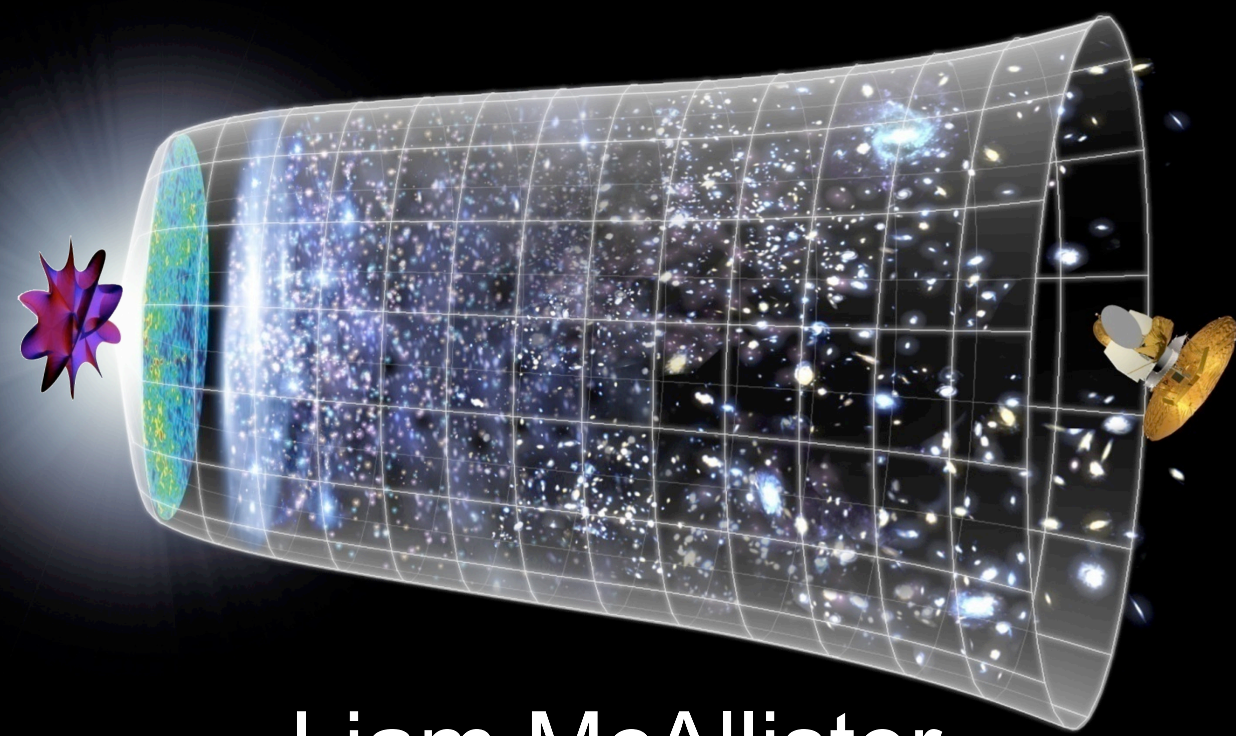
Holographic Systematics of D-Brane Inflation



Liam McAllister
Cornell

ISM08, Pondicherry

December 9, 2008

NASA/WMAP Scienc

Based on:

D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, & L.M., 0808.2811.

Building on:

S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L.M., S. Trivedi, hep-th/0308055.
D. Baumann, A. Dymarsky, I. Klebanov, J. Maldacena, L.M., & A. Murugan, hep-th/0607050.
D. Baumann, A. Dymarsky, I. Klebanov, L.M., & P. Steinhardt, 0705.3837.
D. Baumann, A. Dymarsky, I. Klebanov, & L.M., 0706.0360.

See also:

C. Burgess, J. Cline, K. Dasgupta, & H. Firouzjahi, hep-th/0610320.
O. DeWolfe, L.M., G.Shiu, & B. Underwood, hep-th/0703088.
A. Krause & E. Pajer, 0705.4682.
S. Panda, M. Sami, S. Tsujikawa, 0707.2848.
A. Ali, R. Chingangbam, S. Panda, M. Sami, 0809.4941.

Plan

I. Motivation and background
   I. Inflation
   II. Inflation in string theory
II. Case study: warped D-brane inflation
III. Direct approach: computation of nonperturbative superpotential
IV. Systematic method: enumeration of contributing modes
V. Pre-Conclusions
VI. Remarks on primordial tensor signature
VII. Conclusions

Part I.

Inflation
and
String Inflation

Inflation

A period of accelerated expansion

$$ds^2 = -dt^2 + e^{2Ht} d\vec{x}^2 \qquad H \approx const.$$

- Solves horizon, flatness, and monopole problems. *i.e.* explains why universe is so large, so flat, and so empty.

A.Guth,1981.

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A.Guth,1981.

- Predicts scalar fluctuations in CMB temperature:
  - approximately, but not exactly, scale-invariant
  - approximately Gaussian
- Predicts primordial tensor fluctuations

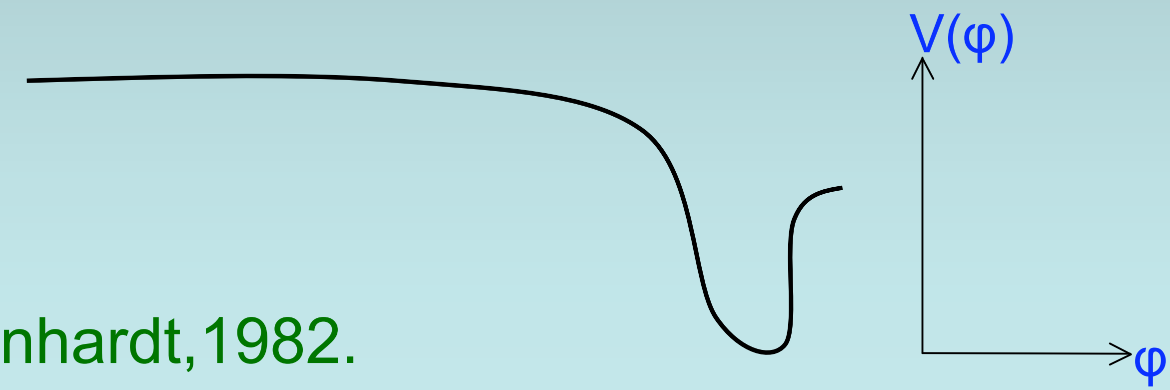
Physics of inflation

- Scalar field φ with a potential. V(φ) ~ 10^{16} GeV

L [illegible]

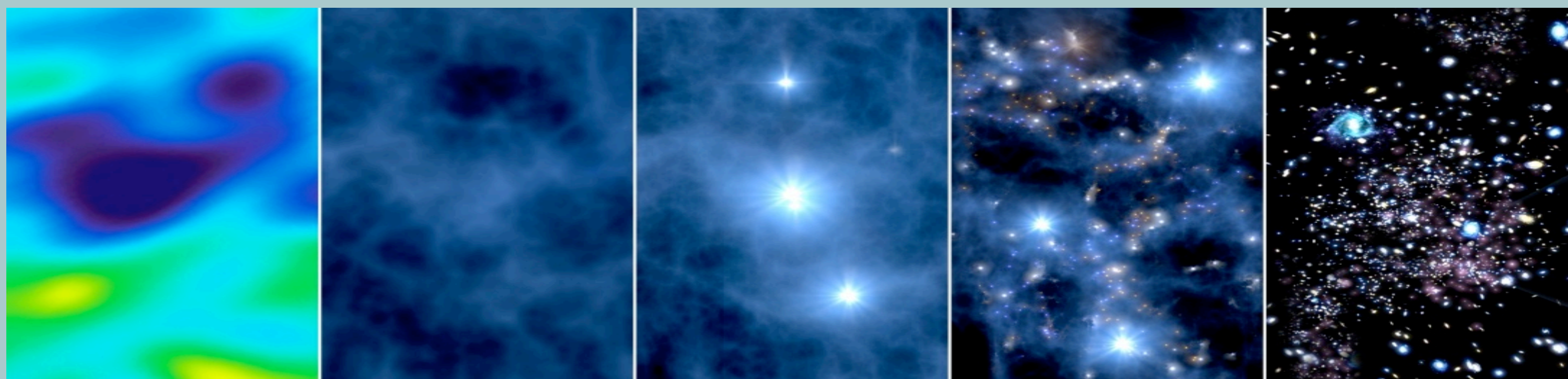
- Potential drives acceleration.
- Acceleration is prolonged if V(φ) is flat in Planck units:

$$\eta \equiv M_{pl}^{2}\frac{V''}{V} \quad 1 \quad \text{and} \quad \varepsilon \equiv \frac{1}{2}M_{pl}^{2}\left(\frac{V'}{V}\right)^{2} \quad 1$$



A.Linde,1982.
A. Albrecht and P.Steinhardt,1982.

Quantum fluctuations seed structure formation



Perturbations are small enough to permit linear analysis, for a while.
This is a very clean and tractable system.

Prediction: these perturbations are approximately scale-invariant.

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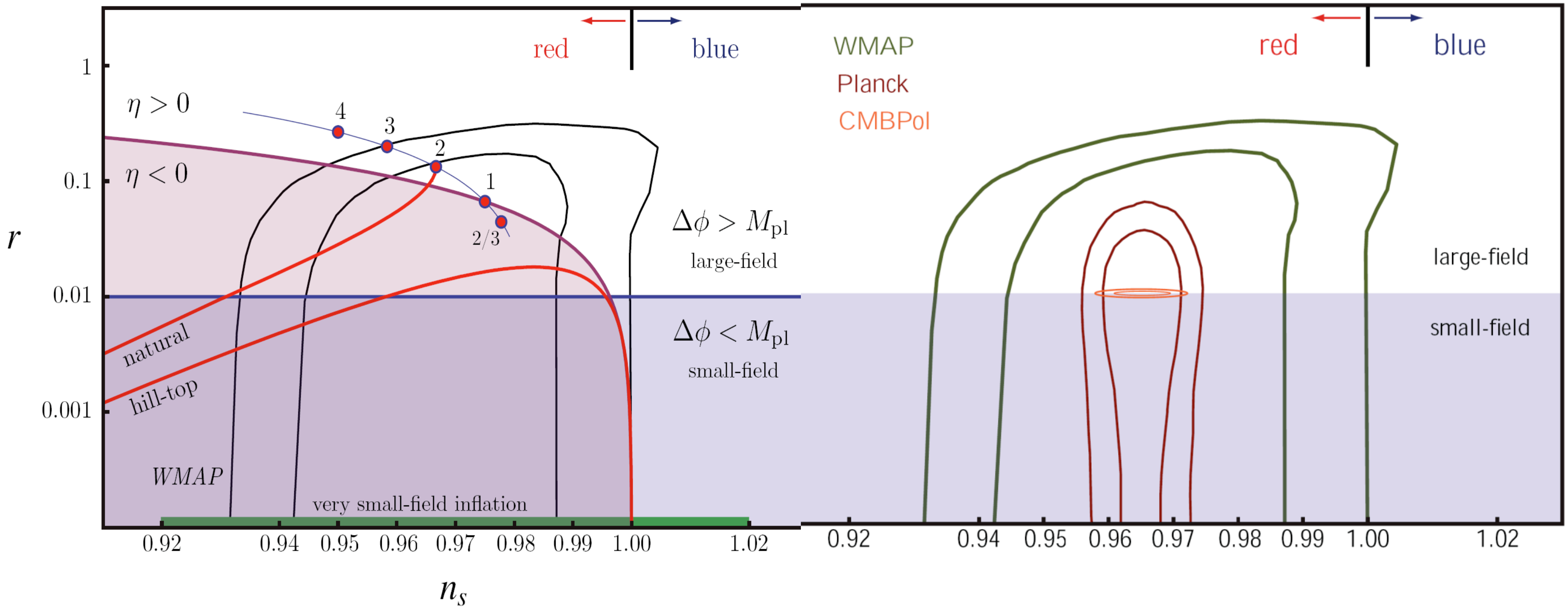
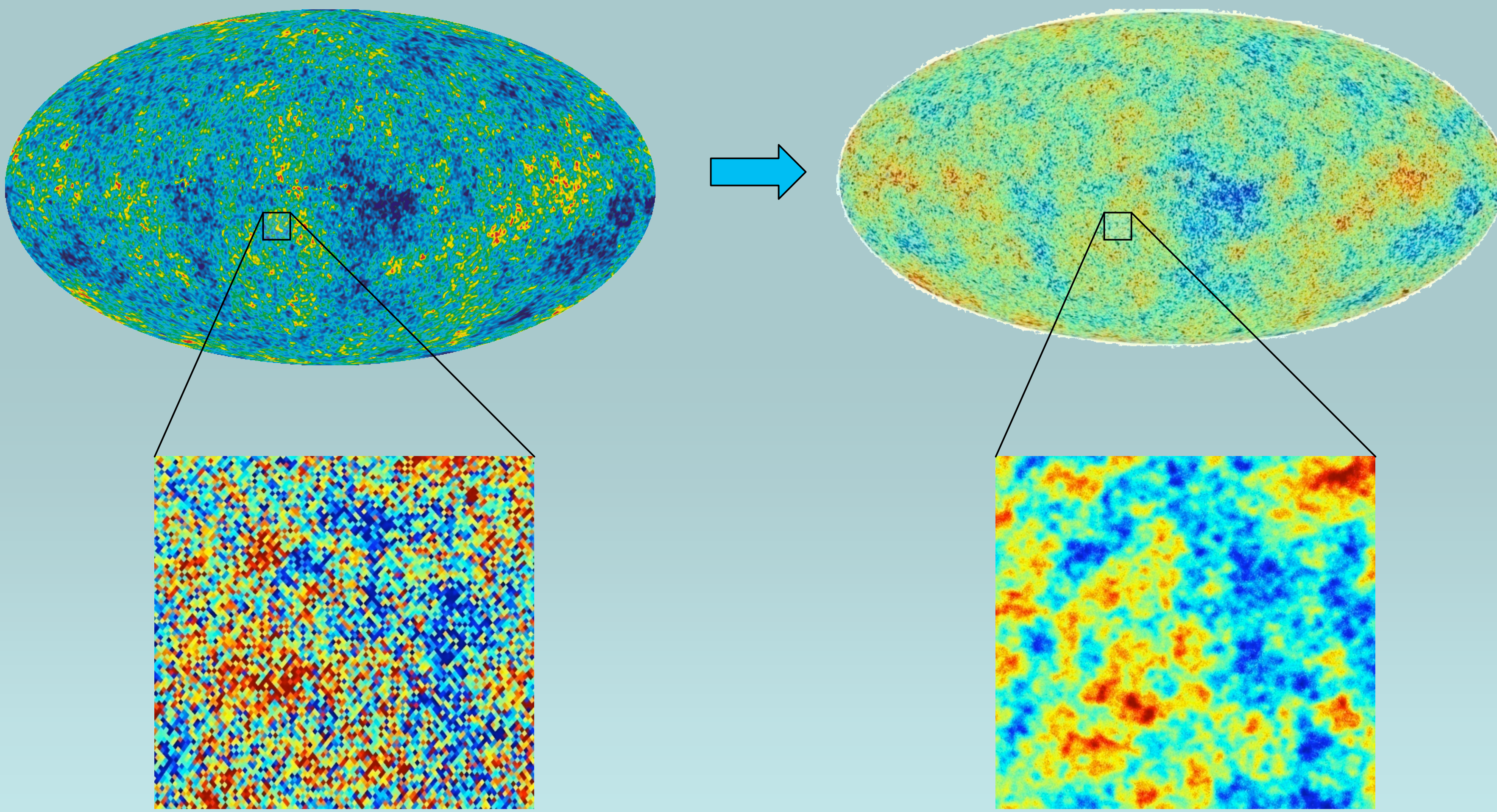
Current Constraints



