Gravity Duals of Non-Relativistic Quantum Critical Points

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Based On

K. Balasubramanian, J. McGreevy Phys. Rev. Lett. **101***, 061601 (2008) hep-th:0804.4053*

A. Adams, K. Balasubramanian, J. McGreevy (2008) hep-th:0807.1111

D.T. Son Phys. Rev. D **78***, 0406003 (2008) hep-th:0804.3972*

C.P. Herzog, M. Rangamani, S. F. Ross hep-th:0807.1099

J. Maldacena, D. Martelli, Y. Tachikawa hep-th:0807.1099

Why are Non-Relativistic Quantum Critical Points interesting to String theorists?

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- String theorists can make use of such systems to learn more about holographic principle (theoretically and experimentally).
- Condensed matter physicists can make use of the holographic principle to learn more about these systems!
- Most of these systems are described by non-relativistic theories. Hence, we need to generalize the AdS/CFT correspondence to NRCFTs.

What else makes non-relativistic theories interesting?

- There are many non-relativistic theories that are scale invariant and they are characterized by a parameter called 'dynamical exponent' z [*P.C. Hohenberg and B.I. Halperin Rev. Mod. Phys, 1977; S. Sachdev, 1999*]

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- Dynamical exponent is the relative scale dimension of time and space. For *e.g* Schrödinger case has $z = 2$.
- NRQCPs many or may not be Galilean Invariant. For *e.g* Lifshitz (or Lifshitz-like) fixed points are not Galilean invariant. [*E. Ardonne, P. Fendley, E. Fradkin, cond-mat/ 0311466; S. Kachru, X. Liu, M. Mulligan, hep-th/ 0808.1725; P. Hořova, hep-th/ 0811.2217*]

Non-relativistic Conformal Group

Commutation relations for a general z

$$[M_{ij},\, N] = [M_{ij},\, D] = 0, [M_{ij},\, P_k] = i(\delta_{ik}P_j - \delta_{jk}P_i),$$

$$[P_i,\, P_j] = [K_i,\, K_j] = 0, [M_{ij},\, K_k] = i(\delta_{ik}K_j - \delta_{jk}K_i)$$

$$[M_{ij},\, M_{kl}] = i(\delta_{ik}M_{jk} - \delta_{jk}M_{il} + \delta_{il}M_{kj} - \delta_{jl}M_{ki})$$

$$[D,\, P_i] = iP_i, [D,\, K_i] = (1-z)iK_i, [K_i,\, P_j] = i\delta_{ij}N,$$

$$[H,\, K_i] = -iP_i, [D,\, H] = ziH, [D,\, N] = i(2-z)N,$$

$$[H,\, N] = [H,\, P_i] = [H,\, M_{ij}] = 0.$$

Schrödinger group

There is an additional conformal generator, C for $z = 2$. C satisfies the following commutation relations

$$[M_{ij}, C] = 0, \quad [K_i, C] = 0, \quad [D, C] = -2iC, \quad [H, C] = -iD.$$

