Microstate dependence of scattering from the D1/D5 system

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Prelude

- Brane systems are described by solutions of supergravity.
- Geometries of black branes or black holes are often associated with some sort of no-hair theorems.
- On the other hand, scattering from the brane system carries information about the quantum state of the system.
- There are geometries (Lunin-Mathur, Maldacena-Maoz, LLM,..) which depend on the “microstate”.
- How do geometries with horizon emerge? How does one define an average geometry and/or a long distance geometry?
- In this talk, we will address some of these in the context of the D1-D5 system.

Outline

- Introduction: D1-D5 system
- Absorption cross-section at finite R:
  microstate dependence
  emergence of the naive D1-D5 geometry for large $R\Delta E$
- Finite R correction to σ_{abs}:
  anayltic bound
  typical states
  comparison with higher derivative corrections
  atypical states
- Summary
- Conclusion

Basic References

- S. Das and G. Mandal, arXiv:0812.1358[hep-th].
- V. Balasubramanian, P. Kraus and M. Shigemori, “Massless black holes and black rings as effective geometries of the D1-D5 system,” arXiv:hep-th/0508110.
- O. Lunin and S. D. Mathur, “The slowly rotating near extremal D1-D5 system as a ’hot tube’,” [arXiv:hep-th/0107113]
- A. Dhar, G. Mandal and S. R. Wadia, “Absorption vs decay of black holes in string theory and T-symmetry,” [arXiv:hep-th/9605234].
- S. R. Das and S. D. Mathur, “Comparing decay rates for black holes and D-branes,” [arXiv:hep-th/9606185];

Related references

- G. Mandal and S. R. Wadia, “Black Hole Geometry around an Elementary BPS String State,” [arXiv:hep-th/9511218].
- N. V. Suryanarayana, “Half-BPS giants, free fermions and microstates of superstars,” [arXiv:hep-th/0411145].
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- V. Balasubramanian, J. de Boer, V. Jejjala and J. Simon, “The library of Babel: On the origin of gravitational thermodynamics,” [arXiv:hep-th/0508023].
- A. Sen, “Black holes and elementary string states in N = 2 supersymmetric string theories,” [arXiv:hep-th/9712150].

D1-D5 system

	t	z^1	z^2	z^3	z^4	y	x^1	x^2	x^3	x^4
$D5$	$-$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$-$	$-$	$-$	$-$	$-$
$D1$	$-$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$-$	$\cdot$	$-$	$-$	$-$

- $z^a \rightarrow$ non-compact. $y \rightarrow$ circle of radius R. $x^i \rightarrow T^4$ of volume V.
- The microscopic description of the system at low energies is a $(4,4)$ superconformal field theory with world-volume $\sigma^\alpha = (t, y)$ and a target space which is a resolution of the orbifold $(T^4)^N/S(N)$, $N = Q_1 Q_5$.

$$S_{\text{orbifold}} = \frac{1}{2} \int d^2\sigma \, \left(\partial_\alpha X^i_A \partial_\alpha X^i_A + \text{fermions} \right)$$

where $i = 1, ..., 4$, $A = 1, ..., N$ (N copies).

- For periodic y, the CFT is in the Ramond sector.

D1-D5 system

Twist sectors and long strings

- An element of $S(N)$ can be described by N_n copies of the cyclic permutation Z_n, $n = 1, \ldots N$ (up to equivalence).
- Z_n twist acts on $X_A(\sigma)$ as $X_A(\sigma + 2\pi R) = X_{A+1}(\sigma)$, $A = 1, \ldots, n$.
  $\Rightarrow$ 'Long string of length $2\pi nR$':
  $\{X_A(\sigma)\} \to X(\sigma) \equiv X(\sigma + 2\pi R)$.
- A twist sector = N_n long strings, each of length n.

$$\sum_n n\, N_n = N = Q_1 Q_5$$

- 2-charge D1-D5 system = Ramond ground states={lowest energy state of each twist sector}. Number of these = Ω, where

$$S_{\text{stat}} = \ln \Omega \approx 2\pi\sqrt{2N}$$

D1-D5 system

Supergravity: standard description

In two derivative supergravity, standard description of the D1-D5 system is in terms of the *naive metric*

$$ds^2 = 1/\sqrt{f_1(r)f_5(r)}\left[-dt^2+dy^2\right] + \sqrt{\frac{f_1(r)}{f_5(r)}}\left[dx_1^2+\cdots+dx_4^2\right]+\sqrt{f_1(r)f_5(r)}\left[dr^2+r^2d\Omega_3^2\right]$$

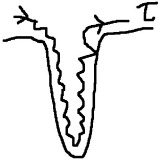
$$f_1(r)=1+\frac{16\pi^4 g_s l_s^6 RQ_1/V}{r^2} \qquad f_5(r)=1+\frac{g_s l_s^2 Q_5}{r^2}$$

Near horizon limit= $AdS_3\times S^3$, with $\ell^2=(\kappa^2N)/(4\pi^3V)$.

