Gauge theories with time-dependent couplings and cosmological singularities

K. Narayan

Chennai Mathematical Institute (CMI), Chennai

[hep-th/0602107, hep-th/0610053,
Sumit Das, Jeremy Michelson, KN, Sandip Trivedi;
arXiv:0711.2994, Adel Awad, Das, KN, Trivedi;
arXiv:0807.1517, Awad, Das, Suresh Nampuri, KN, Trivedi.]

- basic setup: AdS/CFT with cosmological singularities
- gauge theories with time-dep coupling sources
- Spacelike cosmological singularities, BKL etc

Related references:

Craps, Hertog, Turok, arXiv:0712.4180 [hep-th] ,
Chu, Ho, hep-th/0602054, arXiv:0710.2640 [hep-th],
Hertog, Horowitz, hep-th/0503071 : cosmological generalizations of AdS/CFT framework.

B. Craps, S. Sethi, E. Verlinde, hep-th/0506180, and followup work by various people: Matrix theory duals of cosmological singularities.

...

Cosmology, time dependence, ...

Tempting to think very early Universe has deep repercussions on various aspects of physics.

- Big Bang singularities, time, in string theory models? Understand spacelike, null singularities — events in time.

General Relativity breaks down at singularities: curvatures, tidal forces divergent. Want "stringy" description, eventually towards smooth quantum (stringy) completion of classical spacetime geometry.

Previous examples: "stringy phases" in $e.g.$ 2-dim worldsheet (linear sigma model) descriptions (including time-dep versions, e.g. tachyon dynamics in (meta/)unstable vacua), dual gauge/Matrix theories, ...

In what follows, we'll use the AdS/CFT framework.

AdS/CFT and deformations

Nice stringy playground: AdS/CFT. Bulk string theory on $AdS_5 \times S^5$ with dilaton (scalar) $\Phi = const$, and metric

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5} ,$$

(Poincare coords) with 5-form field strength, dual to boundary $d = 4$ $\mathcal{N}{=}4$ (large N) $SU(N)$ Super Yang-Mills theory.

Want: *time-dependent* deformations of AdS/CFT.

Bulk subject to time-dependent sources classically evolves in time (thro Einstein eqns), eventually giving rise to a cosmological singularity, and breaks down. Avoid any bulk investigation near singularity.

Boundary: Gauge theory dual is a sensible Hamiltonian quantum system in principle, subject to time-dependent sources. Response ?

AdS cosmologies

Start with $AdS_5 \times S^5$ and turn on non-normalizable deformations for the metric and dilaton (also nontrivial 5-form):

$$ds^2 = \frac{1}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}\ , \qquad \Phi = \Phi(x^\mu)\ .$$

This is a solution in string theory if

$$\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi\ , \qquad \frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0\ ,$$

$i.e.$ if it is a solution to a 4-dim Einstein-dilaton system.

Time dep: $\Phi = \Phi(t)$ or $\Phi = \Phi(x^+)$.

More later on cosmological solutions.

General family of solutions: ($Z(x^m)$ harmonic function)

$$ds^2 = Z^{-1/2}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}g_{mn}dx^m dx^n\ , \quad \Phi = \Phi(x^\mu),$$

$g_{mn}(x^m)$ is Ricci flat, and $\tilde{g}_{\mu\nu} = \tilde{g}_{\mu\nu}(x^\mu)$. $[\mu = 0123, m = 4\ldots 9.]$

AdS cosmologies cont'd

In many cases, possible to find new coordinates such that boundary metric $ds_4^2 = \lim_{z\to 0} z^2 ds_5^2$ is flat, at least as an expansion about the boundary ($z=0$) if not exactly.

These are Penrose-Brown-Henneaux (PBH) transformations: subset of bulk diffeomorphisms leaving metric invariant (in Fefferman-Graham form), acting as Weyl transformation on boundary.

E.g. null cosmologies $ds^2 = \frac{1}{z^2}(e^{f(x^+)}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)\ ,\ \ \Phi(x^+)$.

The coord. transf. $w = ze^{-f/2}, \quad y^- = x^- - \frac{w^2 f'}{4}$, gives

$$ds^2 = \frac{1}{w^2}[-2dx^+dy^- + dx_i^2 + \frac{1}{4}w^2(\Phi')^2(dx^+)^2] + \frac{dw^2}{w^2}\ ,$$

using $R_{++} = \frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\Phi')^2$, the constraint on these solutions.