[redacted]

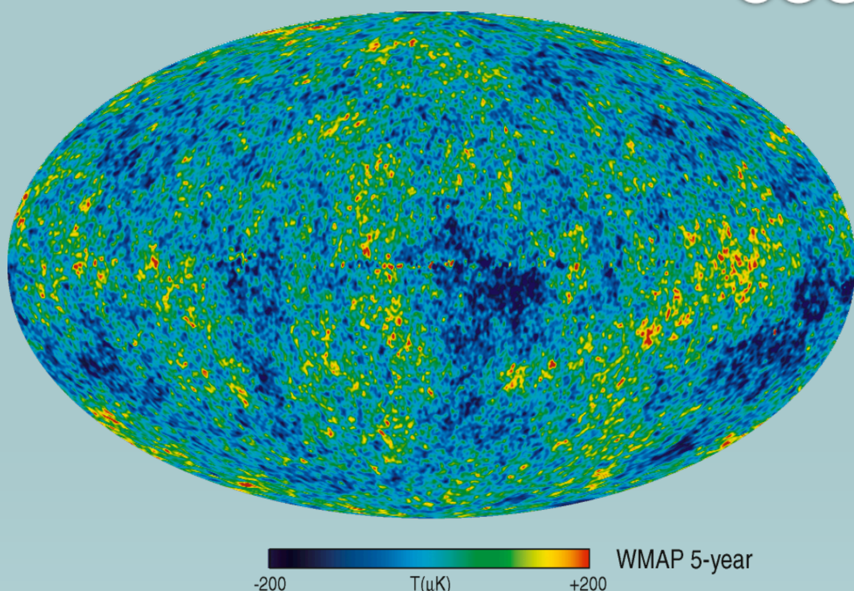
Expect dramatic improvements in these limits in next 5-10 years:
Planck, SPIDER, EBEX, Clover, QUIET, BICEP, PolarBEAR,...

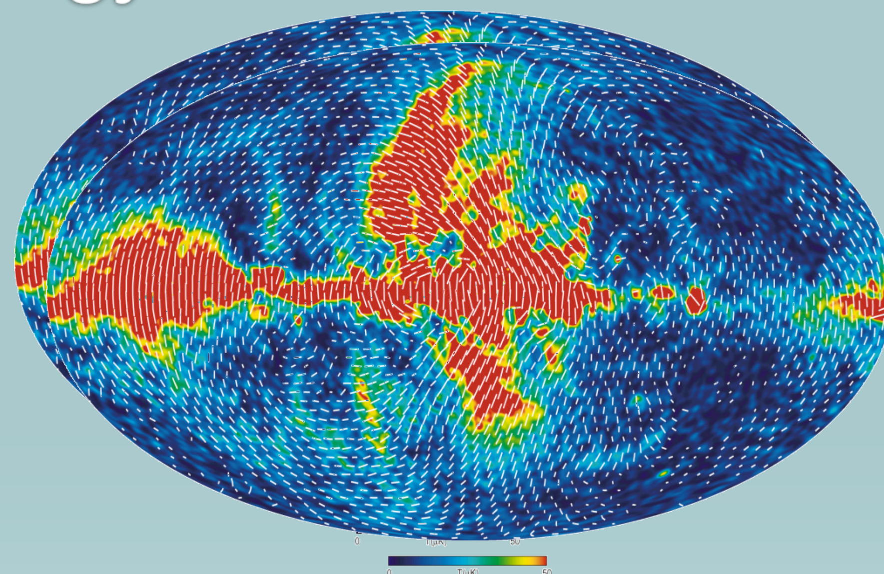
From WMAP to PLANCK (2010?)

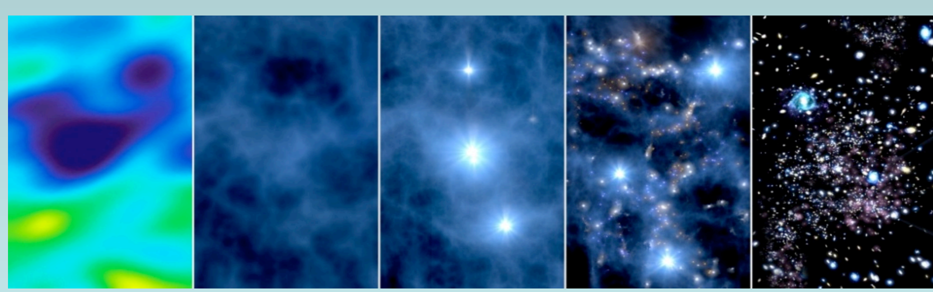


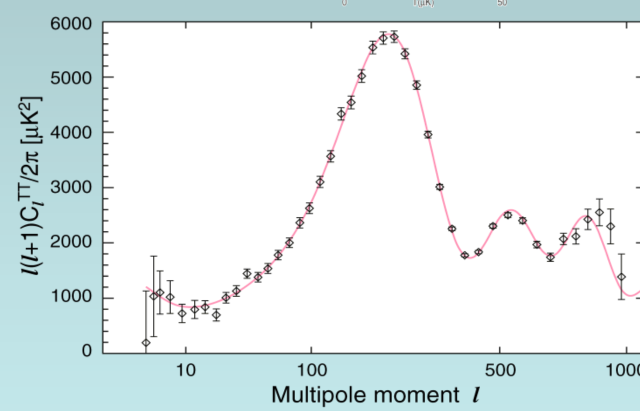
WMAP; C. Cantalupo, J. Borrill, and R. Stompor

Goal: develop theoretical models of the early universe suitable for an era of precision cosmology.









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Since inflation occurred at sub-Planckian energies, why not just use effective QFT?

- Consistent if one is careful. However, inflation is UV-sensitive!
- So, would like toy models (at least) in which Planck-scale theory is specified.
- Moreover, rare opportunity to probe (some aspects of) Planck-scale physics.

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String theory provides:

- many fundamental scalar fields
- considerable UV control

Hope:

- use string theory to clarify UV issues in inflation
- use UV-sensitive aspects of inflation to constrain string theory – or at least, constrain string theory models of inflation

Warning: many aspects of inflation are not UV-sensitive.

Aim:

Use CMB and LSS to probe fundamental physics.

Method:

Propose and constrain mechanisms for inflation in string theory.

Focus on correlated signatures (tilt; tensors; non-Gaussianity; cosmic strings; isocurvature;...).

Task:

Derive inflaton action in consistent, computable string compactifications with stabilized moduli.

Status Report: Inflation in String Theory

- We now have
  - many scenarios that are partially constructed in string theory.
  - a few scenarios that are complete or nearly so
- Modern models are far more realistic than those of 5-10 years ago. Moduli problem explicitly solved in a few cases, implicitly in a few more.
- Making correct predictions is still very difficult!
  - A key lesson across scenarios has been that solving the moduli problem changes the scenario and can shift n_s, r dramatically. So one should be skeptical of predictions for these quantities in the absence of stabilization.
- It is clear that many complete models will soon exist. At present only a few are worked out in enough detail to make correct predictions.
- This talk: one of the best-understood scenarios.

The eta problem

- Successful inflation requires controlling Planck-suppressed contributions to inflaton potential.

e.g. 

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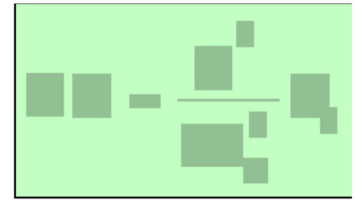
e.g. [illegible] [illegible]

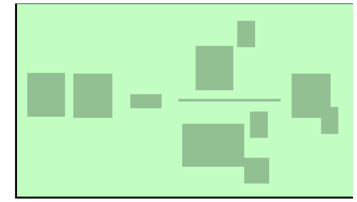
[illegible]

[illegible]

The eta problem from an F-term potential



$$V \approx V_F = e^{K/M_{pl}^2} \left(K^{A\bar{B}} D_A W D_{\bar{B}} \overline{W} - \frac{3}{M_{pl}^2} |W|^2 \right)$$



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$$K = K(0) + K_{,\phi\bar{\phi}}(0)\phi\bar{\phi} + \ldots$$

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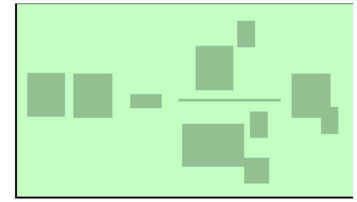

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$$K = K(0) + K_{,\phi\bar{\phi}}(0)\phi\bar{\phi} + \ldots$$

$$\mathcal{L} \approx K_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi} -$$

$$- e^{K(0)/M_{pl}^2}\left(1 + \frac{1}{M_{pl}^2}K_{,\phi\bar{\phi}}\phi\bar{\phi}\right)\left(K^{A\bar{B}} D_A W D_{\bar{B}}\overline{W} - \frac{3}{M_{pl}^2}|W|^2\right) + \ldots .$$

the canonically-normalized inflaton φ obeys $\partial\varphi\partial\bar{\varphi} \approx K_{,\phi\bar{\phi}}(0)\partial\phi\partial\bar{\phi}$,

$$\Delta m_\varphi^2 \approx V_F(0)/M_{pl}^2 = 3H^2$$

Moduli stabilization and the eta problem

- Inflation requires controlling Planck-suppressed contributions to the inflaton potential up to dimension 6.

- Moduli stabilization almost always introduces such terms.
- Knowing (and controlling) the inflaton potential therefore requires detailed information about moduli stabilization.

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- Moduli stabilization almost always introduces such terms.
- Knowing (and controlling) the inflaton potential therefore requires detailed information about moduli stabilization.
- 'Detailed' means one usually needs to know string + α' corrections, backreaction effects, etc.
- Common approximations (classical, noncompact/large-volume, probe...) are often insufficient.
- String incarnation of (misnamed) supergravity eta problem (Copeland, Liddle, Lyth, Stewart, & Wands astro-ph/9401011).

Today's talk

- Focus on a specific, concrete model: warped D-brane inflation.
- Determine form of inflaton potential, incorporating **all relevant contributions.**
  - approached directly, this is highly nontrivial.
  - We have explicitly computed a wide class of contributions.
  - to finish the job, we will take a different tack, inspired by AdS /CFT, that gives the complete answer and reduces to our earlier result in the appropriate limit.
- This gives very substantial progress towards an existence proof.
- It also provides a truly general characterization of possible phenomenological models in this scenario.