- Horizon at $r=0$: $S_{\rm Bek}=0$.
- In similar theories (D1-D5 on $K3$) higher derivative corrections produce a finite horizon and $S_{\rm Wald}=S_{\rm stat}$. We expect here too: area of horizon $=A_h\sim\kappa_5^2 S_{\rm stat}\sim\kappa_5^2\sqrt{N}$.

Fuzzball geometries

- Mathur and collaborators have found smooth horizon-free solutions ("fuzzball") of leading order supergravity corresponding to CFT states of the 2-charge system.
- These geometries agree with the naive geometry at large r, but disagree for $r \lesssim r_0$ where $r_0^3 \sim A_h \sim \kappa_5^2 \sqrt{N}$. At small r the geometries are "capped":



- A fuzzball solution can sometimes mimic the effect of a naive geometry (can show absorption) for times less than a certain time scale τ. Lunin-Mathur argument

Other related works

- Balasubramanian et al showed that for *typical microstates* AND *short time scales* $t \ll O(R\sqrt{N})$, CFT correlation functions are independent of the details of the microstate and agree with the naive supergravity answers at short time scales.
- In typical microstates $\{N_n\}$ are defined to be close to the thermal values

$$N_n = 8/\sinh(\beta n)$$

- This would suggest a role of averaging in the emergence of geometries with horizons.
- These issues have also been studied in the context of LLM and similar geometries.

Absorption cross-section

- We will consider scattering a low energy supergravity mode χ off the D1-D5 system and compute using the orbifold CFT the absorption cross-section. We will study σ_{abs} as a tool to figure out about the geometry.
- We will take χ to be minimally coupled at the leading order supergravity, e.g. $\chi = h_{12}$. The CFT is considered coupled to the supergravity field as an external source.
- A long string in the CFT with length $2\pi nR$ is coupled to a monochromatic wave $\chi(t, y) = e^{-iEt}$ by the term

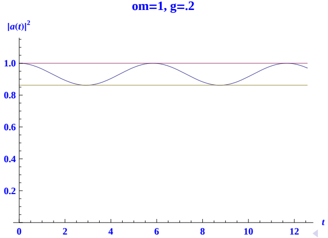
$$S_{int} = \sqrt{2}\kappa \int_{-T}^{T} dt \int_{0}^{2\pi nR} d\sigma \; e^{-iEt} \; \partial_\alpha X^1 \, \partial^\alpha X^2$$

The absorption probability is given roughly by

$$\mathcal{P} \sim \sum_m p_0^2 \frac{\sin^2[(p_0 - E)T]}{(p_0 - E)^2}, \quad p_0 = m/(nR), m = 0, 1, ..$$

Finite R, ΔE

- This computation is similar to the calculation of $\sigma_{\rm abs}$ for the 3-charge D1-D5-P system by Dhar et al and Das et al in 1996, which reproduced the supergravity answer. However, those calculations were obtained in the large R limit.
- We will compute $\sigma_{\rm abs}$ at finite R and deduce the microstate-dependence from *finite size effects*.
- Subtlely: if the spectrum is discrete, σ_{abs} does not exist, unless we consider a wave packet of the incoming wave χ, of a finite width ΔE.



Scales

- Suppose we are in a long string sector of effective length $2\pi nR$.
- Now, if $\Delta E \gg 1/R$, we lose all information about discreteness. On the other hand, if $\Delta E \ll 1/(nR)$, then we are back to an effectively monochromatic wave and hence there is no absorption.
- Furthermore, validity of Fermi's Golden rule demands that $\Delta E \gg 1/T$. This gives a lower bound on T.
- Also, the basic process of absorption at the first order is the splitting of the closed string mode χ into two open strings; in order to avoid recombination of these modes, we must have $T \ll \pi nR$.