M_{ij} generate spatial rotations,

P_i are momenta,

K_i generate Galilean boosts,

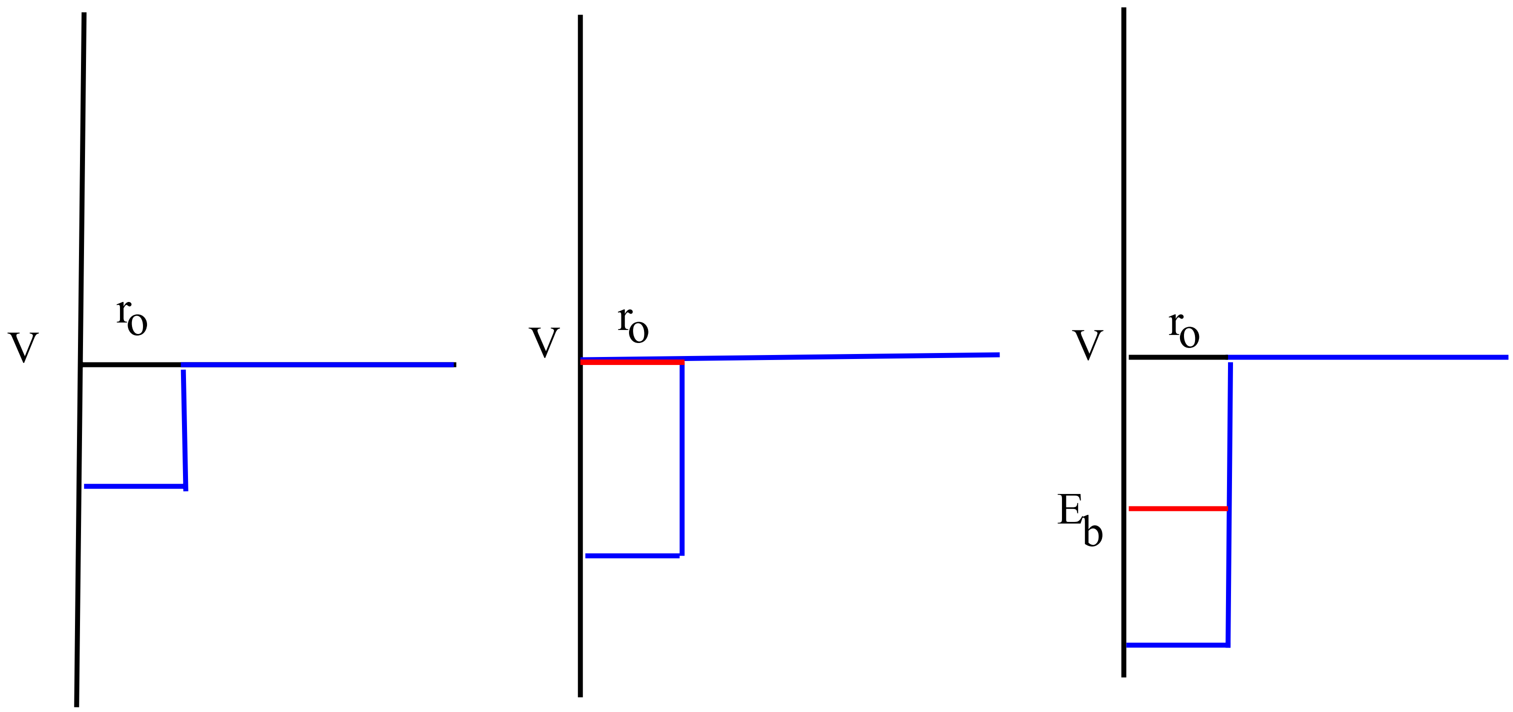
N is a conserved rest mass or particle number,

and D is the dilatation operator.

This NR conformal group with Galilean symmetry and $z = 2$ is the Schrödinger group.

An example of a field theory that has these symmetries

Unitarity Limit



(a) (b) (c)

- a) $V_0 < 1/mr_0^2$
  There is no bound state b) $V_0 = 1/mr_0^2$
  Bound state with zero energy. c) $V_0 > 1/mr_0^2$
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- It doesn't matter what potential we start with before taking the infinite scattering limit.
- The interactions can be tuned to make the scattering length infinite. Scattering length is controlled using Magnetic field..

Y. Nishida and D. T. Son [hep-th/0706.3746] constructed representations of Schrödinger algebra in terms of operators in a NRCFT. In a NR theory described by a second quantized field ψ, the number density and momentum density can be defined as follows

$$n(\vec{x}) = \psi^\dagger(\vec{x})\psi(\vec{x})$$

$$j_i(\vec{x}) = -\frac{i}{2}(\psi^\dagger(\vec{x})\partial_i\psi(\vec{x}) - \partial_i\psi^\dagger(\vec{x})\psi(\vec{x}))$$

From this definition we can construct the rest mass or particle number operator , momentum, rotation, boost, dilatation and special conformal generators as follows.

$$N = \int d\vec{x} n(\vec{x}), P_i = \int d\vec{x} j_i(\vec{x}), M_{ij} = \int d\vec{x} \left(x_i j_j(\vec{x}) - x_j j_i(\vec{x}) \right)$$

$$K_i = \int d\vec{x} x_i n(\vec{x}), D = \int d\vec{x} x_i j_i(\vec{x}), C = \int d\vec{x} \frac{x^2 n(\vec{x})}{2}$$

The above operators satisfy all the commutation relations not involving the Hamiltonian. The computation of $[H,\ D]$ requires scale invariance. Let us consider the following Hamiltonian.

$$H = \int d\vec{x} \frac{1}{2} \nabla_i \psi^\dagger \nabla_i \psi + \frac{1}{2} \int d\vec{x} d\vec{y} \psi^\dagger(\vec{x}) \psi^\dagger(\vec{y}) V(|\vec{x} - \vec{y}|) \psi(\vec{y}) \psi(\vec{x})$$

where ψ is a fermionic field.

Under the action of dilatation operator,

$$V(r) \rightarrow V'(r) = e^{-2\lambda} V\left(e^{-\lambda} r\right)$$

If V corresponds to infinite scattering length, then V' also corresponds to infinite scattering length. Hence,

$$e^{-i\lambda D} H e^{i\lambda D} = e^{-2\lambda} H \implies [H,\, D] = 2iH$$

Note: Infinite scattering length corresponds to saturation of the $s-$ wave unitarity bound and in this limit there is no intrinsic scale associated with the potential.