Part II.

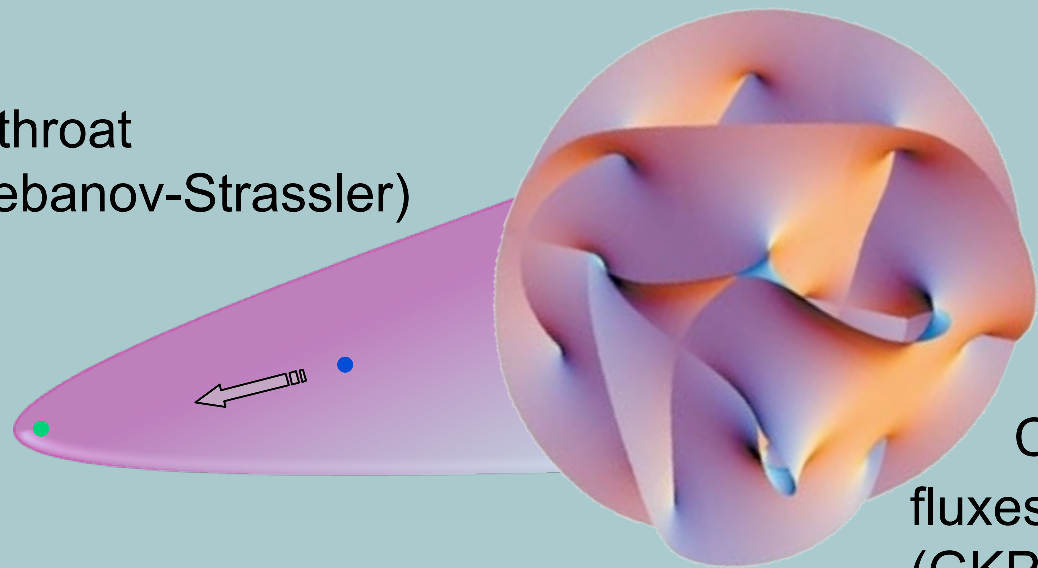
D-brane Inflation

initiated in:

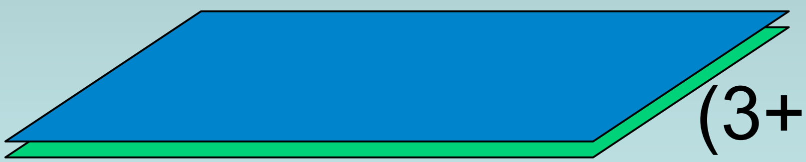
Dvali&Tye 1998
Dvali,Shafi,Solganik 2001
Burgess,Majumdar,Nolte,Quevedo,Rajesh,Zhang 2001

Warped D-brane inflation

warped throat
(e.g. Klebanov-Strassler)



CY orientifold, with
fluxes and nonperturbative W
(GKP 2001, KKLT 2003)



warped throat gives:
- weak Coulomb potential
- control of energy scales
- **explicit local geometry**
- **dual CFT**

Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003

Throat Geometry

Finite-length KS throat, which we approximate by the Klebanov-Tseytlin solution:

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(dr^2 + r^2 ds^2_{T^{1,1}})$$

$$h(r) = \frac{27\pi}{4r^4}\alpha'^2 g_s M\left(K + g_s M\left(\frac{3}{8\pi} + \frac{3}{2\pi}\ln(\frac{\mathrm{r}}{\mathrm{r_{max}}})\right)\right)$$

Can ignore logs and approximate as AdS_5 x $T^{1,1}$:

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + dx_i^2) + \frac{R^2}{r^2}dr^2$$ + angles

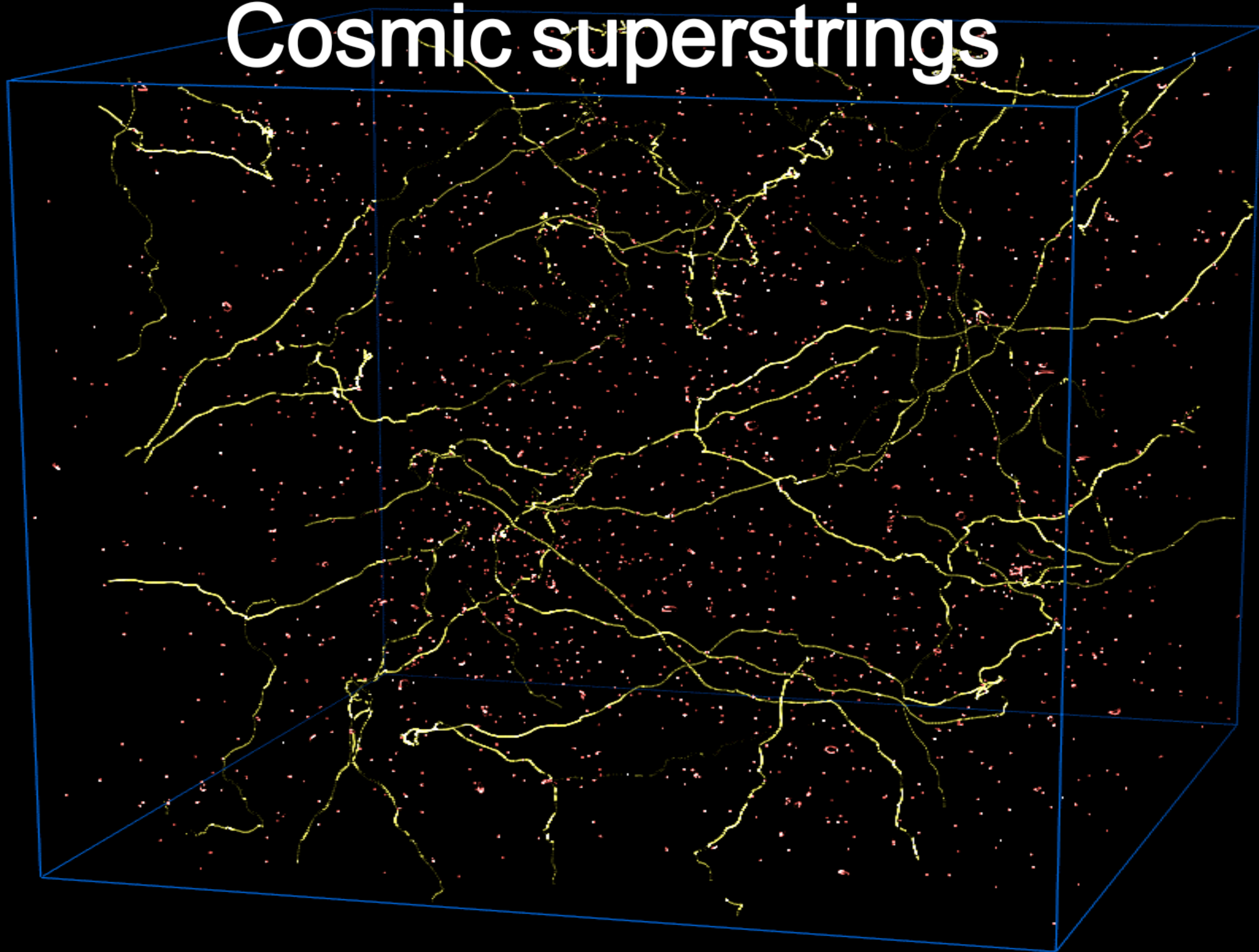
$$R^4 = \frac{27}{4}\pi g_s N\alpha'^2, \quad N \equiv KM$$

Coulomb Potential

$$V = 2T_3 \frac{r_0^4}{R^4}\left(1 - \frac{1}{N}\frac{r_0^4}{r_1^4}\right)$$

- Looks extremely flat, because interaction term is warped.
- If this were the whole story, the model would be perfect
  - dimensional transmutation explains smallness of slow-roll parameters
  - inflation has a natural end, with the possibility of interesting topological defects from tachyon condensation: cosmic superstrings.

Cosmic superstrings



B. Allen and E.P. Shellard

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  - dimensional transmutation explains smallness of slow-roll parameters
  - inflation has a natural end, with the possibility of interesting topological defects from tachyon condensation: cosmic superstrings.
- In reality, many more important contributions to the potential!
- Origin: moduli stabilization and other `compactification effects'
- We need to understand and incorporate these effects.

Does it work?

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + H^2\phi^2 + \Delta V(\phi)$$

?

known in initial proposal ($V_{D3/\overline{D3}}(\phi) + H^2\phi^2$)

introduced by moduli stabilization ($H^2\phi^2 + \Delta V(\phi)$)

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One contribution to ΔV determined in 2006-2007:

D. Baumann, A. Dymarsky, I. Klebanov, J. Maldacena, L.M., & A. Murugan, hep-th/0607050.
C. Burgess, J. Cline, K. Dasgupta, & H. Firouzjahi, hep-th/0610320.
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D. Baumann, A. Dymarsky, I. Klebanov, & L.M., 0706.0360.

Complete parameterization: D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, & L.M., 0808.2811.

Part III.

The D3-Brane Potential

D3-branes in flux compactifications

$$ds^2 = e^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) \mathrm{d}y^m \mathrm{d}y^n$$

$$\tilde{F}_5 = (1 + \star_{10}) \left[\mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3\right]$$

$$V = T_3 \, \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

GKP solutions:

$$G_- = \Phi_- = 0$$

$$G_\pm \equiv (i \pm \star_6) G_3$$

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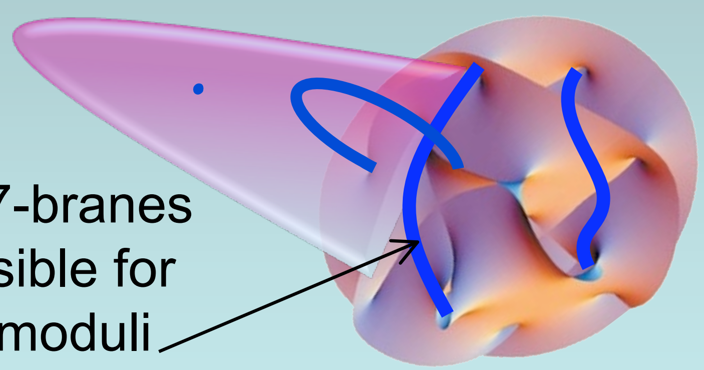
D3-branes feel no potential in GKP solutions (‘no-scale’),
but
nonperturbative stabilization of Kahler moduli will spoil this.

D3-brane vacua in KKLT compactifications

$$W = \int G_3 \wedge \Omega + A(y) e^{-a\rho}\,, \qquad \mathcal{K} = -3\,\log\Big(\rho + \bar{\rho} - k(y,\bar{y})\Big)$$



ED3/D7-branes responsible for Kahler moduli stabilization

I will assume that
  # fermion zero modes = 2.
cf. A. Uranga's talk

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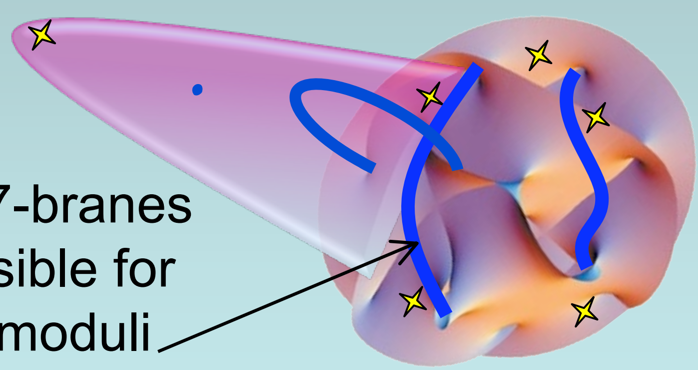
For generic A(y), solutions to $D_\rho W = D_y W = 0$

i.e., supersymmetric D3-brane vacua, are **isolated**. But where are they, and what is the potential in between?



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DeWolfe, L.M., Shiu, & Underwood, hep-th/0703088.

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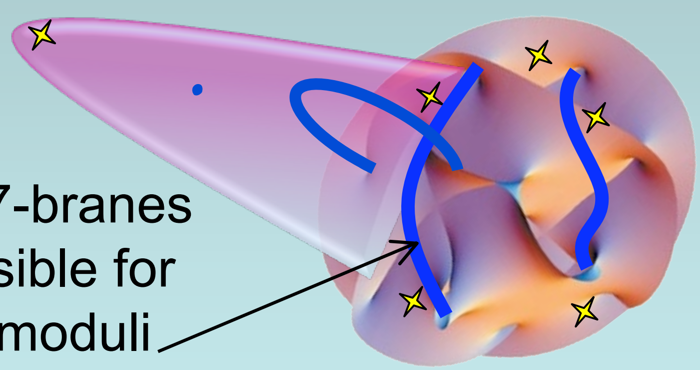

ED3/D7-branes responsible for Kahler moduli stabilization

We need to know A(y).

DeWolfe, L.M., Shiu, & Underwood, hep-th/0703088.