Scales

- Combining all, in a specific long string sector

$$R \ll \frac{1}{\Delta E} \ll T < \pi n R$$

- Note that the upper bound on T is the same as the one found by Lunin and Mathur.
- In a typical state, with $\langle n \rangle \sim \sqrt{N}$, we have

$$R \ll \frac{1}{\Delta E} \ll T < \pi R \sqrt{N}$$

  The upper bound on T here is the same as the one found by Balasubramanian et al.
- We will now compute the absorption crosssection $\sigma_{\rm abs}$ using finite $R, \Delta E$ keeping the above bounds in mind.

Finite R absorption crosssection

With an incoherent wave-packet and summing over various long strings, the absorption probability is now given by

$$\mathcal{P} \sim \sum_n N_n \int dE \sum_m \left(\frac{m}{nR}\right)^2 \frac{\sin^2(p_0 - E)T}{(p_0 - E)^2} \rho_{E_0,\Delta E}(E)$$

Take a Lorentzian wave packet

$$\rho_{E_0,\Delta E}(E) = \mathcal{N} \frac{E}{[(E - E_0)^2 + (\Delta E)^2]^2}$$

This gives

$$\sigma(E_0, \Delta E) = 2\kappa_5^2 \mathcal{N} \sum_{n=1}^{N} N_n \sum_{m=1}^{\infty} (\frac{m}{nR})^2 \frac{1}{[(\frac{2m}{nR} - E_0)^2 + (\Delta E)^2]^2}$$

Large R limit

$R\Delta E \gg 1$

- In this limit the sum over m becomes an integral: replace $2m/(nR) \to x, \sum_m \to (n\,R/2) \int dx$

$$\sigma = F(E_0, \Delta E) \times \left(\sum_{n=1}^{N} n\, N_n \right) = F(E_0, \Delta E)\, N$$

$F \sim E_0, E_0 \gg \Delta E, F \sim \Delta E, E_0 \ll \Delta E.$

- Classical σ_{abs}, calculated from the naive metric with singular horizon and for a monochromatic wave (frequency E), is $\sigma_{\text{abs}}(E) = \pi^3 \ell^4 E$. For a wave-packet

$$\sigma_{\text{abs}}(E) = \pi^3 \ell^4 \mathcal{N} \int dE\; E\; \rho_{E_0, \Delta E}(E) \equiv N\; F(E_0, \Delta E)$$

Hence the naive geometry (zero horizon) emerges at low resolution, defined by $R\Delta E \gg 1$. For any microstate!

Emergence of black hole geometry

- As long as $R\Delta E \gg 1$, even atypical microstates, e.g. the untwisted state, behave as black holes.
- Will see that for $R\Delta E \ll 1$, σ_{abs} strongly depends on the microstate. *Typical microstates* still behave as black holes, as long as $R\Delta E \gg 1/\sqrt{N}$.

CFT	Bulk	Observation
$R\Delta E \gg 1$ (coarse-grain)	$r \gg r_{\text{stretch}}$	BH (zero area)
$R\Delta E \ll 1$ (fine-grain)	$r \ll r_{\text{stretch}}$	LM
Typical states		
$R\Delta E \gg 1/\sqrt{N}$ (medium)	$r \gg r_{\text{stretch}}/\sqrt{N}$	BH (finite horizon)
$T \ll T_{\text{recombn.}} \sim \sqrt{N}R$	$T \ll T_{\text{trapping}} \sim nR$	Absorption and emergent T-asymmetry

Finite R correction: sum minus integral

McLaurin integral approximation: if a function $f(x)$ is positive and monotonically decreasing in $P \leq x \leq Q$, the following is true

$$\int_P^Q dx\, f(x) + f(P) > \sum_{n=P}^{Q} f(i) > \int_P^Q dx\, f(x) + f(Q),$$

We can rewrite the above in the form of an estimate for the sum:

$$\sum_{n=P}^{Q} f(i) = \int_P^Q dx\, f(x) + f(Q) + \eta_1 \left(f(P) - f(Q)\right),\ 0 < \eta_1 < 1$$

Similarly, for a positive monotonically increasing function $f(x)$ in $P' \leq x \leq Q'$, we get

$$\sum_{n=P'}^{Q'} f(i) = \int_{P'}^{Q'} dx\, f(x) + f(P') + \eta_2 \left(f(Q') - f(P')\right),\ 0 < \eta_2 < 1$$