Geometric Realization

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$N = -i\partial_\xi$, corresponds to number operator (rest mass). For N to have a discrete spectrum, $\xi \sim \xi + L_\xi$. Metric is invariant under $t \to -t, \xi \to -\xi$, which can be interpreted as the composition of charge conjugation and time-reversal.

Isometries

Translation in space:

$$x^i \to x^i + a^i,\; t \to t,\; \xi \to \xi,\; r \to r$$

Time Translation:

$$x^i \to x^i,\; t \to t + b,\; \xi \to \xi,\; r \to r$$

Galilean boosts:

$$x^i \to x^i - v^i t,\; t \to t,\; \xi \to \xi + \frac{1}{2}(2\vec{v}\cdot\vec{x} - v^2 t),\; r \to r$$

Special Conformal Transformation:

$$x^i \to \frac{x^i}{1+ct},\; t \to \frac{t}{1+ct},\; \xi \to \xi + \frac{c}{2}\frac{(\vec{x}.\vec{x} + r^2)}{(1+ct)},\; r \to \frac{r}{1+ct}.$$

Einstein's Equations

The Einstein tensor for this metric is,

$$G_{ab} = -\Lambda g_{ab} - \mathcal{E}\delta_a^0\delta_b^0 g_{00}$$

where $\Lambda = -\frac{(d+1)(d+2)}{2L^2}$. metric is sourced by the ground state of an Abelian Higgs model in its broken phase. The model

$$S = \int d^{d+3}x\sqrt{g}\left(-\frac{1}{4}F^2 + \frac{1}{2}|D\Phi|^2 - V\left(|\Phi|^2\right)\right)$$

with $D_a\Phi \equiv (\partial_a + ieA_a)\Phi$, with a Mexican-hat potential

$$V\left(|\Phi|^2\right) = g\left(|\Phi|^2 - \frac{z(z+d)}{e^2}\right)^2 + \Lambda$$

produces the 'dust' stress tensor sourcing this metric.

Correlators

Correlators of field theor operators are calculated by solving the wave equation in this background. Wake equation in this background is given by,

$$\left(-r^{d+3}\partial_r\left(\frac{1}{r^{d+1}}\partial_r\right)+r^2(2l\omega+\vec{k}^2)\ +r^{4-2z}l^2+m^2\right)f_{\omega,\vec{k},l}(r)=0.$$

The behavior of the solution near the boundary is given by,

$$f\propto r^{\Delta}$$

where, $\Delta_{\pm}=1+\frac{d}{2}\pm\sqrt{\left(1+\frac{d}{2}\right)^2+m^2+\delta_{z,2}l^2}$.

For $d = 3$, $z = 2$

$$f_{\omega,\vec{k},l}(r) = Ar^{5/2}K_\nu(\kappa r), \nu = \sqrt{\left(\frac{5}{2}\right)^2 + l^2 + m^2}, \;\; \kappa^2 = 2l\omega + \vec{k}^2$$

On-Shell action:

$$S[\phi_0] = \frac{1}{2}\left[\int d^{d+2}X\sqrt{g}g^{rr}\phi(X)\partial_r\phi(X)\right]_{r=\epsilon}$$

In momentum space, the on-shell action can be written as,

$$S[\phi_0] = \frac{1}{2}\int dp\phi_0(-p)\mathcal{F}(\kappa,\epsilon)\phi_0(p)$$

where, the ‘flux factor’ is defined as,

$$\mathcal{F}(\kappa,\epsilon) = \lim_{r\rightarrow\epsilon} \sqrt{g} g^{rr} f_\kappa(r)\partial_r f_\kappa(r) = \sqrt{g} g^{rr}\partial_r r^{1+\frac{d}{2}} \ln K_\nu(\kappa r)|_{r=\epsilon}$$

Hence, boundary Green’s function is given by,

$$\langle \mathcal{O}_1(x,t)\mathcal{O}_2(0,0)\rangle \propto \frac{\Gamma(1-\nu)}{\Gamma(\nu)}\delta_{\Delta_1,\Delta_2}\theta(t)\frac{1}{|\epsilon^2 t|^\Delta}e^{-ilx^2/2|t|}$$

The expression for 2-point function is consistent with NR conformal Ward identities.

Is it possible to embed this geometry into String theory?

Null Melvin Twist

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- Boost back by $-\gamma$ along y
- Scaling limit: $\gamma \to \infty$, $\alpha \to 0$ keeping $\beta = \frac{1}{2}\alpha e^{\gamma}$ fixed.