Q. Can one compute A(y) in a special case?

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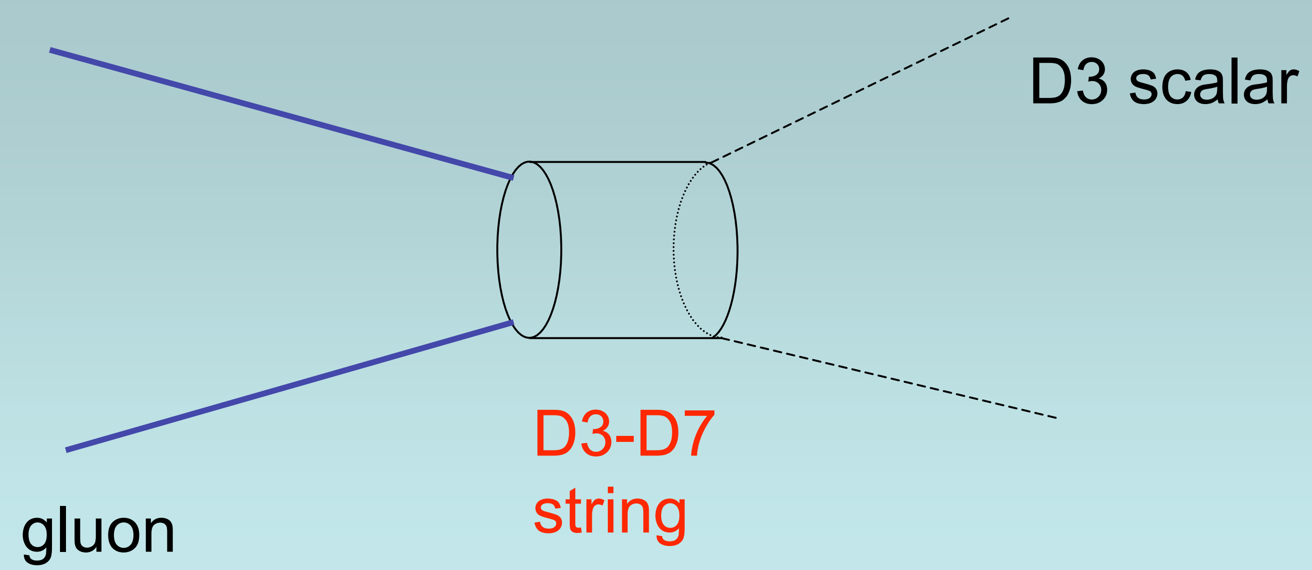
In case of gaugino condensation, they compute dependence on D3 position y as threshold correction due to 3-7 strings.



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Q. What about a case directly applicable to a D3-brane in a warped throat?

Nonperturbative Effects

[illegible]

[illegible]

- Gaugino condensation on N D7-branes wrapping a four-cycle Σ_4

[illegible]

- Euclidean D3-branes wrapping a four-cycle Σ_4

[illegible]

Either case: can write

[illegible]

Corrected Warped Volumes

[illegible]

[illegible] (probe approximation)

Including D3-brane backreaction:

h = h(Y,X)

D3-brane position

[illegible] $\delta h(X,Y)$

[illegible] $\delta h(X,Y)$

V_0 δV

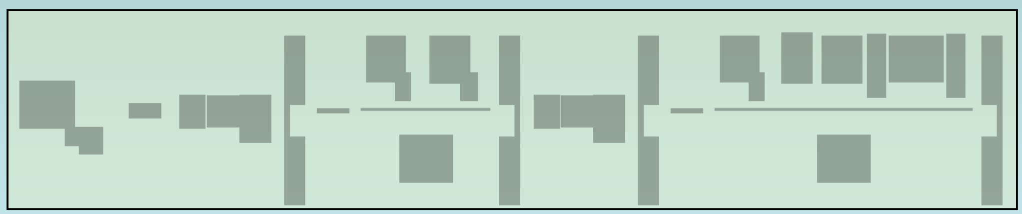
D3-brane Backreaction

$$h = h_0(Y) + \delta h(X,Y)$$

$$V_{\Sigma_4}^w = \underbrace{\int_{\Sigma_4} d^4Y \sqrt{g}\, h_0(Y)}_{V_0} + \underbrace{\int_{\Sigma_4} d^4Y \sqrt{g}\, \delta h(X,Y)}_{\delta V}$$

$$\nabla_Y^2 \delta h(X,Y) = -2\kappa_{(10)}^2 T_3 \left[\delta^6(X-Y) - \rho_{bg}(Y)\right]$$

- Solve for δh
- Integrate over Σ_4 to get δV(X)
- Read off δW(X)



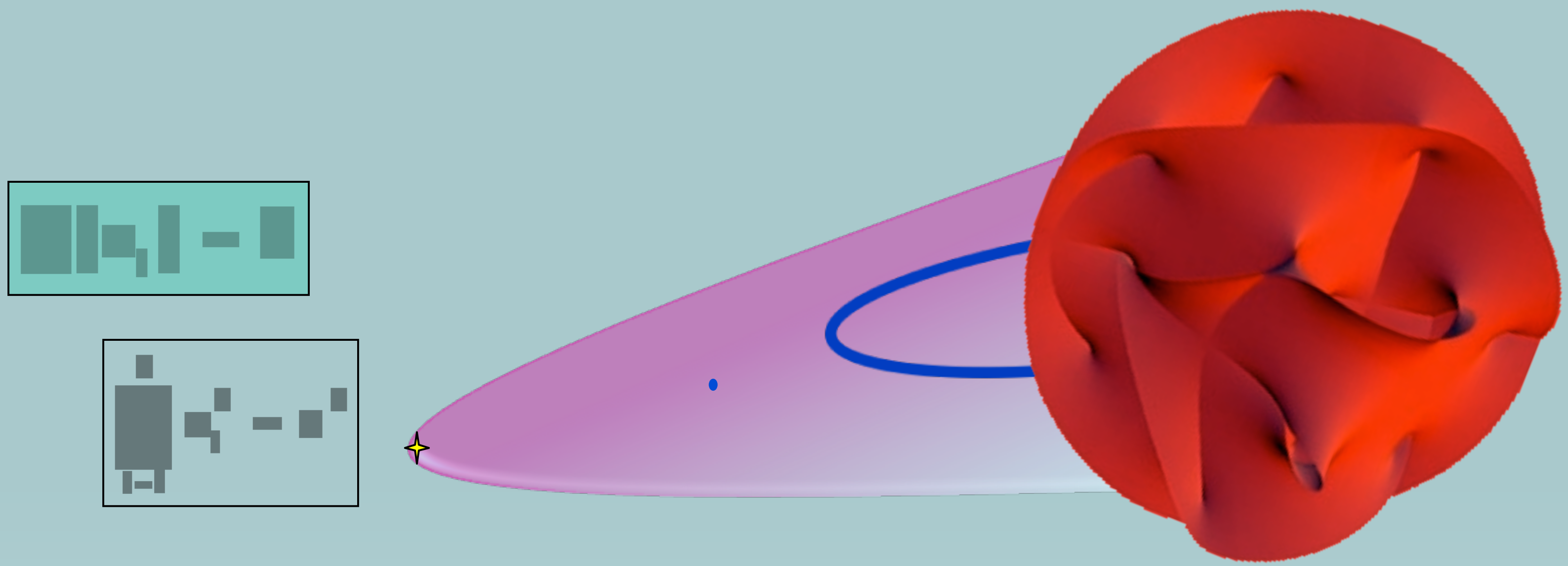
Baumann, Dymarsky, Klebanov, Maldacena, L.M., & Murugan, hep-th/0607050.

Comparison

Open String BHK	Closed String Giddings-Maharana; Us
One-loop open string	Tree-level SUGRA
Threshold correction	Backreaction on warping
Gaugino only	Gaugino or ED3
Hard	Comparatively easy
Toroidal cases only	More general geometries
Unwarped only	Warping ok

Perfect agreement where comparison is possible.

A tractable background for computing A(y): a four-cycle embedded in the throat



Ideally, embedding should be SUSY in KS throat.

Result for a general warped throat

Ganor, hep-th/9612007
Baumann, Dymarsky, Klebanov, Maldacena, L.M., & Murugan, hep-th/0607050.

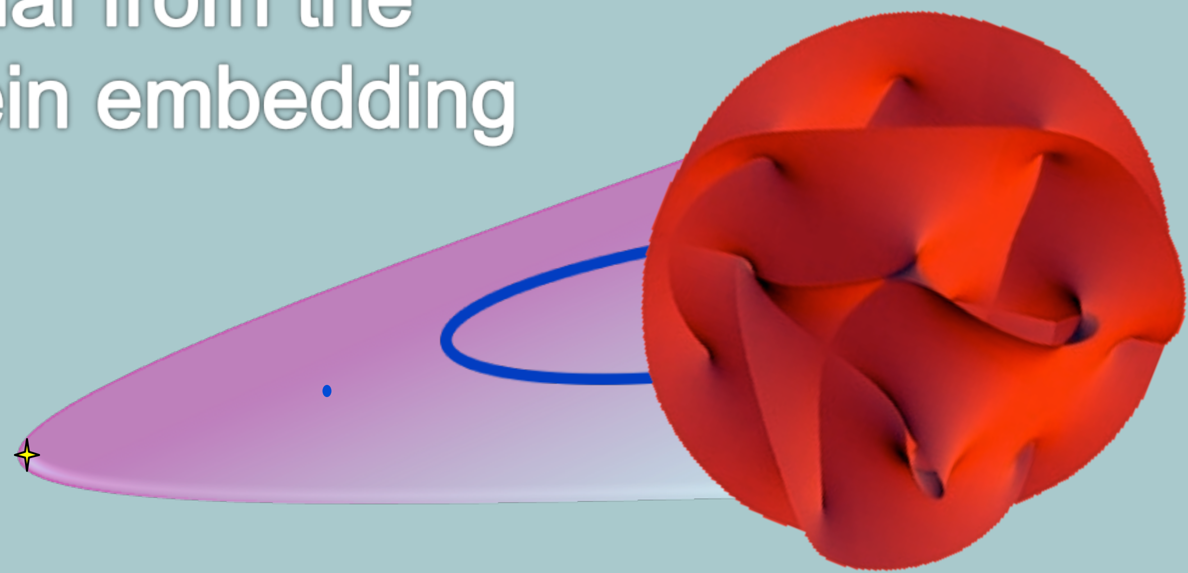
If N wrapped branes are embedded along

then the superpotential correction is

so the superpotential is

$$f(z_i)^{1/N}$$

Potential from the Kuperstein embedding

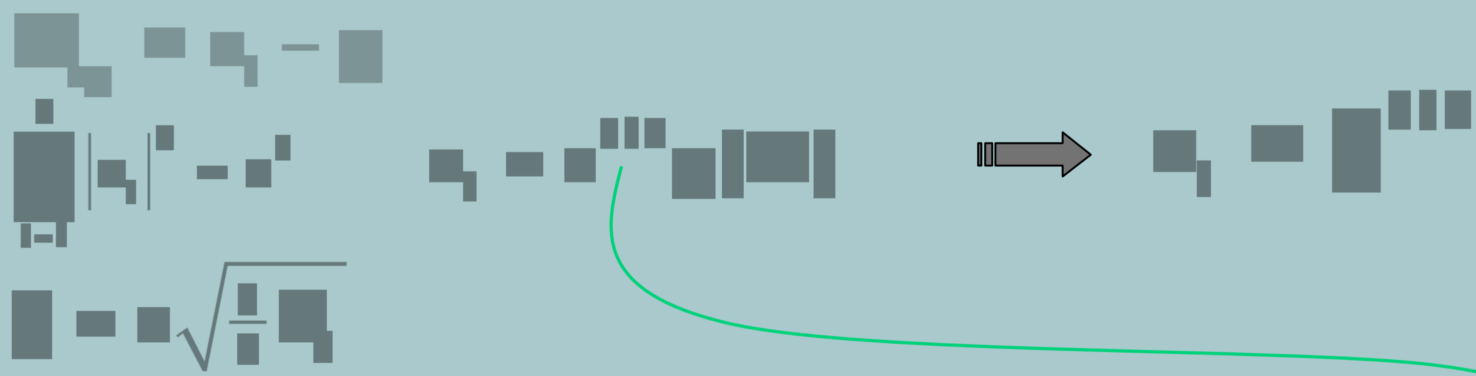


S. Kuperstein, hep-th/0411097

$(z_1-\mu)^{1/N}$

Holomorphy + throat geometry constrain corrections to potential

Holomorphy + throat geometry constrain corrections to potential



$$V(\phi) \; = \; V_{D3/\overline{D3}}(\phi) \; + \; M_{\rm pl}^2 H^2 \left[\left(\frac{\phi}{M_{\rm pl}} \right)^2 - a_{3/2} \left(\frac{\phi}{M_{\rm pl}} \right)^{3/2} \right]$$