Analytic upper bound for $\sigma - \sigma_{\text{classical}}$

By using this analysis we find an upper bound of the difference between the sum over m and its integral approximation (recall the latter gives the classical value)

$$|\Delta\sigma(E_0, \Delta E)|_{\max} = \frac{\kappa^2}{2VR}\tilde{G}(E_0/\Delta E)\sum_{n=1}^{N} N_n$$

Microstate dependence is in the sum $\sum_n N_n$.
For a Lorentzian wave-packet,

$$\tilde{G}(x) = \frac{x^2+1}{(1+(\sqrt{x^2+1}-x)^2)^2}\left[\frac{1}{2}+\frac{x}{2}\left(\frac{\pi}{2}+\tan^{-1}x\right)\right]^{-1}$$

Typical state: non-zero E_0

Analytic

A typical microstate is defined by $\{N_n\}$ which closely approximate a canonical ensemble, given by

$$\langle N_n \rangle = 8/\sinh(\beta n), \ \beta \approx \pi\sqrt{2/N}$$

e.g. $N_n = [\langle N_n \rangle]$ (nearest integer).

$$\sum_n N_n \approx (8/\beta)\log\beta \sim \sqrt{N}\log N$$

Hence

$$|\Delta\sigma(E_0, \Delta E)|_{\max} \sim \kappa_5^2 \tilde{G}(E_0/\Delta E)\sqrt{N}\log N$$

Typical state: non-zero E_0

Analytic

$E_0 \ll \Delta E$:

$$\sigma_{classical}(E_0, \Delta E) = N\, R \Delta E\, \frac{\kappa_5^2}{4} \left[\frac{\pi}{2} + (2 - \frac{\pi^2}{4}) \frac{E_0}{\Delta E} + O(\frac{E_0}{\Delta E})^2 \right]$$

$$\Delta\sigma_{\max}(E_0, \Delta E) = \sqrt{N}\, \frac{\kappa_5^2}{2} \frac{2\sqrt{2}}{\pi} \left[\frac{1}{2} + (1 - \frac{\pi}{4}) \frac{E_0}{\Delta E} + O(\frac{E_0}{\Delta E})^2 \right] \times \left[\frac{1}{2} \log(N) - \log \frac{\pi}{\sqrt{2}} + \eta \right]$$

$E_0 \gg \Delta E$:

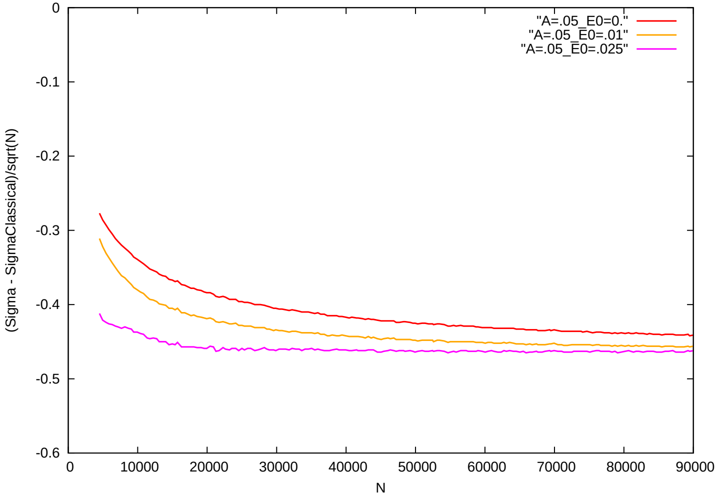
$$\sigma_{classical}(E_0, \Delta E) = N\, R E_0 \frac{\kappa_5^2}{4} \left[1 + O(\frac{\Delta E}{E_0})^2 \right]$$

$$\Delta\sigma_{\max}(E_0, \Delta E) = \sqrt{N}\, \frac{\kappa_5^2}{2} \frac{2\sqrt{2}}{\pi} \frac{E_0}{\Delta E} \left[\frac{2}{\pi} + O(\frac{\Delta E}{E_0})^2 \right] \left[\frac{1}{2} \log(N) - \log \frac{\pi}{\sqrt{2}} + \eta \right]$$

Note the perturbation parameter $1/(R\, \Delta E\, \sqrt{N})$.