Now boundary at $w=0$ manifestly flat 4D Minkowski spacetime.

Gauge theories with time-dep couplings

Thus dual gauge theory lives on flat space. So sharp sub-question: Gauge theory with time-dependent coupling $g_{YM}^2 = e^{\Phi}$. Response?

We would like to study sources that are trivial in the far past (bulk is $AdS_5 \times S^5$) and smoothly turn on: this means the gauge theory begins in vacuum state and is subject to Hamiltonian time evolution through this external time-dependent source. Basic expectation: time-dep source excites vacuum to higher energy state.

Want to consider sources that approach $e^{\Phi} \rightarrow 0$ at some finite point in time: e.g. $g_{YM}^2 = e^{\Phi} \ \rightarrow \ (-t)^p \ , \ p > 0 \ \ [t < 0]$.

We'd specially like to understand gauge theory response near $t = 0$.

This point in time corresponds to a singularity in the bulk:

$R_{tt} = \frac{1}{2}\dot{\Phi}^2 \ \sim \frac{1}{t^2}$. Curvatures, tidal forces diverge near $t = 0$.

Gauge theories, time-dep couplings

Gauge theory kinetic terms $\int e^{-\Phi} F^2$ not canonical.
In usual perturbation theory, we absorb the coupling g_{YM}^2 into the definition of the gauge field A_μ so that g_{YM} appears only in the interaction terms. Let's do something similar here.

First, consider a simpler toy scalar theory
$L = -e^{-\Phi}\left(\frac{1}{2}(\partial\tilde{X})^2 + \tilde{X}^4\right)$. Redefining $\tilde{X} = e^{\Phi/2}X$ gives

$$L = -(\partial X)^2 - m^2(\Phi)X^2 - e^{\Phi}X^4 \,,$$

dropping a boundary term. The mass term is

$$m^2(\Phi) = \frac{1}{4}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}\partial_\mu\partial^\mu\Phi \,.$$

Null time-dependence

Null cosmologies: $\Phi = \Phi(x^+)$. No nonzero contraction so the mass term vanishes i.e. $m^2(\Phi) = 0$.
Similar story for gauge theory using lightcone gauge for convenience.
Suppressing many details, but briefly, cubic/quartic interaction terms: multiplied by powers of $g_{YM} = e^{\Phi/2}$, unimportant near $e^\Phi \to 0$.

Thus we obtain weakly coupled Yang-Mills theory at the location in null time ($x^+ = 0$) corresponding to the bulk singularity [e.g. $e^\Phi = g_s(-x^+)^p$].
This suggests that lightcone Hamiltonian time evolution of the gauge theory is sensible.
In other words, null cosmological singularities seem to be an artifact of using bad (classical bulk sugra) variables.

Null time-dependence

Is energy pumped in by null-time-dependent source ? No.
Since x^--translations are symmetries, there is in fact no particle production. This suggests that continuing past singularity at $x^+ = 0$ is OK, and late-time state is vacuum.

Thus late-time bulk is $AdS_5 \times S^5$ (dual to vacuum state of $\mathcal{N}=4$ gauge theory, with $\Phi \rightarrow const$ for large x^+).

Bulk: since $e^\Phi \rightarrow 0$ near singularity, no large g_s effects. Preliminary calculations suggest stringy (α') effects (beyond GR) are becoming important.
In some still simpler toy models with no RR-flux, dilaton, the singularity is purely gravitational so possibly more tractable.
I am studying these to understand worldsheet effects near null singularities.

Time-dependent couplings

The story is very different with time-dependent couplings.
Toy scalar theory $L = -e^{-\Phi}\left(\frac{1}{2}(\partial\tilde{X})^2 + \tilde{X}^4\right)$. The redefinition $\tilde{X} = e^{\Phi/2}X$ gives $L = -(\partial X)^2 - m^2(\Phi)X^2 - e^{\Phi}X^4$,

dropping a boundary term, the mass term being

$$m^2(\Phi) = -\frac{1}{4}(\dot{\Phi})^2 + \frac{1}{2}\ddot{\Phi}\ .$$

For $g_{YM}^2 = e^{\Phi} \ \rightarrow \ (-t)^p\ ,\ p > 0\ \ [t < 0]$, we have $m^2 = -\frac{p(p+2)}{4\ t^2}$.