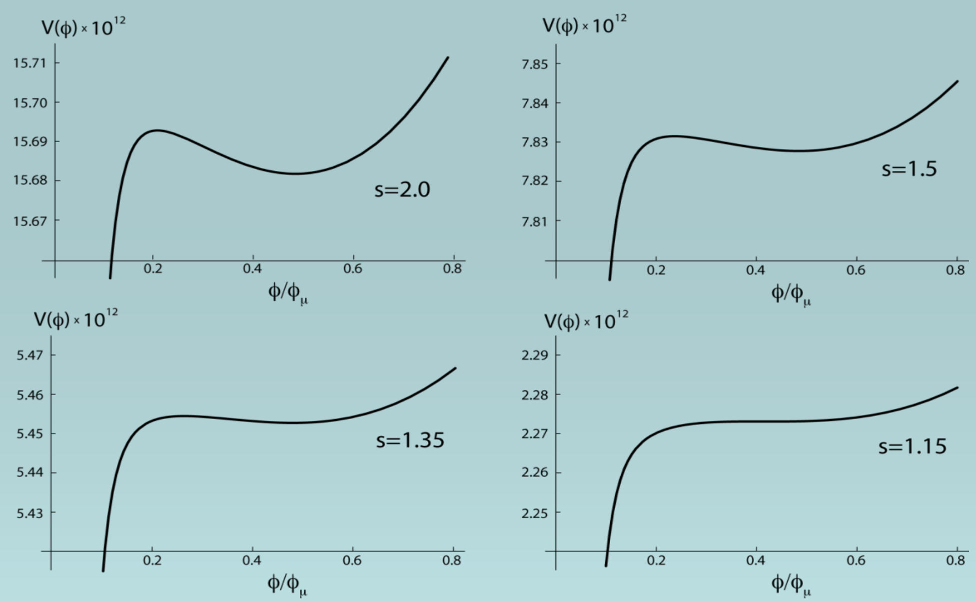
new term in ΔV

No quadratic correction from this embedding!

i.e. still cannot fine-tune inflaton mass per se

A Nearly Explicit Model of D-brane inflation

Baumann, Dymarsky, Klebanov, L.M., & Steinhardt, 0705.3837;
Baumann, Dymarsky, Klebanov, & L.M., 0706.0360.
Krause & Pajer, 0705.4682.
Dynamics:
Panda, Sami, Tsujikawa, 0707.2848.



ΔV does not include a quadratic term.
Can fine-tune only locally.

D7-brane embedding:

For fine-tuned parameters, an inflection point appears.
Inflation can occur near this inflection point.

Part IV.

Holographic Systematics

How can we be more complete and systematic?

- We had to assume that the divisor bearing nonperturbative effects sags into the throat.
  - This is likely not generic, and it imposes strong constraints on the parameters that only reflect our wish to compute, not the underlying physics.
- We have assumed that all other effects are negligible!
  - We have thereby neglected more distant divisors, bulk fluxes, distant antibranes, etc.
- Our result may be sensitive to the gluing of the throat into the compact space.
- Purely practically, while it is easy to compute the superpotential for a general embedding in a general throat, it is slightly subtle and rather involved to read off the complete potential that results.

A Simple Idea

The D3-brane potential comes from Φ_- alone. So we are interested in the profile of Φ_-.

$$V = T_3 \, \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

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The warped geometry filters these effects; the leading contributions are those that diminish least rapidly towards the IR (~ lowest few multipoles).

Normalizable Φ_- profiles arise from supersymmetry-breaking at the tip and are already incorporated.

cf. DeWolfe, Kachru, & Mulligan 0801.1520

Gauge Theory version

Arbitrary compactification effects can be represented by allowing arbitrary deformations of the Lagrangian of the dual (approximate) CFT.

Supersymmetric deformations:
Aharony, Antebi, & Berkooz, **hep-th/0508080**

Gauge Theory version

Arbitrary compactification effects can be represented by allowing arbitrary deformations of the Lagrangian of the dual (approximate) CFT.

These deformations incorporate couplings to bulk moduli X that get F-term vevs.

$$\Delta\mathcal{K} = c \int d^4\theta \; M_{\rm UV}^{-\Delta} \; X^\dagger X \, \mathcal{O}_\Delta \qquad \Rightarrow \qquad \Delta V = c \; M_{\rm UV}^{-\Delta} |F_X|^2 \, \mathcal{O}_\Delta$$

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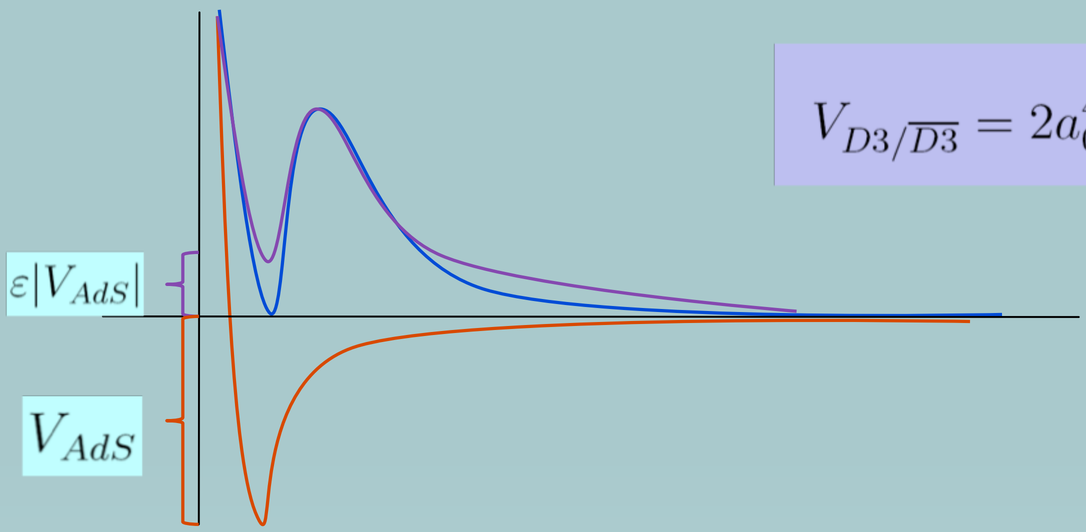
The RG flow filters these effects; the leading contributions are those that diminish least rapidly towards the IR, i.e. the most relevant contributions.

Deformations of the CFT state arise from supersymmetry -breaking at the tip and are already incorporated.

Scale of the moduli F-terms



$$V_{D3/\overline{D3}} = 2a_0^4 T_3 = \varepsilon |V_{AdS}| \,, \qquad \varepsilon \lesssim \mathcal{O}(1)$$

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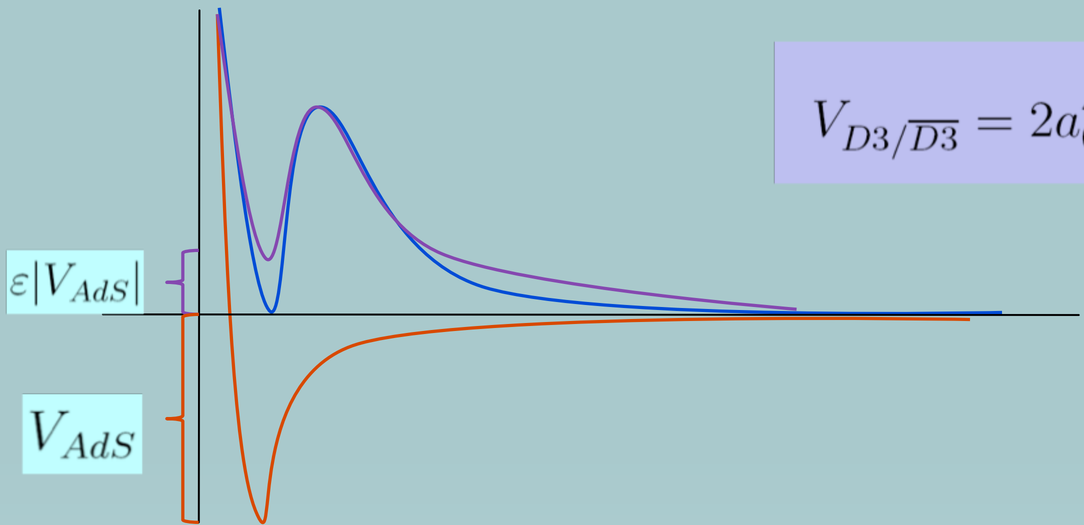

$$V_{D3/\overline{D3}} = 2a_0^4 T_3 = \varepsilon |V_{AdS}| \,, \qquad \varepsilon \lesssim \mathcal{O}(1)$$

$$\Delta V_F(D3) \sim |V_{AdS}| \sim \frac{1}{\varepsilon} \times 2a_0^4 T_3$$

$$V_{\rm UV}(D3) = \frac{c_F}{\varepsilon} 2a_0^4 T_3 \equiv c\, a_0^4 T_3$$

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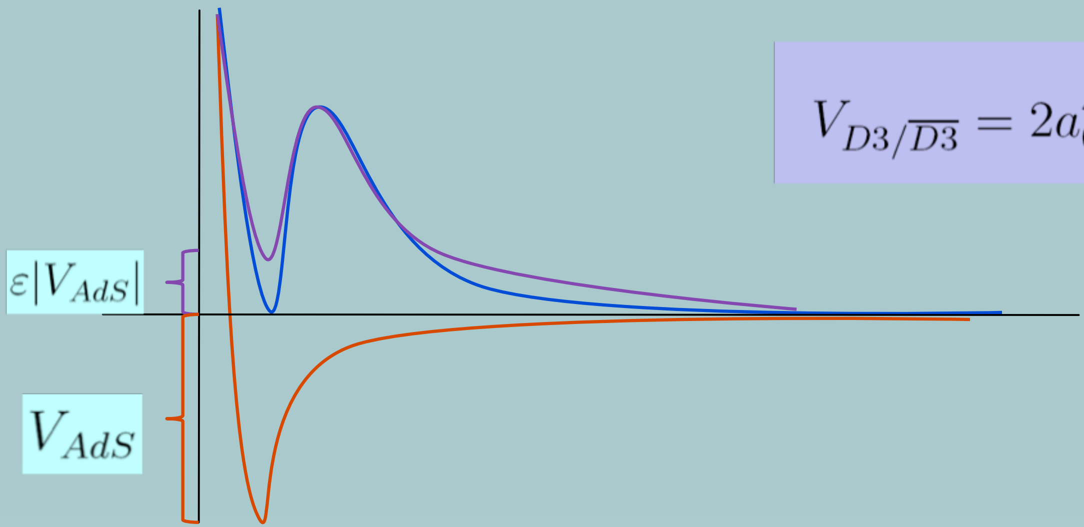

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$$V_{\rm UV}(D3) = \frac{c_F}{\varepsilon} 2a_0^4 T_3 \equiv c\, a_0^4 T_3$$

$$F_X \sim \xi\, a_0^2$$

Method

Consider linearized Φ_- perturbations around a finite-length KS throat, which we approximate by $AdS_5 \times T^{1,1}$.

$$ds^2 = h^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(dr^2 + r^2 ds^2_{T^{1,1}})$$

$$h(r) = \frac{27\pi}{4r^4}\alpha'^2 g_s M \left(K + g_s M \left(\frac{3}{8\pi} + \frac{3}{2\pi}\ln(\frac{\mathrm{r}}{\mathrm{r_{max}}}) \right) \right)$$

In general, many other modes are turned on, but at the linear level they do not couple to D3-branes!

CFT version: interested only in operators that generate a potential on the Coulomb branch.

Linearization around ISD compactifications

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n$$

$$\tilde{F}_5 = (1 + \star_{10}) \left[d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right]$$

$$G_\pm \equiv (i \pm \star_6) G_3$$

$$V = T_3 \, \Phi_-$$

$$\Phi_\pm \equiv e^{4A} \pm \alpha$$

$$\widetilde{\nabla}^2 \Phi_\pm = \frac{e^{8A+\phi}}{24} |\widetilde{G_\pm}|^2 + e^{-4A} |\widetilde{\nabla} \Phi_\pm|^2 + \text{local}$$

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So, solve the Laplace equation in $AdS_5 \times T^{1,1}$.