Typical state: non-zero E_0

Numerical



$\Delta\sigma \propto \sqrt{N}$ for large N, and is negative for small E_0.

Typical state: $E_0 = 0$

Analytic

Here the sum over m can be done:

$$\sum_{m=1}^{\infty} \frac{m^2}{(m^2+a^2)^2} = \frac{\pi}{4a}\left[1 + H(2\pi a)\right]$$

where

$$H(x) \equiv \frac{2}{(e^x-1)^2}\{(1-x)\,e^x - 1\} = \frac{d}{dx}\frac{2x}{e^x-1}$$

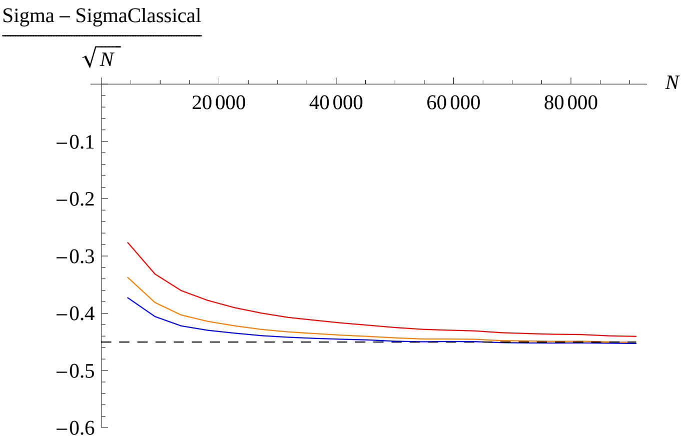
For $R\Delta E \gg 1$, we again recover the classical calculation from the naive geometry. For $R\Delta E \ll 1$, we get

$$\Delta\sigma_L(E_0=0,\Delta E) \approx -\sqrt{N}\,\kappa_5^2\,\frac{\sqrt{2}}{\pi}\left[1 - \frac{\eta'\pi R(\Delta E)}{2} + O(R\Delta E)^2\right]$$

Note the $-$ sign.

Typical state: $E_0 = 0$

Numerical



$\Delta\sigma/\sqrt{N}$ is plotted in units of κ_5^2 for $E_0 = 0$ and for $A = \pi R \Delta E =$ $.05, .075, .1, .5$. The dotted line at $-\sqrt{2}/\pi \approx -.45$ is the analytic estimate.

Typical states: Summary

- Microstate-dependent absorption crosssection appears when $1 \gg R\Delta E \gg 1/\sqrt{N}$, that is, when the energy resolution lies between the largest energy gap and the average energy gap.
- For $E_0 \ll \Delta E$

$$|\Delta\sigma|_{\max} \propto A_h \log(N), \; A_h \sim \kappa_5^2 \sqrt{N}$$

where A_h is the area of the stretched horizon. The propoortionality is up to a pure number.

- Compared to the (microstate independent) classical answer, the correction is suppressed by a power of $1/(R\Delta E\sqrt{N})$.

Typical states: Summary

- For $E_0 \ll \Delta E$, the sign of the correction is negative

$$\Delta\sigma \sim -A_h$$

  This can be seen numerically for $E_0 \neq 0$ and analytically for $E_0 = 0$. Of course, the total cross-section is explicitly positive since it is a sum of positive terms. In our approximation there is never a domain where the correction is bigger than the classical term.

- For generic $E_0 R \ll 1$, numerical calculations show $\Delta\sigma < 0$.
- For $E_0 \lesssim 1/R$, $\Delta\sigma > 0$. Plot

Atypical States

Untwisted sector

- Consider an atypical twist sector with p equal long strings, each with winding number q: $N_n = p\,\delta_{n\,q}$, $N = p\,q$.

$$\Delta\sigma_{\rm max} = \frac{1}{2}\kappa_5^2\,\frac{N}{q}\,\tilde{G}(E_0/\Delta E)$$

- Untwisted sector ($p = N, q = 1$): we find

$$\Delta\sigma = \kappa_5^2\,N\,f(E_0,\Delta E) \sim \sigma_{\rm classical}/(R\Delta E)$$

The correction to the crosssection is *as large as the classical answer* in N-counting, but is suppressed at large $R\Delta E$. This is consistent with the fact that these states are very discrete *even at large* N. At large R, however, the energy spectrum becomes continuous and approches the classical answer.