Schrödinger spacetime from AdS

- $AdS_5 \times S_5$ is a solution of type II supergravity.

$$ds^2 = \frac{-d\tau^2 + dy^2 + dx_1^2 + dx_2^2 + dr^2}{r^2} + ds_{S_5}^2$$

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- $ds_{S_5}^2 = ds_{\mathbb{P}^2}^2 + (d\chi + \mathcal{A})^2$ can be written as
- Melvin twist produces the following metric

$$ds^2 = \frac{1}{r^2}\Bigg(- \left(1 + \frac{\beta^2}{r^2}\right) d\tau^2 + \left(1 - \frac{\beta^2}{r^2}\right) dy^2 + 2\frac{\beta^2}{r^2} d\tau dy$$

$$+ dx_1^2 + dx_2^2 + dr^2 \Bigg) + ds_{S_5}^2$$

- Defining $\xi = \frac{1}{2\beta}(y-\tau)$, $t = \beta(\tau + y)$, and reducing on the 5-sphere we get, small

$$ds^2 = \frac{2d\xi dt + d\vec{x}^{\,2} + dr^2}{r^2} - \frac{dt^2}{r^4}$$

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$$ds^2 = \frac{2d\xi dt + d\vec{x}^{\,2} + dr^2}{r^2} - \frac{dt^2}{r^4}$$

- The ten-dimensional metric is sourced by a five form flux and NS-NS 2-form field.

$$B = \beta/r^2(d\chi + \mathcal{A}) \wedge (d\tau + dy),$$

$$F_5 = (1 + \star)\Omega_5 d\theta \wedge d\phi d\psi \wedge d\mu \wedge d\chi$$

Field Theory Interpretation

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- This modified DLCQ (DLCQ$_\beta$) lifts the zero modes and makes the theory non-relativistic.

Solutions with Finite temperature and Density

10-D Black Hole solution

- Melvinization of planar AdS Black Hole yields the following metric.

$$ds^2 = \frac{1}{r^2 K}\left(\frac{-f}{r^2}dt^2 - 2d\xi dt - \frac{g}{4}\left(\frac{dt}{2\beta} - \beta\xi\right)^2 + K d\vec{x}^2 + \frac{K dr^2}{f}\right) +$$

$$\frac{1}{K}(d\chi + \mathcal{A})^2 + ds^2_{\mathbb{P}^2}$$

where, $f = 1 + g = 1 - \frac{r^4}{rH^4}$ and $K = 1 + \beta^2 \frac{r^2}{rH^4}$

- The ten-dimensional metric is sourced by a five form flux and NS-NS 2-form field and a Dilaton field.

$$B = \beta/r^2(d\chi + \mathcal{A}) \wedge ((1+f)/2dt + 2(1-f)\beta^2 d\xi)$$

$$F_5 = (1+\star)\Omega_5 d\theta \wedge d\phi d\psi \wedge d\mu \wedge d\chi, \; e^{-2\Phi} = K$$

5-D reduction

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$$ds^2 = \frac{K^{-2/3}}{r^2}\left(\frac{-f}{r^2}dt^2 - 2d\xi dt - \frac{g}{4}\left(\frac{dt}{2\beta} - \beta\xi\right)^2 + Kd\vec{x}^2 + \frac{Kdr^2}{f}\right)$$

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- An Effective action ($8\pi G = 1$):

$$S = \frac{1}{2}\int d^5x\sqrt{-g}\left(R - \frac{4}{3}(\partial\Phi)^2 - \frac{1}{4}F^2 - 4A^2 - V(\Phi)\right)$$

where $V(\Phi) = 4e^{2\Phi/3}(e^{2\Phi} - 4)$.

Black Hole Thermodynamics

- Temperature & Chemical Potential:

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- Free energy ($S_{onshell} T$):

$$S_{onshell} = \frac{L_{x_1} L_{x_2} L_y}{8 T \pi r_H^3 G_5} \implies F = L_{x_1} L_{x_2} L_\xi \frac{\pi^2 N^2 T^4}{32\mu^2}$$

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- The on-shell action is regularized by adding ’counter-terms’, which makes the variational principle well defined.

Boundary stress tensor

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- Density:

$$N = \int \sqrt{\gamma} T^{\xi}_t = \frac{\beta^2}{16\pi G r_H^4} = \frac{\pi^2 N^2 T^4}{32\mu^3} L_{x_1} L_{x_2} L_{\xi}$$

- Pressure:

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$$E + P = TS + \mu N$$

- Consistent with conformal Ward identities

$$E = P = F$$

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- Stress tensor is an operator with particle number zero.

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- T_{12} couples to h_{12} in the bulk and it solves the bulk wave equation.
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- Hence, η/s is same as that of $\mathcal{N}=4$ theory and it saturates the viscosity bound.

Conclusions and Outlook

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Thank You