Solution:

Kim, Romans, & van Nieuwenhuizen, 1985.
Gubser, 1998.
Ceresole, Dall'Agata, D'Auria, Ferrara, 1999.

$$\Phi_-(r,\Psi) = \sum_{L,M} \Phi_{LM} \left(\frac{r}{r_{\rm UV}}\right)^{\Delta(L)} Y_{LM}(\Psi) + c.c.$$

$$L \equiv \{J_1, J_2, R\} \quad \Longleftrightarrow \quad SU(2)\times SU(2)\times U(1)_R$$

$$\Delta \equiv -2 + \sqrt{6\Big[J_1(J_1+1) + J_2(J_2+1) - R^2/8\Big] + 4}$$

$$\Box_5 Y_{LM} = -\Lambda Y_{LM}$$

$$\Lambda \equiv 6\Big[J_1(J_1+1) + J_2(J_2+1) - R^2/8\Big]$$

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$$V(\phi) \;=\; V_{D3/\overline{D3}}(\phi) \;+\; H^2\phi^2 \;+\; \sum_{L,M} c_\Delta \left(\frac{\phi}{\phi_{\rm UV}}\right)^{\Delta(L)} \alpha_{LM} Y_{LM}(\Psi) + c.c.$$

Simple case: one mode

$$\Phi_{-}^{(\Delta)} = \left(\frac{r}{r_{\mathrm{UV}}}\right)^{\Delta} f_L(\Psi)$$

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$$\Delta V = -c\, a_0^4\, T_3 \left(\frac{\phi}{\phi_{\mathrm{UV}}}\right)^{\Delta}$$

What are the lowest modes?

$$\{J_1, J_2, R\} = \{\tfrac{1}{2}, \tfrac{1}{2}, 1\}$$

Δ=3/2 chiral mode $\Phi_{-}^{(3/2)}$

$$\{J_1, J_2, R\} = \{1, 0, 0\} \text{ and } \{0, 1, 0\}$$

Δ=2 nonchiral mode $\Phi_{-}^{(2)}$

Gauge Theory Description

Klebanov-Witten CFT:

SU(N) x SU(N) gauge group

SU(2) x SU(2) x $U(1)_R$ global symmetry

bifundamentals A_i, B_i

Klebanov & Witten, hep-th/9807080

ultraviolet deformation:

$$c \int d^4\theta \; M_{\rm UV}^{-\Delta} \, X^\dagger X \, \mathcal{O}_\Delta$$

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contributing chiral operators:

$$\mathcal{O}_\Delta = \mathrm{Tr}\Big(A^{(i_1} B_{(j_1} A^{i_2} B_{j_2} \ldots A^{i_R)} B_{j_R)} \Big) + c.c.$$

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$$\mathcal{O}_\Delta = \mathrm{Tr}\left(A^{(i_1} B_{(j_1} A^{i_2} B_{j_2} \ldots A^{i_R)} B_{j_R)} \right) + c.c.$$

most relevant chiral operators:

$$\mathcal{O}_{3/2} = \mathrm{Tr}\,(A_i B_j) + c.c.$$

Discrete symmetries can forbid chiral perturbations

e.g., $A_i \rightarrow -A_i$ forbids $\mathcal{O}_{3/2} = \mathrm{Tr}\left(A_i B_j\right) + c.c.$

Moreover, the following nonchiral operators are in the supermultiplets of global symmetry currents, and have protected dimension, Δ=2:

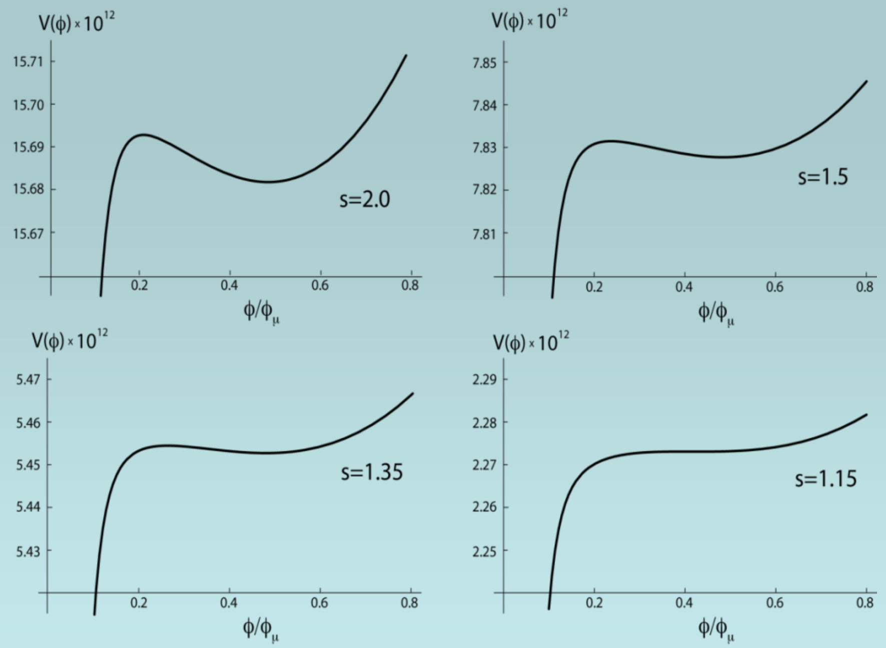
$$\mathcal{O}_2 = \quad \mathrm{Tr}\left(A_1 \bar{A}_2\right) \ , \quad \mathrm{Tr}\left(A_2 \bar{A}_1\right) \ , \quad \frac{1}{\sqrt{2}}\mathrm{Tr}\left(A_1 \bar{A}_1 - A_2 \bar{A}_2\right)$$

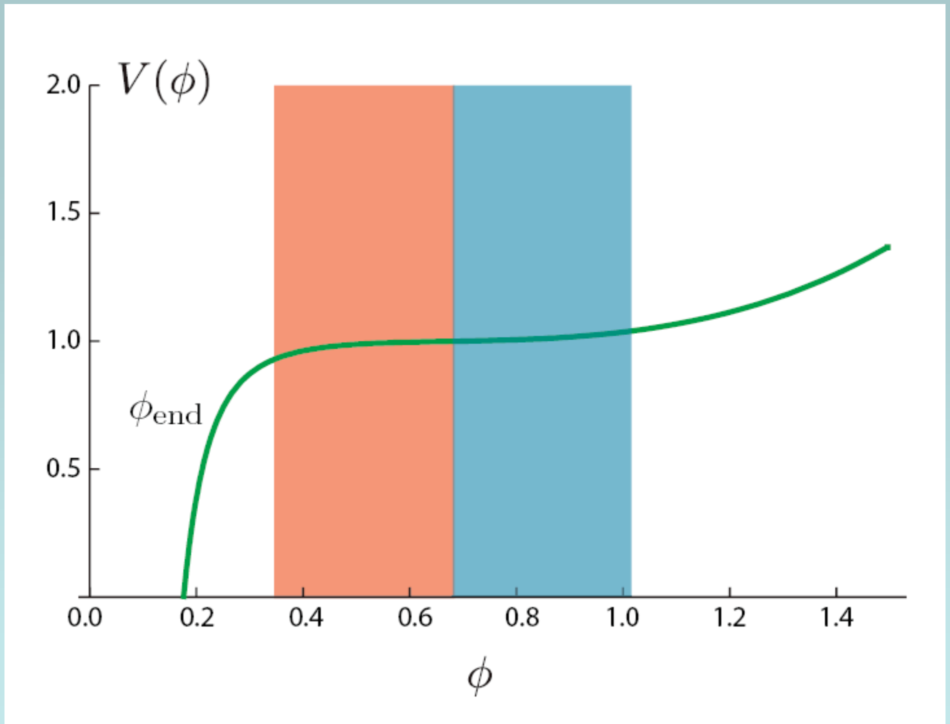
If the compactification preserves a discrete symmetry that forbids the leading chiral perturbation $\mathbf{O_{3/2}}$, the dominant compactification effect comes from $\mathbf{O_2}$.

Scenario I:

Inflection point inflation again!

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + M_{\rm pl}^2 H^2 \left[\left(\frac{\phi}{M_{\rm pl}} \right)^2 - a_{3/2} \left(\frac{\phi}{M_{\rm pl}} \right)^{3/2} \right]$$



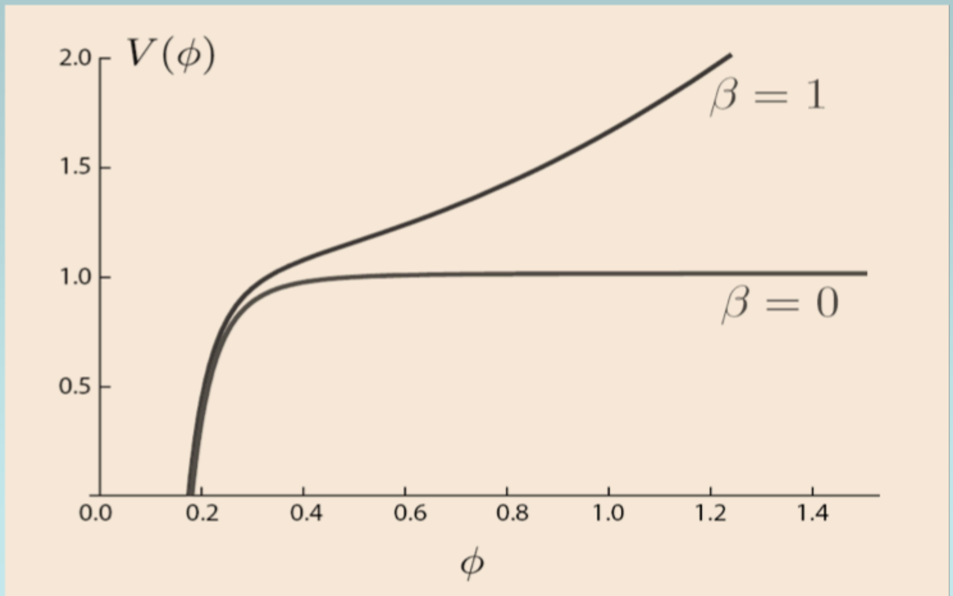


Scenario II:

wherein the inflaton mass admits fine-tuning

$$V(\phi) = V_{D3/\overline{D3}}(\phi) + H^2 \underbrace{\left[1 - c_2\left(\frac{M_{\rm pl}}{M_{\rm UV}}\right)^2\right]}_{\equiv\, \beta} \phi^2$$

(This is the leading term if discrete symmetries forbid the chiral perturbation.)



Phenomenology:
Firouzjahi & Tye, hep-th/0501009

Part IV.

Pre-Conclusions

Conclusions, Part I: Method

- We have an easy and completely systematic approach to computing the inflaton potential in warped brane inflation.
- Method: consider generic perturbation of ultraviolet region, sourced by supersymmetry-breaking compactification effects, and focus on most relevant terms.
- Equivalently, perturb dual CFT Lagrangian by most relevant operators.
- Our approach reproduces, extends, and simplifies the results of direct computation of W from wrapped D7-branes.
- We reliably capture totally general effects of compactification, provided that the D3-brane is far from the top and far from the tip.

Conclusions, Part II: Implications

- Two scenarios:
  - ❖ inflection point inflation is most generic possibility.
  - ❖ discrete symmetry leads instead to terms that allow direct fine-tuning of the inflaton mass.

- Study of multifield effects is now straightforward.
- Moreover, since we know the structure of the most general potential for this system, one could try to understand generic predictions.

Part V.

Primordial Gravitational Waves from String Inflation

Vacuum Fluctuations: Tensors

$$\delta g_{ij} \equiv h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times$$

$$\langle h(k) h(k') \rangle = \frac{2\pi^2}{k^3} \delta^3(k - k') \mathcal{P}_T$$

$$\mathcal{P}_T = \frac{8}{M_P^2} \left(\frac{H}{2\pi} \right)^2$$

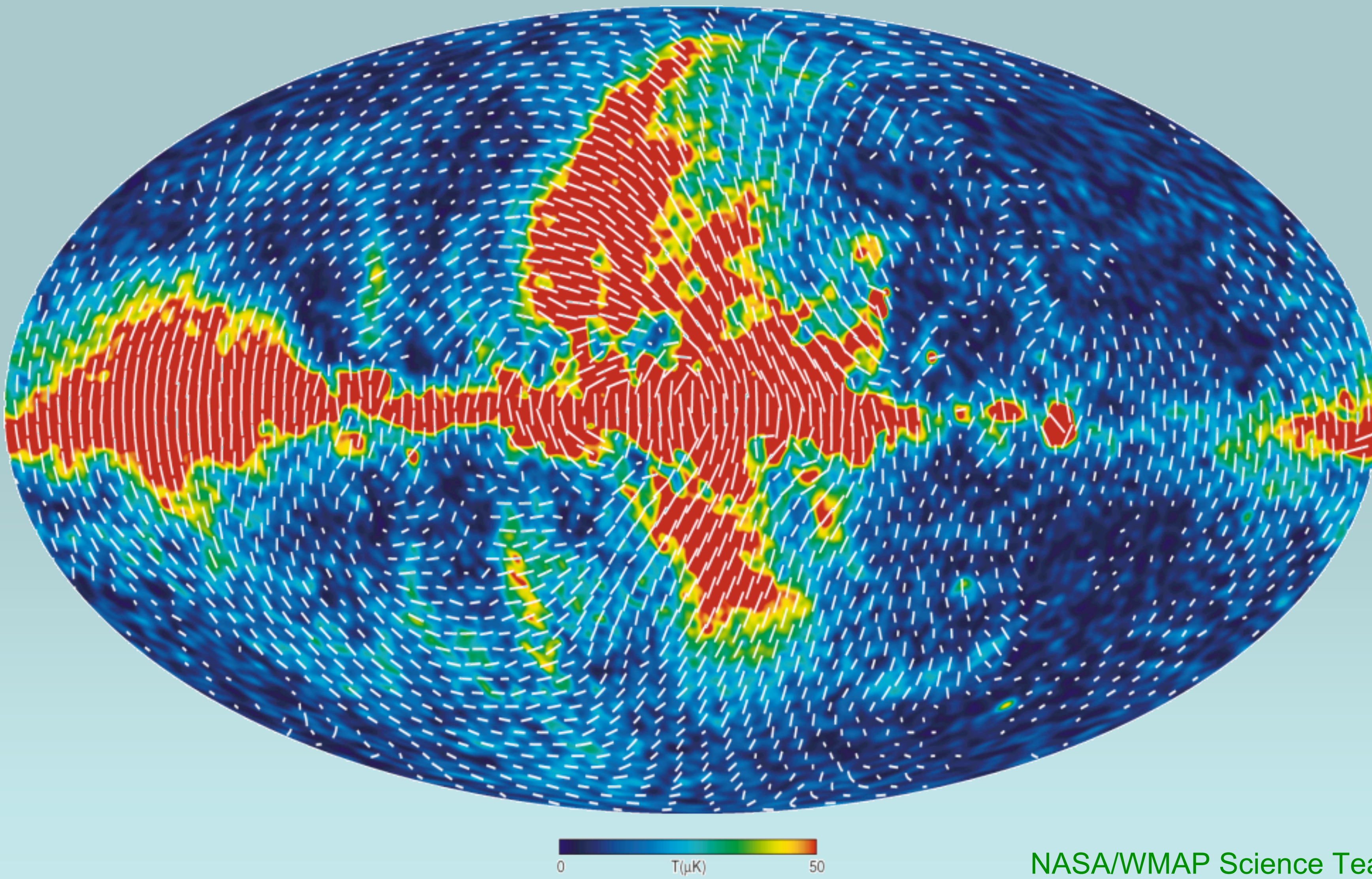
$$\mathcal{P}_S = \left(\frac{H}{2\pi} \right)^2 \left(\frac{H}{\dot{\varphi}} \right)^2$$