Atypical states

Maximally twisted sector

- Maximally twisted sector ($p = 1, q = N$):

$$\Delta\sigma_{\rm max} = \frac{1}{2}\kappa_5^2\,\tilde{G}(E_0/\Delta E) \sim \sigma_{\rm classical}/(N\,R\Delta E)$$

  $\Delta\sigma_{\rm max}$ is independent of N, but numerically ▸ Plot $\Delta\sigma$ is found to vanish exponentially in N. For $E_0 = 0$ one can show explicitly that $E_0 \sim N\exp[-N]$.

- Lesson: although even this state is generic, in the range $\Delta E \gg 1/(R\sqrt{N})$, the discreteness is not perceived by the incoming wave and hence the state appears as classical.

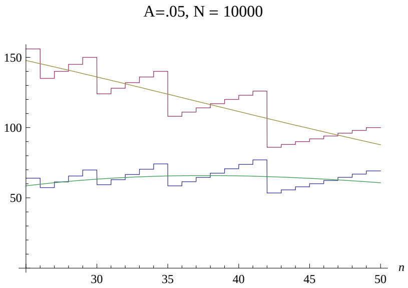
Robustness

- The above analytic and numerical results have used a Lorentzian profile for the incoming wave.
- We have generalized the analytic results to a Gaussian profile, with similar results. In particular, for typical states the bound is again proportional to $\sqrt{N}$. The proportionality constant becomes a pure number for $E_0 \ll \Delta E$ and linear in $\frac{E_0}{\Delta E}$ for $E_0 \gg \Delta E$. It is likely that there is a large class of energy profiles for which similar results will hold.

Averaging and horizon

- We recovered the naive geometry with zero horizon with coarse graining but no averaging. When we use relatively finer resolution and averaging, we get a result related to a finite horizon area A_h.
- It is known that in case of the 3-charge system, the absorption crosssection sees the horizon area *only when* we average over the microstates.
- In case of elastic scattering off a massive heterotic string state, the scattering amplitude reproduces properties of the putative black hole state only after averaging over a microcanonical ensemble of states of.

Two continuum's



We have plotted on the y-axis $\tilde{\sigma}(n)$ for $E_0 = 0$, in units of $\pi\ R/(32\ \Delta E)$. The upper histogram and curve refer to $\tilde{\sigma}_{\text{classical}}(n)$ and its continuum approximation, respectively. The lower histogram refer to the full expression, $n\ N_n\ (1 + H(A\ n))$, while the lower curve plots the function $[8x/\sinh(\beta\ x)]\ (1 + H(A\ x))$. The upper curve corresponds to the naive geometry which arises due to low resolution (large $A \equiv R\ \Delta E$) while the lower curve refers to a different continuum which arises due to averaging over microstates.

Conclusion and Open problems

We have computed finite R absorption crosssection for the 2-charge D1-D5 system on T^4 and studied the conditions for microstate (in)dependence. Under suitable conditions, we have obtained finite R corrections related to the area A_h of the stretched horizon.

Open problems:

- Find precise bulk interpretation of the A_h terms we have obtained.
- Compare our results with scattering in LM geometries.
- Generalize to D1-D5 on K3. In the latter case, classical σ_{abs} has been calculated in the derivative corrected geometry.
- Move away from the orbifold limit. In case of the 3-charge thermal state, there are non-renormalization theorems for our supergravity mode.
- Generalize to 3-charge geometries. Compare with 3-charge LM type states.

Absorption by a fuzzball solution

The geometry is a rotating D1-D5 system with angular momentum J with a 6 dimensional metric

$$\begin{aligned} ds^2 &= -\frac{1}{h(r,\theta)}(dt^2 - dy^2) + h(r,\theta)f(r,\theta)\left(d\theta^2 + \frac{dr^2}{r^2+a^2}\right) \\ &- \frac{2ar_1r_5}{h(r,\theta)f(r,\theta)}\left(\cos^2\theta\, dy\, d\psi + \sin^2\theta\, dt\, d\phi\right) \\ &+ h(r,\theta)\left[\left(r^2 + \frac{a^2r_1^2r_5^2\cos^2\theta}{(h(r,\theta)f(r,\theta))^2}\right)\cos^2\theta\, d\psi^2 + \right. \\ &+ \left.\left(r^2 + a^2 - \frac{a^2r_1^2r_5^2\sin^2\theta}{(h(r,\theta)f(r,\theta))^2}\right)\sin^2\theta\, d\phi^2\right] \end{aligned}$$

where

$$f(r,\theta) = r^2 + a^2\cos^2\theta \qquad h(r,\theta) = \left[(1+\frac{r_1^2}{f(r,\theta)})(1+\frac{r_5^2}{f(r,\theta)})\right]$$

The radius of the y direction is R and the angular momentum J is given by

$$J = \frac{1}{2}Q_1Q_5\,\frac{R}{r_1r_5}\,a$$

For $a = 0$ we get back the naive geometry with an infinite throat. For nonzero a the throat is replaced by a cap.