Tachyonic mass term, divergent as $t \rightarrow 0$.

* X variables canonical: analysing them shows that the mass term forces $X \sim \frac{1}{t^{p/2}}$, so that extra information as $X \rightarrow \infty$ is required: X description not good.

* $\tilde{X}$ variables finite near $t = 0$: interaction terms $e^{-\Phi}\tilde{X}^4|_{t\sim 0}$ large (unlike the null case).

Time-dep quantum mechanics

In more detail: first ignore interactions, quantize $quadratic$ theory.

For a single momentum-k mode, this is time-dep quantum mechanics:

$S_k = \int dt\ (\frac{1}{2}\dot{X}^2 - \omega^2(t)X^2)$, $\omega^2(t) = k^2 + m^2(t) \ \longrightarrow^{t\to-\infty}\ \omega_0^2$.

Generic classical solutions: $X = \sqrt{-t}\ [AJ_\nu(-t) + BN_\nu(-t)]$,

$\nu = \frac{p+1}{2}$. Diverge as $t \to 0$: i.e. generic trajectory driven to large X.

Take $f(t) = \sqrt{\frac{\pi\omega_0}{2}}\sqrt{-t}H^1_\nu(-\omega_0 t)$ as the solution of $\ddot{f} + \omega^2 f = 0$,

with $f \to e^{-i\omega_0 t}$, $t \to -\infty$. Expand $X = \frac{1}{\sqrt{2\omega_0}}[af(t) + a^\dagger f^*(t)]$.

Using the Schrodinger equation: the ground state wave-function is

$$\psi(t,x) = \frac{A}{\sqrt{f^*(t)}}e^{i(\frac{\dot{f}^*}{f^*})\frac{x^2}{2}} \ .$$

Time-dep quantum mechanics

X: wave-fn $\psi(t,x) = \frac{A}{\sqrt{f^*(t)}} e^{i(\frac{\dot{f}^*}{f^*})\frac{x^2}{2}}$,

Wave-fn phase $\sim \frac{1}{t}$, "wildly" oscillating near $t = 0$.

$t \longrightarrow 0^-$: $f \sim (-t)^{-p}$.

Probability density: $|\psi(t,x)|^2 = \frac{|A|^2}{|f|} e^{-\frac{\omega_0 x^2}{|f|^2}}$.

Gaussian, width $|f|^2 \rightarrow \infty$ as $t \rightarrow 0$.

Wave packet infinitely spread out as $t \rightarrow 0$.

X variables spread out infinitely: need extra information at $X \sim \infty$.

X description not good.

Time-dep quantum mechanics

Original $\tilde{X} = e^{\Phi/2} X$ variables better defined: finite near $t = 0$.

$\tilde{X} = e^{\Phi/2}\sqrt{-t}[AJ_\nu(-t) + BN_\nu(-t)] \sim^{t\to 0} \ t^{p/2} t^{1/2} t^{-\nu/2}$.

Wave-fn, probability:

$$\psi(t,\tilde{x}) = \frac{A}{\sqrt{f^*(t)e^{\Phi/2}}} e^{i(\frac{\dot{f}^*}{f^*} + \frac{\dot{\Phi}}{2})\frac{\tilde{x}^2}{2e^\Phi}} , \qquad |\psi(t,\tilde{x})|^2 = \frac{|A|^2}{|f|e^{\Phi/2}} e^{-\frac{\omega_0 \tilde{x}^2}{|f|^2 e^\Phi}} .$$

$t \to 0^-$: $f \sim \ (-t)^{-p} , \qquad |f|^2 e^\Phi \sim \ const$.

$\tilde{X}$: wave-fn phase $\sim \ \frac{1}{(-t)^{p-1}}$, prob. width $const$.

$p > 1$: wave-fn ill-defined near $t \sim 0$. “Wildly” oscillating phase.
$p < 1$: $\tilde{X}$ wave fn phase regular near $t \sim 0$, $|\psi(t,\tilde{x})|^2$ finite.

Quadratic approximation shows interactions are important near $t = 0$.
Perturbation theory insufficient.

Time-dep field theory wave-fn

More general Schrodinger picture analysis in full field theory is possible near $t = 0$.

Lagrangian $\quad L = \int d^3x\, e^{-\Phi}(\frac{1}{2}(\partial_t \tilde{X})^2 - \frac{1}{2}(\partial_i \tilde{X})^2 - \tilde{X}^4)$.