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_S}$$

tensor-to-scalar ratio, a measure of the primordial tensor signal

May be visible through induced curl of CMB photons' polarization (B-mode): SPIDER, Clover, QUIET, BICEP, EBEX, PolarBEAR,...

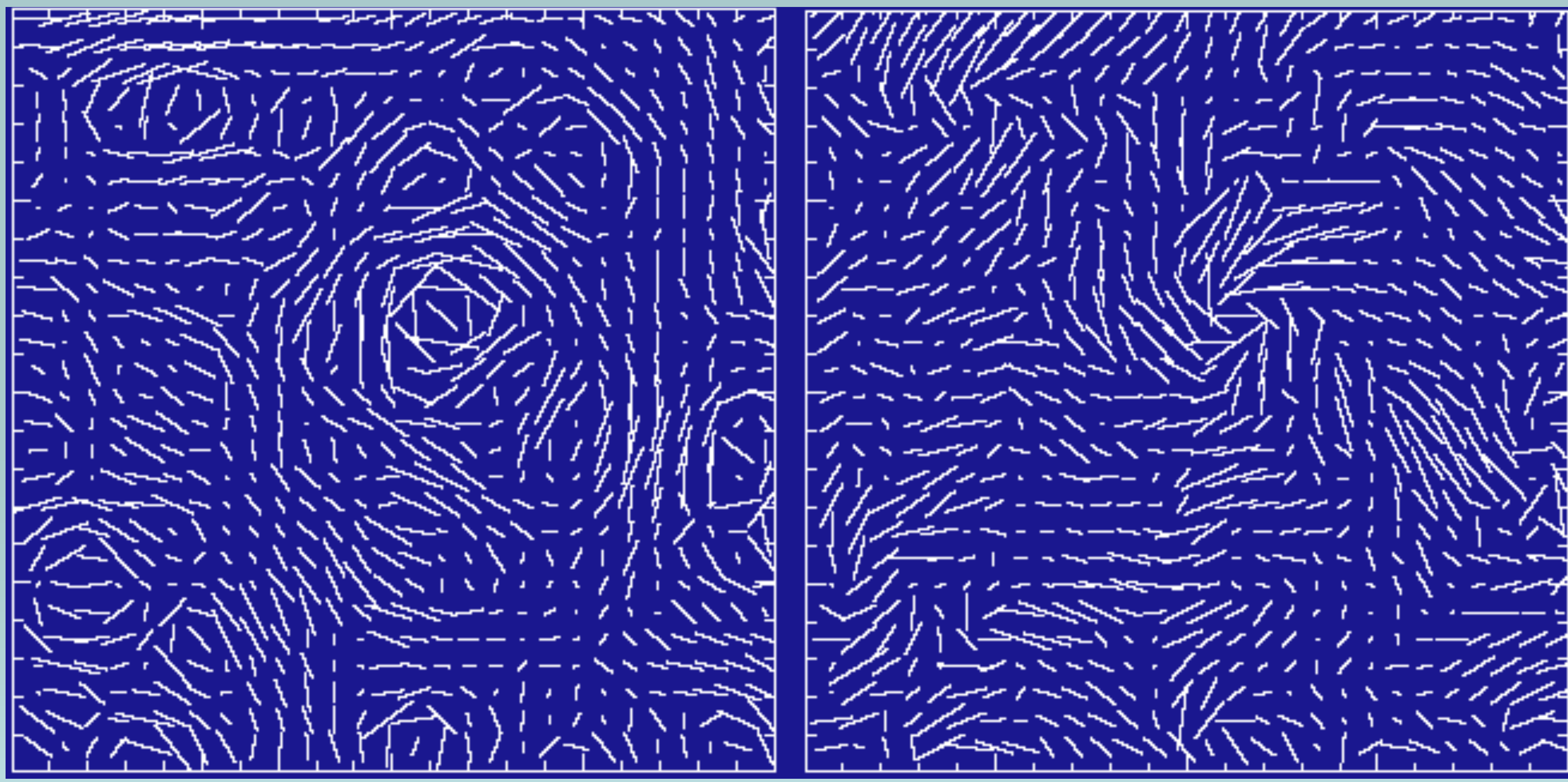
Polarization seen by WMAP



NASA/WMAP Science Team

E-mode vs. B-mode

(curl-free) (curl)

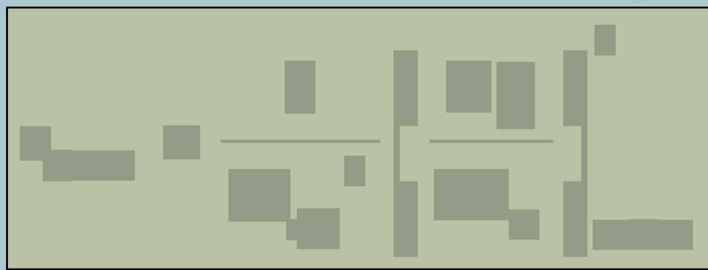


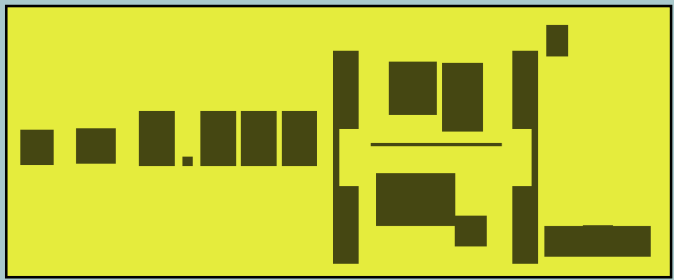
Primordial tensors induce curl of CMB photons' polarization (B-mode).

Image: Seljak and Zaldarriaga

Lyth Bound

D.H. Lyth, 1996

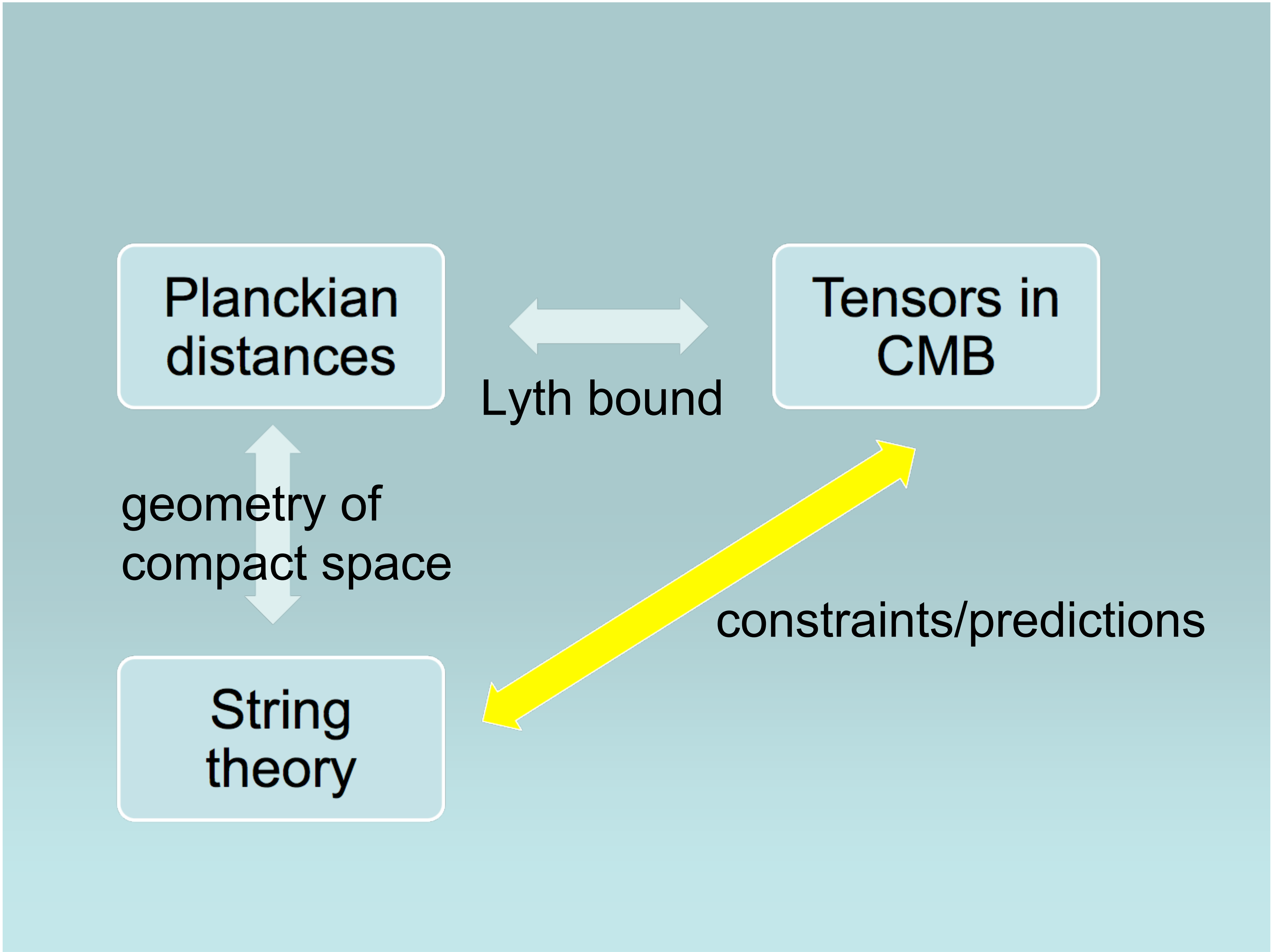




Threshold for detection: $r \quad 10^{-2}$ next decade

$r \quad 10^{-3}$? ultimate

OBSERVABLE TENSORS REQUIRE TRANS-PLANCKIAN FIELD VARIATION WHICH REQUIRES ULTRAVIOLET INPUT.

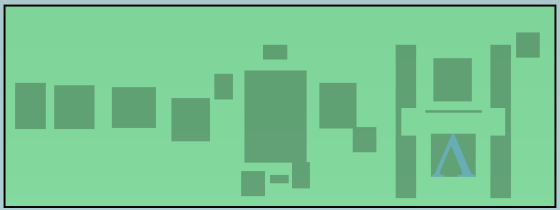

Planckian distances
Tensors in CMB
Lyth bound
geometry of compact space
String theory
constraints/predictions

The challenge:

- Small-field inflation requires controlling Planck-suppressed corrections up to dimension 6

Reasonable to enumerate and fine-tune these terms.

- Large-field inflation requires control of Planck-suppressed contributions of arbitrary dimension!

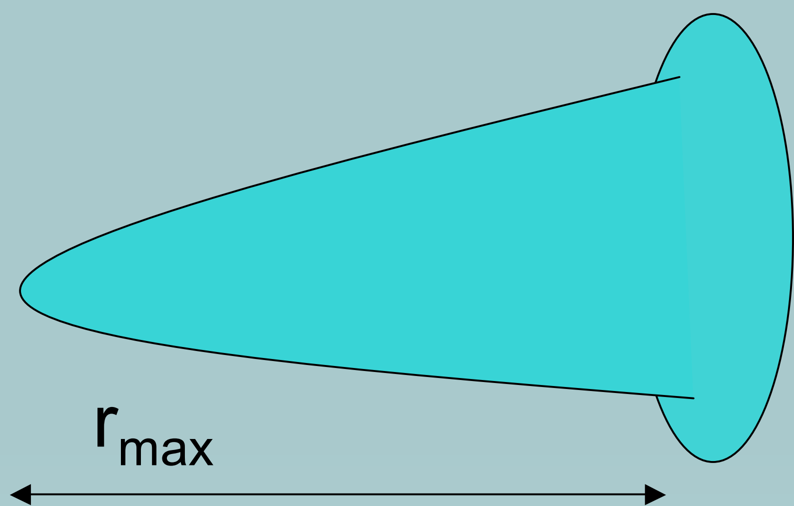


Symmetry clearly essential.