Absorption by a fuzzball solution

Solve the wave equation of a massless minimally coupled scalar in this geometry. Since there is a cap, the reflection coefficient $\mathcal{R}$ at infinity satisfies $|\mathcal{R}| = 1$.

However, $\mathcal{R}$ may be written as an infinite series of terms which may be interpreted as arising from the wave that enters the throat and repeatedly undergoes the process of reflection by the cap and part reflection and part outward transmission at the throat.

For the s-wave and for $w\ell^2 \ll R, w^2(r_1^2 + r_5^2) \ll 1$, this expansion is

$$R \sim e^{-i\pi\epsilon} - 2\pi^2 \frac{(w\ell)^4}{16} - 4\pi^2 \frac{(w\ell)^4}{16} \sum_{m=1}^{\infty} e^{2\pi i m \frac{wRN}{4J}}$$

Absorption by a fuzzball solution

The above expression is an infinite series of terms representing waves with successive time delays of

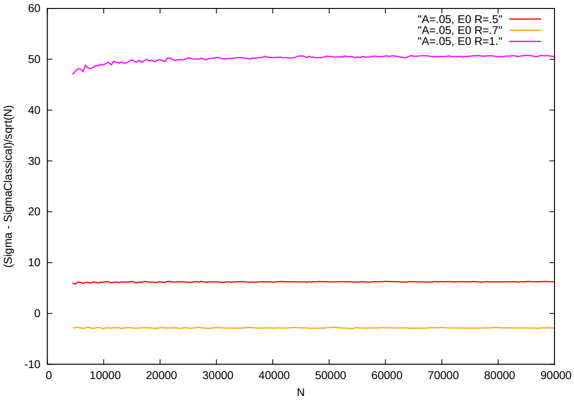
$$t_{delay} = 2\pi \frac{\partial}{\partial w}\left(\frac{wRN}{4J}\right) = \frac{\pi RN}{2J}$$

This expression was interpreted by Lunin and Mathur as follows. The *m*-th term is the contribution for a wave which went into the throat, and re-emerged after going back and forth between the cap and the mouth of the throat *m* times. The probability for entering the throat can be then read off

$$\mathcal{P}_{throat} = 4\pi^2 \frac{(w\ell)^4}{16}$$

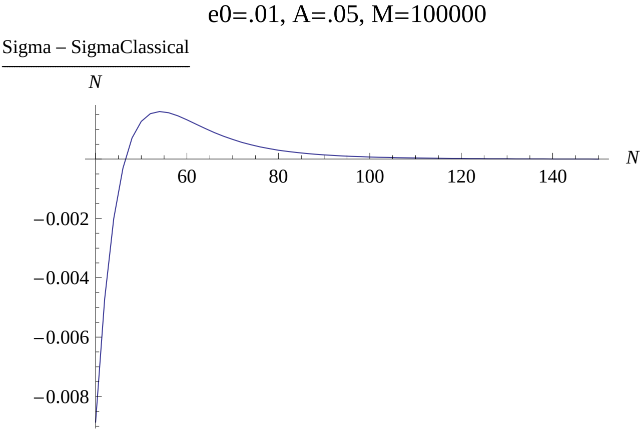
This is in precisely the same as the probability for *absorption* in the naive geometry. Back

Plot for $E_0R \geq 0.5$



We have plotted $\Delta\sigma_L(E_0, \Delta E)$ (in units of κ_5^2) on the y-axis. $A = \pi\,\Delta ER$. All plots have $E_0R \geq .5$. Back

Plot for maximally twisted state



We show $(\sigma - \sigma_{L,\text{classical}})/N$ vs N in units κ_5^2. M denotes the upper limit in the sum over m. $A = \pi R \Delta E, e_0 = RE_0$. The McLaurin upper bound in this case is 0.0278431. Back