Field theory Hamiltonian: $\quad H = e^{-\Phi} V[\tilde{X}] + e^{\Phi} \int d^3x (-\frac{1}{2}\frac{\delta^2}{\delta \tilde{X}^2})$,

where $V[\tilde{X}] = \int d^3x\, (\frac{1}{2}(\partial_i \tilde{X})^2 + \tilde{X}^4)$ [replacing $\Pi(x) \to \frac{1}{i}\frac{\delta}{\delta \tilde{X}}$] .

Schrodinger eqn: $\quad i\partial_t \psi[\tilde{X}(x), t] = H\psi[\tilde{X}(x), t]$.

Near $t = 0$, the potential term $e^{-\Phi} V$ dominates in the Hamiltonian $\Rightarrow$ $i\partial_t \psi = e^{-\Phi(t)} V[\tilde{X}(x)]\psi$. This gives the wave-fn (generic state)

$$\psi[\tilde{X}(x), t] = e^{-i(\int dt\, e^{-\Phi(t)}) V[\tilde{X}(x)]}\, \psi_0[\tilde{X}(x)] .$$

Phase as before $\sim \frac{(-t)^{1-p}}{1-p} V[\tilde{X}(x)]$. If $p > 1$, "wildly" oscillating ($t \to 0$).

Energy divergence

Analyzing KE terms shows they are indeed subleading near $t = 0$.

This means energy pumped in by time-dep source diverges as

$$\langle H \rangle \simeq e^{-\Phi} \langle V \rangle = \frac{1}{(-t)^p} \int D\tilde{X}\, V[\tilde{X}]\, |\psi_0[\tilde{X}(x)]|^2 \ ,$$

since no time-dep in $\langle V \rangle$. Oscillating phase cancels in $|\psi_0|^2$.
This holds for generic states. For special states with $\langle V \rangle = 0$, energy may be finite (subleading KE terms do not diverge unless $p > 2$).

Even for these special states, $\langle H^2 \rangle$ will diverge (if $\langle V \rangle = 0$, generically $\langle V^2 \rangle$ does not vanish).
Thus fluctuations non-negligible about states with $\langle V \rangle = 0$.

Note: this is not perturbation theory. Interactions important.
Diverging energy since coupling strictly vanishes near $t = 0$.

The gauge theory

Scalars, fermions: no dilaton coupling in KE terms. Fermion Yukawa and scalar quartic terms come with powers of $g_{YM} = e^{\Phi/2}$, vanish near $t = 0$.

Gauge fields: KE terms have dilaton coupling $\int e^{-\Phi}\, \mathrm{Tr} F^2$.

Since $e^{\Phi} = (-t)^p$ near $t = 0$, the gauge field terms determine the behaviour of the system near $t \sim 0$. Focus on this.

Consider non-interacting theory first.

Convenient (Coulomb) gauge $A_0 = 0\ ,\ \partial_j A_j = 0$ (longitudinal part of gauge field time-indep from Gauss law: $\partial_0(\partial_j A_j) = 0$).

Residual action for two physical transverse components A^i becomes $\int e^{-\Phi}(\partial A^i)^2$, (i.e. two copies of the scalar theory earlier).

The gauge theory

Cubic/quartic interactions: no time derivatives.

Contribute only to potential energy terms (from magnetic field), not to KE terms (from electric field). PE $V[A^i(x)] = \frac{1}{4}\int d^3x\, \mathrm{Tr} F_{ij}^2$.

$$L_g = \frac{1}{4}\int d^3x\, e^{-\Phi}\, \mathrm{Tr}\left((\partial_t A^i)^2 - F_{ij}^2\right) .$$

Schrodinger quantization: $E^i \longrightarrow \frac{1}{i}\frac{\delta}{\delta A^i}$. Then wave-fn near $t = 0$:

$$\psi[A^i(x), t] = e^{-i(\int dt\, e^{-\Phi})V[A^i(x)]}\, \psi_0[A^i(x)] .$$

Wave-fn phase as before: "wildly" oscillating as $t \to 0$ (for $p > 1$).

Energy diverges $\langle H\rangle \simeq e^{-\Phi}\int DA^i\, V[A^i(x)]\, |\psi_0[A^i]|^2$ (if $\langle V\rangle \neq 0$) .