- Detectable tensors require super-Planckian displacements, so observations will distinguish these cases.

Lyth 1996

Computing the Field Range



warped cone over X_5

$$\approx \mathrm{AdS}_5 ?\, \mathrm{X}_5$$

$$\left(\frac{r}{R}\right)^2$$

$$R^4 = 4\pi\, g_s\, (\alpha')^2 \quad \frac{\pi^3}{\mathrm{Vol}(X_5)}$$

Gubser

Compactification Volume determines the 4d Planck Mass

$$\kappa_{10}^2 = \tfrac{1}{2}(2\pi)^7 g_s^2 (\alpha')^4$$

$$M_P^2 > \frac{2\pi^4 g_s \quad r_{\max}^2 (\alpha')^2}{\kappa_{10}^2}$$

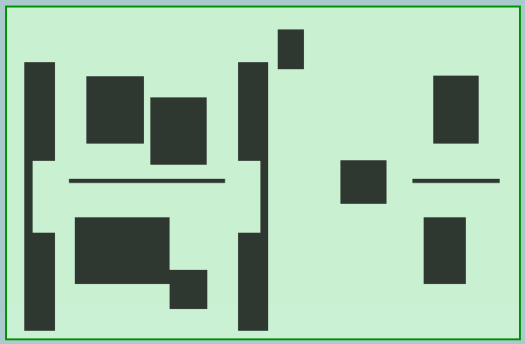
Compactification Constraint on the Field Variation

4d Planck Mass

+

Canonical Inflaton Field

Resulting field range, for any scenario involving a D3-brane in a throat:



N = the number of colors in the dual gauge theory

D. Baumann and L.M., hep-th/0610285.

Resulting field range,
for any scenario involving a D3-brane
in a throat:

[redacted]

N = the number of colors in the dual gauge theory

[redacted]

D. Baumann and L.M., hep-th/0610285.

However, large field ranges, and hence tensor signals, are possible in string inflation – just not in warped D3-brane inflation.

Mechanism:

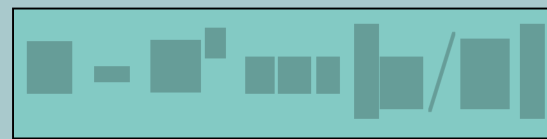
Linear Inflation from Axion Monodromy

L.M., Silverstein,& Westphal, 0808.0706

see also:

Silverstein & Westphal, 0803.3085.

Natural Inflation in string theory?

Freese, Frieman, & Olinto, 1990:  $f > M_P$

Axion shift symmetry protects inflaton potential.

Banks, Dine, Fox, & Gorbatov, hep-th/0303252:

$f > M_P$ not attainable in string theory?

(cf. also Svrcek & Witten, hep-th/0605206)

Dimopoulos, Kachru, McGreevy, & Wacker, hep-th/0507205

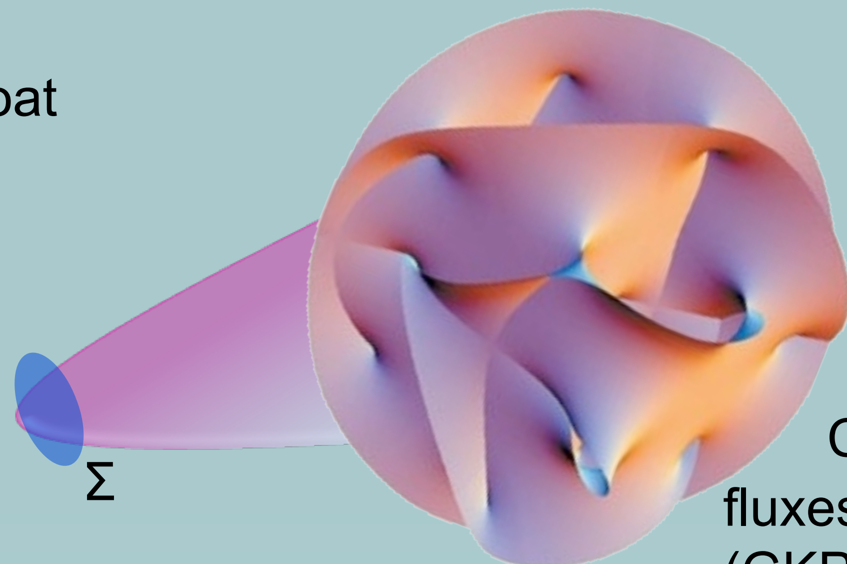
"N-flation": use $N \sim 10^3$ axions at once,

as a collective excitation.

Our idea: recycle a single axion N times.

Axion Inflation from Wrapped Fivebranes

warped throat

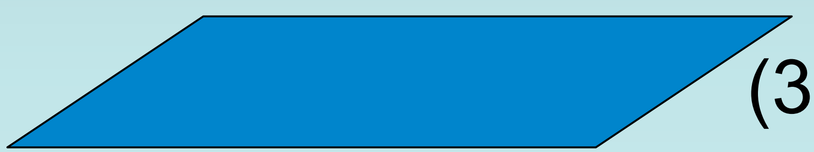


CY orientifold, with
fluxes and nonperturbative W
(GKP 2001, KKLT 2003)

D5-brane/NS5-brane

$$\int_{\Sigma} B_2 \qquad \int_{\Sigma} C_2$$

warped throat gives:
control of energy scales



(3+1)d

Axion monodromy from wrapped fivebranes

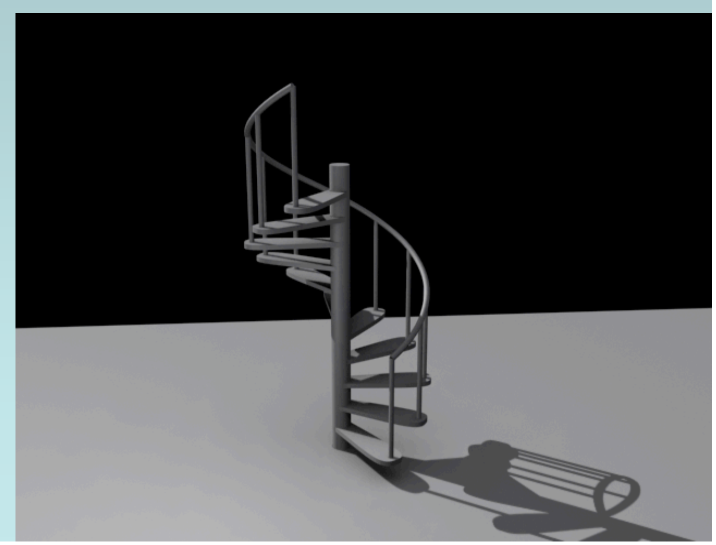
[illegible]

- Fivebrane contribution not periodic: as axion shifts by a period, potential undergoes a monodromy

  cf. inflation from D-brane monodromy Silverstein&Westphal, 0803.3085.

- This unwraps the axion circle and provides a linear potential over an *a priori* unlimited field range.

- In practice, controllable over large ($>> M_P$) but finite range.

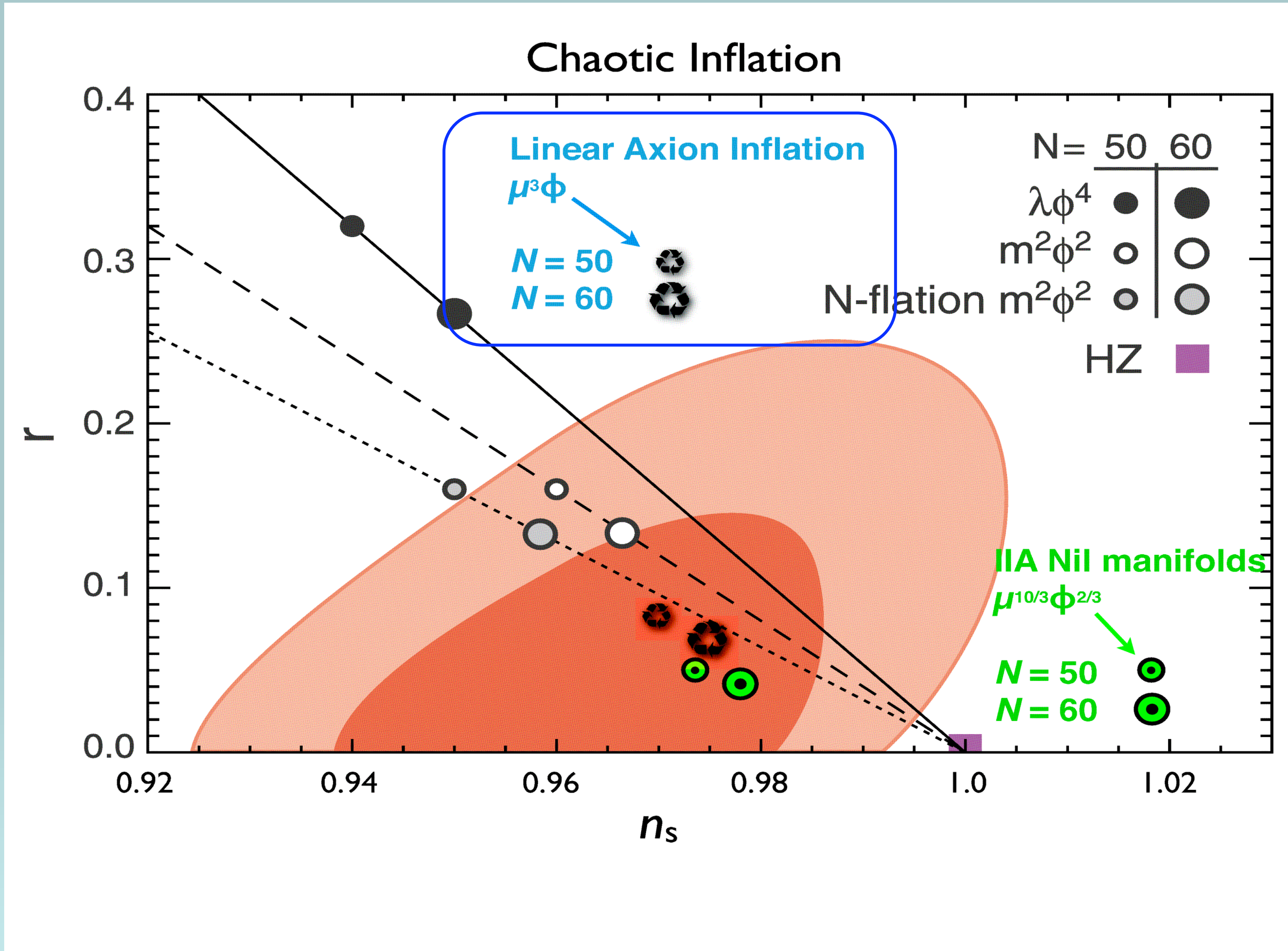


It Works

- Backreaction on local geometry can be made small.
- Checked consistency with moduli stabilization (KKLT scenario).
- Renormalization of Planck mass from many light species is small.

  For very reasonable parameter choices, the instanton contributions to the effective action can be neglected, and the additional problems above are addressed.

- Assumptions Summarized:
  - NS5-brane wrapping a curve Σ that is in a warped region and stabilized at finite volume.
  - Remaining moduli stabilized as in KKLT, by fluxes and strong gauge dynamics (ED3 stabilization may be manageable as well).
  - Tadpole canceled by e.g. anti-NS5-brane on distant cycle homologous to Σ
- Reasonable to expect more efficient/minimal realizations.
- Core idea is very simple: the axionic shift symmetry protects the inflaton potential, just as in Natural Inflation.


Chaotic Inflation
Linear Axion Inflation
μ³φ
N = 50
N = 60
N= 50 60
λφ⁴
m²φ²
N-flation m²φ²
HZ
IIA Nil manifolds
μ^{10/3}φ^{2/3}
N = 50
N = 60
r
0.4
0.3
0.2
0.1
0.0
0.92
0.94
0.96
0.98
1.0
1.02
n_s

Conclusions

- We can construct string inflation models that are realistic, reasonably explicit, and falsifiable.
- Well-understood class of small-field hybrid models: warped D3-brane inflation.
  - ❑ Computable local model with systematic incorporation of compactification effects from moduli stabilization.
  - ❑ Signatures include cosmic strings but not tensors.
- New class of large-field models: axion monodromy.
  - ❑ Axion shift symmetry naturally controls contributions to the inflaton action.
  - ❑ Robust tensor prediction r=.07 can be falsified in 5-10 years.