Thus if $e^{\Phi} \longrightarrow 0$ strictly, gauge theory response singular.

For cutoff e^{Φ}, large energy production due to time-dep source.

AdS cosmologies with spacelike singularities

Recall: $ds^2 = \frac{1}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}\ , \quad \Phi = \Phi(x^\mu)\ .$

Solution if: $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi\ , \quad \frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu(\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0\ .$

Solutions with spacelike Big-Bang (Crunch) singularities:

* $ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \sum_{i=1}^{3} t^{2p_i}(dx^i)^2\right]\ ,$

  $e^\Phi = |t|^{\sqrt{2(1-\sum_i p_i^2)}}, \qquad \sum_i p_i = 1\ .$ [Kasner cosmologies]

* $ds^2 = \frac{1}{z^2}\left[dz^2 + |\sinh(2t)|(-dt^2 + \frac{dr^2}{1+r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2))\right],$

  $e^\Phi = g_s\,|\tanh t|^{\sqrt{3}}\ .$ [$k = -1$ (hyperbolic) FRW boundary]

Dilaton bounded, approaching constant at early/late times: asymptotic spacetime is $AdS_5 \times S^5$ (using a coord transformation).

The $k = 0$ (flat) FRW is the same as symmetric Kasner ($p_i = \frac{1}{3}$).
(There is also a $k = +1$ (spherical) FRW solution.)

AdS BKL-cosmologies

In fact, larger family of cosmological solutions where spatial metric is one of the homogenous spaces in the Bianchi classification:

$$ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \eta_{ab}(t)(e^a_\alpha dx^\alpha)(e^b_\beta dx^\beta)\right] , \quad e^\Phi = e^{\Phi(t)} .$$

$e^a_\alpha dx^\alpha$ are a triad of 1-forms defining symmetry directions. Spatially homogenous dilaton means spatial $R^a_{(a)}$ vanish, and $R^0{}_0 = \frac{1}{2}(\partial_0\Phi)^2$.

Bianchi-IX: $ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \eta_i^2(t) e^i_\alpha e^i_\beta dx^\alpha dx^\beta\right] , \; e^\Phi = |t|^\alpha$.

Approximate Kasner-like solution $\eta_i(t) \simeq t^{p_i}$ with

$$\sum_i p_i = 1 , \; \sum_i p_i^2 = 1 - \frac{\alpha^2}{2} .$$

If all $p_i > 0$, cosmology "stable". Else, spatial curvatures force BKL bounces between distinct Kasner regimes. With each bounce, α increases — dilaton-driven attractor-like behaviour.

Attractor basin: generic Kasner-like solution with all $p_i > 0$.

More on AdS BKL-cosmologies

Bianchi IX: symmetry algebra of $X_a = e_a^\alpha \partial_\alpha$ is $SU(2)$.

Spatial Ricci, decomposing along triad $R^a_{(a)} = R^a_\alpha e^\alpha_a$:

$$R^1_{(1)} = \frac{\partial_t(\eta_2\eta_3\partial_t\eta_1)}{\eta_1\eta_2\eta_3} - \frac{1}{2(\eta_1\eta_2\eta_3)^2}[(\eta_2^2 - \eta_3^2)^2 - \eta_1^4] = 0\,, \quad \ldots .$$

Say $p_1 < 0$: then $\eta_1^4 \sim t^{-4|p_1|}$ non-negligible at some time. This forces metric to transit from one Kasner regime to another. As long as some $p_i < 0$, these bounces continue as:

$$p_i^{(n+1)} = \frac{-p_-^{(n)}}{1+2p_-^{(n)}}\,, \quad p_j^{(n+1)} = \frac{p_+^{(n)}+2p_-^{(n)}}{1+2p_-^{(n)}}\,, \quad \alpha_{(n+1)} = \frac{\alpha_n}{1+2p_-^{(n)}}\,,$$

for the bounce from the (n)-th to the $(n+1)$-th Kasner regime.

If $p_- < 0$, then $\alpha_{n+1} > \alpha_n$. Also $\alpha_{n+1} - \alpha_n = \alpha_n(\frac{-2p_-}{1+2p_-})$, i.e., α increases slowly for small α: attractor-like behaviour. Finite number of bounces. If all $p_i > 0$, no bounce: cosmology "stable". For no dilaton ($\alpha = 0$), BKL bounces purely oscillatory.

More on AdS BKL-cosmologies

Parametrization: $p_1 = x,\ p_{2,3} = \frac{1-x}{2} \pm \frac{\sqrt{1-\alpha^2+2x-3x^2}}{2}$.

Lower bound: $p_1 \geq \frac{1-\sqrt{4-3\alpha^2}}{3}$. Solution existence forces $\alpha^2 \leq \frac{4}{3}$.

Under bounces, α increases, window of allowed p_i shrinks. Lower bound hits $p_1 \geq 0 \Rightarrow \alpha^2 \geq 1$. Bounces stop, cosmology "stabilizes".

Attractor-like behaviour: e.g.: $\{p_1^0 = x_0 = 0.3,\ \alpha_0 = 0.001\}$, flows (initially slowly) to $\{p_i > 0\}$ after 15 oscillations ($\alpha_{15} = 1.0896$).

E.g.: $(-\frac{1}{5}, \frac{9}{35}, \frac{33}{35}) \ \rightarrow\ (-\frac{5}{21}, \frac{7}{21}, \frac{19}{21}) \ \rightarrow\ (-\frac{3}{11}, \frac{5}{11}, \frac{9}{11}) \ \rightarrow$ $(-\frac{1}{5}, \frac{3}{5}, \frac{3}{5}) \ \rightarrow\ (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. [multiple flows with same endpoint]

Chaotic behaviour: 7% change to smallest exponent $-\frac{1}{5}$ gives $(-\frac{13}{70}, \frac{9}{35}, \frac{65}{70}) \ \rightarrow\ (-\frac{2}{11}, \frac{13}{44}, \frac{39}{44}) \ \rightarrow\ (-\frac{3}{28}, \frac{2}{7}, \frac{23}{28}) \ \rightarrow\ (\frac{1}{11}, \frac{3}{22}, \frac{17}{22})$, drastically different endpoint.

Note also that dilatonic ($\alpha \neq 0$) [attractor-like] and non-dilatonic ($\alpha = 0$) [oscillatory] flows drastically different.

Universal behaviour near singularities

Consider symmetric Kasner-like AdS BKL-cosmologies. Near singularity, spatial curvatures unimportant. Leading singular behaviour is essentially dilaton-driven, symmetric Kasner spacetime. Holographic stress tensor has similar leading behaviour ($T_{\mu\nu} \sim \frac{N^2}{t^4}$).

Consider families of such AdS cosmologies which are of the form of the symmetric Kasner-like solution i.e. $p_i = \frac{1}{3}$: (ds_3^2 spatial metric)

$$ds^2 = \frac{1}{z^2}\left[dz^2 + |2t|(-dt^2 + ds_3^2)\right] , \qquad e^{\Phi} = |t|^{\sqrt{3}} .$$

Ignoring subleading curvature effects, spatial metric approximately flat i.e. $ds_3^2 \sim flat$. Then boundary metric is conformally flat, to leading order. [we've used a different time coordinate here.]

Can use PBH transformations to recast boundary metric to be flat spacetime.

The gauge theory

Now $g_{YM}^2 = e^{\Phi} = (-t)^{\sqrt{3}}$ $(t < 0)$. That is, $p = \sqrt{3} > 1$.

From earlier: wave-fn phase "wildly" oscillating, ill-defined.

Energy production divergent if coupling vanishes strictly near $t = 0$.

* In gauge theory, deform gauge coupling so that $g_{YM}^2 = e^{\Phi}$ is small but nonzero near $t = 0$. Now finite but large phase oscillation, finite but large energy production.

Eventual gauge theory endpoint ? Depends on details of energy production at coupling $\mathcal{O}(1)$.

On long timescales, expect gauge theory thermalizes: then reasonable to imagine that late-time bulk is AdS-Schwarzschild black hole.

Conclusions, open questions

* If $g_{YM}^2(t) \rightarrow 0$ strictly, then gauge theory response singular: energy diverges. Deform g_{YM}^2 to be small but nonzero near $t = 0$. Now finite but large phase oscillation and energy production.
$\dot{\Phi} \sim \frac{\dot{g}_{YM}}{g_{YM}}$ finite now, so bulk also nonsingular.
Sugra may still not be valid of course.

* Gauge theory on S^3 : We're investigating this and other issues currently (in part with Archisman Ghosh, Jae Oh).

* Explore AdS BKL-cosmologies/duals further

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